Exam 1
Physics 132

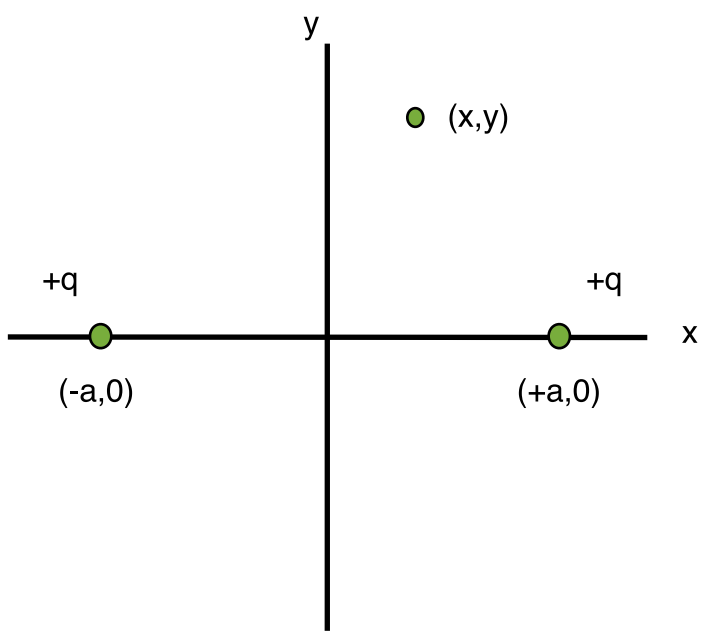
Short Answer Section. Please answer all of the questions.

1. 1. What magnitude force do the "up" quarks and "down" quarks in a proton exert on each other? Assume that the "up" quark has charge $q=+\frac{2}{3}e$ and the "down" quark has a charge $q=-\frac{1}{3}e$. The separation of the quarks is $0.5\times10^{-15}m$. Is the force attractive or repulsive?

$$F=\frac{q_1q_2}{4\pi\varepsilon_0 r^2}=\frac{\frac{2}{3}e\cdot\frac{1}{3}e}{4\pi\varepsilon_0 r^2}=205.1N$$

The force is attractive.

2. What is the electric field at the location (x, y) shown below? Hint: You will need to write out $\vec{r},r,\hat{r}$ for each charge.



$$E=\frac{q}{4\pi\varepsilon_0\left((x+a)^2+y^2\right)}\cdot\frac{(x+a)\hat{i}+y\hat{j}}{\sqrt{(x+a)^2+y^2}}+\frac{q}{4\pi\varepsilon_0\left((x-a)^2+y^2\right)}\cdot\frac{(x-a)\hat{i}+y\hat{j}}{\sqrt{(x-a)^2+y^2}}$$

3. What is the electric field inside a conductor and why?

The static field is zero. Charges will move until the field is zero.

4. A particle experiences an acceleration of $2.0\times10^{6} m/s^2$. If the particle's charge is $q = 2.0\times10^{-6} m/s^2$ and its mass is $m = 1.0\times10^{-8} kg$, what electric field caused this acceleration?

$$ma = qE$$

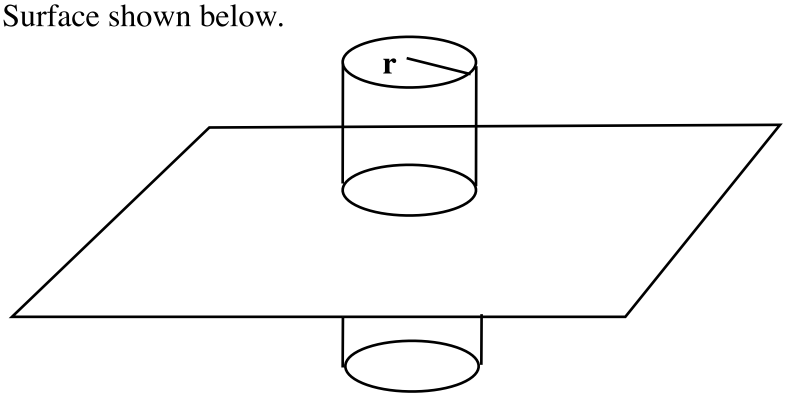
$$E = \frac{ma}{q} = \frac{1.0\times10^{-8} kg \cdot 2.0\times10^{6} m/s^2}{2.0\times10^{-6} C} = 10{,}000 N/C$$

5. A positive point charge charge +10e is placed at the center of a cube. What is the electric flux through the right face of the cube?

$$\varphi = \frac{1}{6}\cdot\frac{10e}{\varepsilon_0}$$

6. Using Gauss' law, show that the field due to an infinite plane of charge is $E = \dfrac{\sigma}{2\varepsilon_0}$ where σ.

Use the Gaussian Surface shown below.



$$\oint \vec{E}\cdot d\vec{A} = \int_{curved} \vec{E}\cdot d\vec{A} + \int_{caps} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

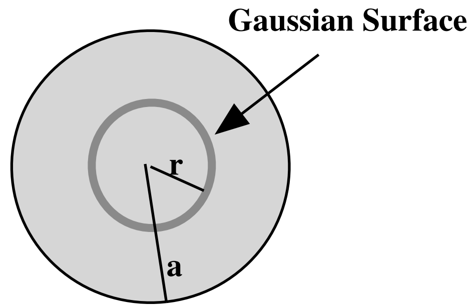
$$= 0 + \int_{caps} E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$= 2EA = \frac{q_{enc}}{\varepsilon_0}$$

$$= 2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

7. A uniformly charged sphere has a charge density ρ and radius a. How much charge is enclosed by a Gaussian surface of radius r where r< a?



$$q_{enc} = \rho \cdot \frac{4}{3}\pi r^2$$

8. A dipole is composed of a charge +e and -e separated by a distance $d = 0.5 \times 10^{-10} m$. What is the electric dipole moment p of this dipole? If it is oriented at an angle $\theta = 60°$ with respect to a uniform electric field E, what is the energy U of the dipole?

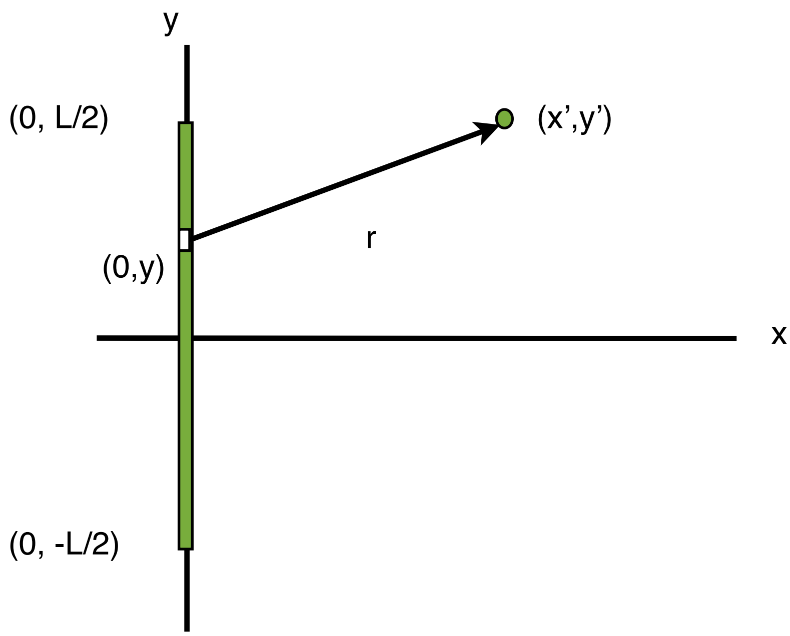
$$p = qd = e \cdot 0.5 \times 10^{-10} = 8.01 \times 10^{-30} Cm$$
$$U = -pE\cos 60$$

9. In a beam, 100×10^9 particles by per second. If each particle has a charge of $10 \times 10^{-12} C$ what is the current of the beam?

$$i = \frac{100 \times 10^9 \, particles}{s} \cdot \frac{10 \times 10^{-12} C}{particle} = 1A$$

Problems: Please work 2 of the 3 problems.

1. Consider a line of charge along the y axis as shown below and the field at a point (x', y')



a) Write dq for a little length of charge shown.

$$dq = \lambda dy$$

b) Write the $r, \vec{r}, \hat{r}$ for the charge dq.

$$\vec{r} = (x' - 0)\hat{i} + (y' - y)\hat{j}$$
$$r = \sqrt{(x'-0)^2 + (y'-y)^2}$$
$$\hat{r} = \frac{(x'-0)\hat{i} + (y'-y)\hat{j}}{\sqrt{(x'-0)^2 + (y'-y)^2}}$$
$$= \frac{(x')\hat{i} + (y'-y)\hat{j}}{\sqrt{x'^2 + (y'-y)^2}}$$

c) **<u>Set up</u>** the integral to calculate both the x and y components of the Electric Field but **<u>do not do the the integrals.</u>**

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$= \frac{\lambda dy}{4\pi\varepsilon_0\left((x'-0)^2+(y'-y)^2\right)} \cdot \frac{(x'-0)\hat{i}+(y'-y)\hat{j}}{\sqrt{(x'-0)^2+(y'-y)^2}}$$

$$\vec{E} = \int_{-L/2}^{L/2} \frac{\lambda dy}{4\pi\varepsilon_0\left((x'-0)^2+(y'-y)^2\right)} \cdot \frac{(x'-0)\hat{i}+(y'-y)\hat{j}}{\sqrt{(x'-0)^2+(y'-y)^2}}$$

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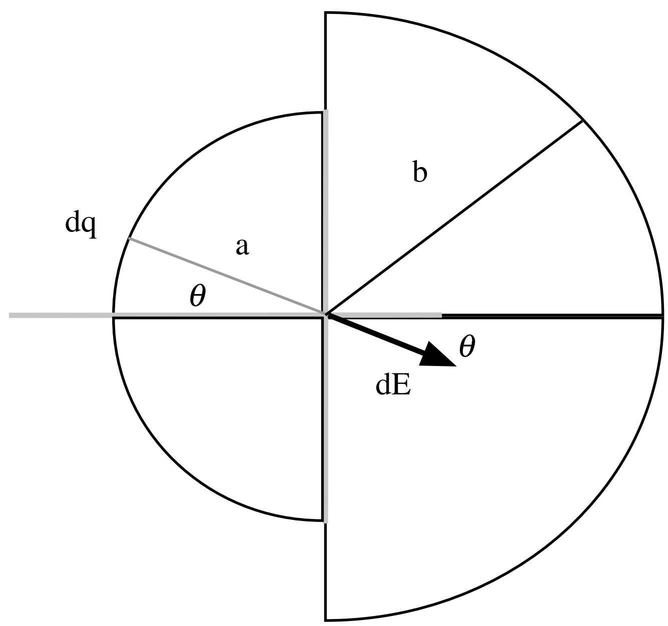
2. Consider the two have circles of charge below or charge below. One half circle has radius a and charge per unit length λ_a and the second has radius b charge per unit length λ_b. A little charge dq produces a dE as shown from the left half circle. You may assume that both charge densities are positive.



Consider just the left half circle with radius a first.

a) What direction will the field point after integrating?

To the right along the x axis.

b) Write dq for a little arc length of charge? Write the arc length in terms of the radius a and an angle $d\theta$?

$$dq = \lambda_a a d\theta$$

c) Write the magnitude of dE that is produced by dq?

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$

d) Write the component of dE that will survive in terms of $\cos\theta$

$$\begin{aligned} dE_x &= \frac{dq}{4\pi\varepsilon_0 r^2}\cos\theta \\ &= \frac{\lambda_a a d\theta}{4\pi\varepsilon_0 a^2}\cos\theta \\ &= \frac{\lambda_a d\theta}{4\pi\varepsilon_0 a}\cos\theta \end{aligned}$$

e) Integrate θ from $-\pi/2$ to $+\pi/2$ to find the field. Remember that

$$\int_{-\pi/2}^{\pi/2} \cos d\theta = \sin\theta\Big|_{-\pi/2}^{\pi/2} = \sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right) = 2$$

$$\begin{aligned} E_x &= \int_{-\pi/2}^{\pi/2} \frac{\lambda_a d\theta}{4\pi\varepsilon_0 a}\cos\theta \\ &= \frac{\lambda_a}{4\pi\varepsilon_0 a}\sin\theta\Big|_{-\pi/2}^{\pi/2} \\ &= \frac{\lambda_a}{2\pi\varepsilon_0 a} \end{aligned}$$

f) Now knowing the field due to the left half circle, what is the field due to both half circles. Hint: You can just write the field due to the right half circle down with appropriate substitution and then add the fields together as appropriate.

$$E_x = \frac{\lambda_a}{2\pi\varepsilon_0 a} - \frac{\lambda_b}{2\pi\varepsilon_0 b}$$

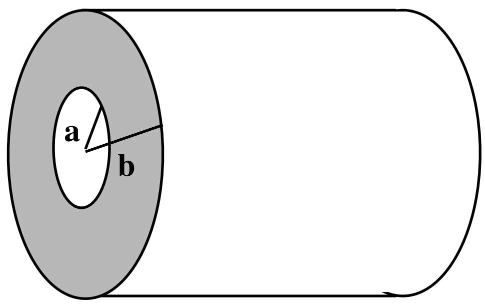
Bonus) What is the relationship between λ_a and λ_b such that the field is zero?

$$0 = \frac{\lambda_a}{2\pi\varepsilon_0 a} - \frac{\lambda_b}{2\pi\varepsilon_0 b}$$

$$0 = \frac{\lambda_a}{a} - \frac{\lambda_b}{b}$$

$$\lambda_a = \lambda_b \frac{a}{b}$$

3. Consider the **nonconducting** infinite cylindrical shell shown below . It has inner radius a and outer radius b. You may assume uniform charge density ρ .



a) What is the field for radii less than a and why?

The field is zero. No charge is enclosed

Choose a Gaussian surface for a < r < b.

b) What is the charge enclosed by this surface? (Hint: The volume of a cylinder is at the end of the exam. Don't forget to subtract the hole).

$$q_{enc} = \rho \cdot (\pi r^2 - \pi a^2) L$$

c) Use Gauss' Law to find the electric field for **a < r < b**.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curved} \vec{E} \cdot d\vec{A} + \int_{ends} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curved} \vec{E} \cdot d\vec{A} + 0 = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curved} E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \int_{curved} dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r L = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r\ L = \frac{\rho \cdot (\pi r^2 - \pi a^2) L}{\varepsilon_0}$$

$$E = \frac{\rho \cdot (r^2 - a^2) L}{2\pi \varepsilon_0 r}$$

Choose a Gaussian surface for r > b.

d) What is the charge enclosed in this surface? (Hint: The volume of a cylinder is at the end of the exam. Don't forget to subtract the hole).

$$q_{enc} = \rho \cdot (\pi\, b^2 - \pi\, a^2) L$$

e) Use Gauss' Law to find the field for **r > b.**.

$$E \cdot 2\pi r L = \frac{\rho \cdot (\pi\, b^2 - \pi\, a^2) L}{\varepsilon_0}$$

$$E = \frac{\rho \cdot (b^2 - a^2)}{2\ \varepsilon_0\, r}$$

Some useful formulae

Charge on the proton: $+1.6\times10^{-19}C$

Charge on the electron $-1.6\times10^{-19}C$

$$\varepsilon_0 = 8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N m}^2}$$

Surface area of a sphere: $A = 4\pi r^2$

Surface area of cylinder: $A = 2\pi a L + 2\pi a^2$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Volume of a cylinder: $V = \pi a^2 L$