Sample Exam 3

1. A spring stores 50 J when it is compressed by 10 cm. What is the spring constant k?

$$U = \frac{1}{2}kx^2$$
$$k = \frac{2U}{x^2} = \frac{2 \cdot 50J}{(0.1m)^2} = 10{,}000 N/m$$

2. A conservative force has the form F = b x^3 **i**. What potential energy is associated with this force?

$$\begin{aligned} U &= -\int \vec{F} \cdot d\vec{r} \\ &= -\int \vec{F} \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}) \\ &= -\int bx^3\,\hat{i} \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}) \\ &= -\int bx^3\,dx \\ &= -\frac{b}{4}x^4 \end{aligned}$$

3. Is it possible to write a potential energy for the frictional force? Why or why not?

No. Frictional forces are not conservative.

4. A ball is dropped from of a height of 10 m. What is its kinetic energy just before it hits the ground? What is its velocity?

$$\begin{aligned} v_i &= 0 \\ \Delta K &= K_f - K_i = W \\ K_f - 0 &= mgh \\ K_f &= mgh \\ \Delta K &= \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2} = W \\ \frac{1}{2}mv_f^{\,2} - 0 &= mgh \\ v_f &= \sqrt{2gh} \\ &= \sqrt{2 \cdot 9.8m/s^2 \cdot 10m} \\ &= 14.0m/s \end{aligned}$$

5. A potential has the form V = b x^2. What force can be associated with this potential?

$$F = -\frac{dV}{dx} = -2bx$$

6. Identical masses are placed at (-2,0), (2, 0) and (0,2.0). Compute the position of the center of mass. Suddenly the masses exert repulsive forces on each other and fly off at high speed. What is the position of the center of mass after this event?

$$r_{cm} = \frac{m\cdot(-2\mathbf{i}+0\mathbf{j}) + m\cdot(2\mathbf{i}+0\mathbf{j}) + m\cdot(0\mathbf{i}+2\mathbf{j})}{m+m+m} = 0\mathbf{i} + \frac{2}{3}\mathbf{j}$$

The position remains unchanged since the forces are internal.

7. A 5000 kg automobile has a momentum of $\vec{p} = 10{,}000\, kg\, m/s\, \hat{i}$ What velocity does it have?

$$\vec{p} = m\vec{v}$$
$$\vec{v} = \frac{\vec{p}}{m} = \frac{10{,}000\, kg\, m/s\, \hat{i}}{5{,}000\, kg} = 2\, m/s\ \hat{i}$$

8. A 2000 kg car is heading westward with a velocity of 20 m/s. It collides with a 4000kg car heading eastward at 10 m/s. Compute the momentum after the collision and the change in the kinetic energy.

$$\vec{p}_i = 2000kg\cdot(-20m/s\,\hat{\mathbf{i}}) + 4000kg\cdot(10m/s\,\hat{\mathbf{i}}) = 0$$
$$\vec{p}_f = \vec{p}_i = 0$$
$$E_i = \frac{1}{2}\cdot 2000kg\cdot(20m/s)^2 + \frac{1}{2}\cdot 4000kg\cdot(10m/s)^2 = 6.0\times10^5 J$$
$$E_f = 0J$$
$$\Delta E = 0J - 6.0\times10^5 J = -6.0\times10^5 J$$

9. A 10 kg ball with an initial velocity of 50 m/s is stopped in a time of 0.1 second. What average force does it experience?

$$F_{Avg} = \frac{\Delta p}{\Delta t} = \frac{0kg\, m/s - 10kg\cdot 50\, m/s}{0.1s} = -5000N$$

10. A 100 kg mass initially at rest explodes, breaking into a 75 kg piece and a 25 kg piece. If the final velocity of the 75 kg piece is $10m/s\hat{i}$, find the final velocity of the 25 kg piece? What is the change in kinetic energy? Hint: This is a conservation of momentum problem.

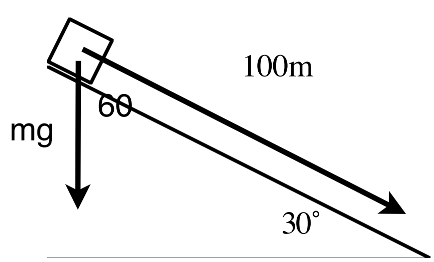
$$
\begin{aligned}
&\vec{p}_i = 0 \\
&\vec{p}_f = 75kg \cdot 10m/s + 25kg \cdot v \\
&\vec{p}_f = \vec{p}_i \\
&75kg \cdot 10m/s + 25kg \cdot v = 0 \\
&v = -30m/s
\end{aligned}
$$

11. A force of 10 N is exerted on a 5 kg mass through a horizontal distance of 20 m. What work does the force do and what is the change in the kinetic energy of the mass?

$$
\begin{aligned}
W &= \int \vec{F} \cdot d\vec{r} \\
&= \vec{F} \cdot \vec{d} = Fd\cos\theta \\
&= 10N \cdot 20m \cdot \cos 0 \\
&= 200J \\
\Delta K &= W = 200J
\end{aligned}
$$

12. A 50 kg mass slides down an incline. The incline makes an angle of 30 degrees with horizontal and the mass slides a distance of 100 m. How much work did the weight of the mass do? What is its velocity at the bottom of the incline? *(multiplication corrected!)*

The work done by gravity only depends on the change in vertical height.



$$
\begin{aligned}
W &= \vec{F} \cdot \vec{d} = Fd\cos\theta \\
&= mgd\cos 60 \\
&= 24500J \\
v_i &= 0 \\
\Delta K &= \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2} = W \\
\frac{1}{2}mv_f^{\,2} &= W \\
v_f &= \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \cdot 9800J}{50kg}} = 31.3m/s
\end{aligned}
$$

13. A tension acts inward to keep a mass of 20 kg moving in a perfect circle with constant speed of 1 m/s. How much work does the tension do?

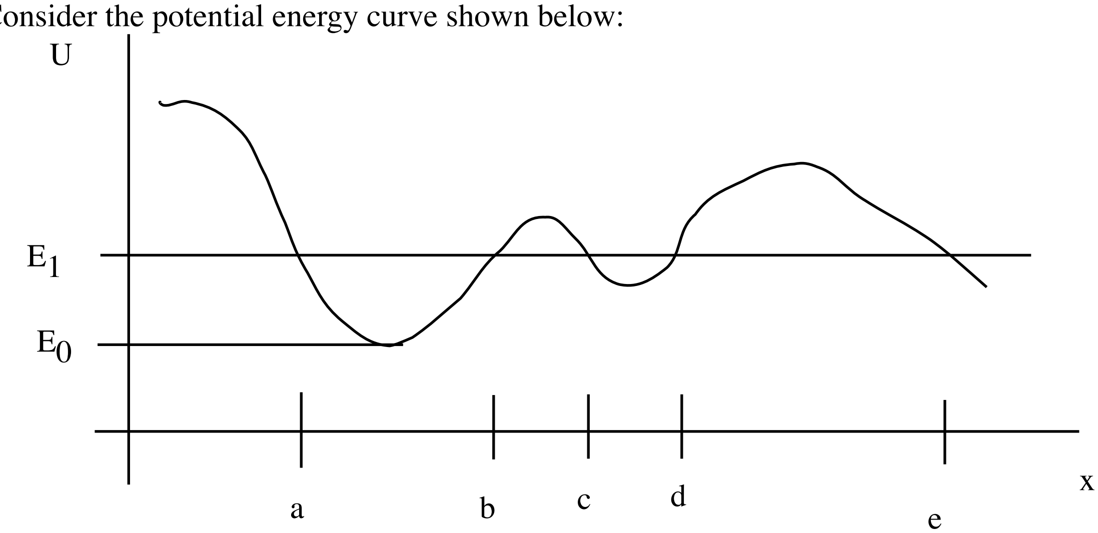
None. The force is perpendicular to the displacement.

14. A 100 kg mass moving at 10 m /s is stopped in a distance of 10 m by friction. How much work did the frictional force do on the block and what was the coefficient of friction?

$$
\begin{aligned}
W &= K_f - K_i \\
&= 0 - \frac{1}{2} m v_i^2 \\
&= -\frac{1}{2} \cdot 100kg \cdot (10m/s)^2 = -5000J \\
W &= F\,d \\
&= -\mu N\,d \\
-\mu N\,d &= -\frac{1}{2} m v_i^2 \\
\mu &= \frac{\frac{1}{2} m v_i^2}{N\,d} = \frac{\frac{1}{2} m v_i^2}{mg\,d} \\
\mu &= \frac{v_i^2}{2g\,d} = \frac{(10m/s)^2}{2 \cdot 9.8m/s \cdot 10} = 0.51
\end{aligned}
$$

Problems.

1. Consider the potential energy curve shown below:



a. Describe the motion that a mass with energy E0 can undergo.

The mass sits at the bottom of the well at x0. It has no kinetic energy--all of its energy at that point is potential.

b. Describe the motion that a mass with energy E1 can undergo. Note be sure to describe all of the motions and turning points.

If the mass begins between a and b it can oscillate between a and b2. It may also oscillate between c and d but it cannot travel from the first region (a-b) to the second region (c-d).
If the mass starts at a postion greater than e, it can approach the origin, stop at e, and turn around.

c. What is the direction of the force at points a and b? How do you know this?

The force at a is positive(right). The force at b is negative (left). We know this because we know that the force is -slope.

2. A deuterium (known as a deuteron) nucleus consists of a proton and neutron. It collides with a proton and the two stick together to form an isotope of helium.

a. The deuteron's speed is $1\times 10^{8}\, m/s$ and the proton's speed is $2\times 10^{8}\, m/s$. What is the total initial momentum. (Note: $m_d = 3.34\times 10^{-27} kg,\ m_p = 1.67\times 10^{-27} kg$) and the total initial kinetic energy.

$$\begin{aligned}\vec{p}_i &= m_d\,\vec{v}_d + m_p\,\vec{v}_p\\ &= 3.34\times 10^{-27} kg\cdot 1\times 10^{8}\, m/s - 1.67\times 10^{-27} kg\cdot 2\times 10^{8}\, m/s\\ &= 0\, kg\, m/s\end{aligned}$$

$$\begin{aligned}K_i &= \frac{1}{2}m_d v_d{}^2 + \frac{1}{2}m_p v_p{}^2\\ &= \frac{1}{2}\cdot 3.34\times 10^{-27} kg\cdot (1\times 10^{8}\, m/s)^2 + \frac{1}{2}\cdot 1.67\times 10^{-27} kg\cdot (2\times 10^{8}\, m/s)^2\\ &= 5.01\times 10^{-11} J\end{aligned}$$

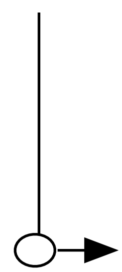
b. What is the final momentum? What is the final velocity of the helium?

$$\vec{p}_f = 0$$
$$v_f = 0$$

c. What is the change in kinetic energy?

$$K_f = 0$$
$$\Delta K = K_f - K_i$$
$$= 0 - 5.01 \times 10^{-11} J$$
$$= -5.01 \times 10^{-11} J$$

3. A mass of 10 kg is fixed on the end of a light rigid rod so that it swings as a pendulum. The length of the rod is 1 m. It has an initial velocity of 0.001 m/s



a. What are the initial Kinetic, initial Potential, and total energy?

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} \cdot 10 kg \cdot (0.001 m/s)^2 = 5.0 \times 10^{-6} J$$
$$U_i = 0J$$
$$E_i = K_i + U_i = 5.0 \times 10^{-6} J$$

b. What are the Kinetic, Potential and total energy at the highest point that the pendulum reaches? How high does it go?

$$E_f = E_i = 5.0 \times 10^{-6} J$$
$$K_f = 0$$
$$U_f = 5.0 \times 10^{-6} J$$
$$U_f = mgy_f$$
$$y_f = \frac{U_f}{mg} = \frac{5.0 \times 10^{-6} J}{10kg \cdot 9.8m/s^2} = 5.1 \times 10^{-8} m$$

c. What angle with respect to vertical does it reach?

$$\cos\theta = \frac{1m - 5.1 \times 10^{-8} m}{1m}$$
$$\theta = 3.19 \times 10^{-4} radians$$

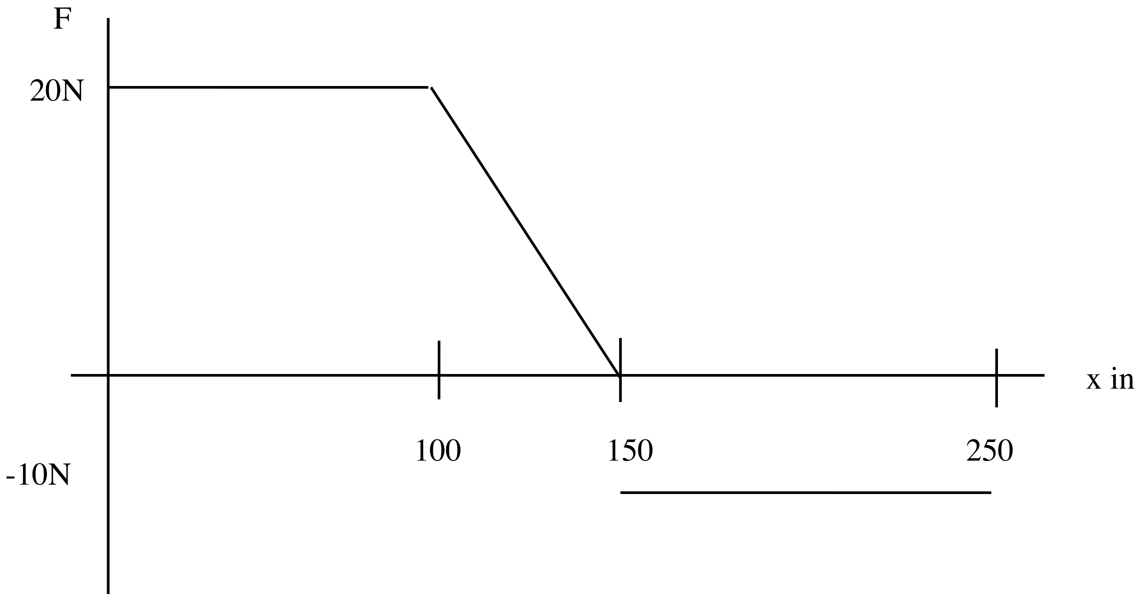
d. What velocity would the mass need at the bottom to just reach it to the top of the circle?

$$E_f = K_f + U_f = 0 + mg(2l)$$
$$E_i = K_i + U_i = \frac{1}{2} m v_i^2 + 0$$
$$E_i = E_f$$
$$\frac{1}{2} m v_i^2 = mg(2l)$$
$$v_i = \sqrt{4gl} = \sqrt{4 \cdot 9.8m/s \cdot 1m} = 6.26 m/s$$

e. What work does the Weight do when the mass moves from the bottom to the top of the circle?

$$\begin{aligned} W &= -\Delta U = -(mgy_f - mgy_i) \\ &= -(mg(2l) - 0) \\ &= -2mgl \\ &= -2 \cdot 10kg \cdot 9.8m/s^2 \cdot 1m \\ &= -196.0J \end{aligned}$$

4. A mass is subjected to a force over a distance as shown below...



a. How much work was done on the mass as it moved from 0 m to 100 m.

The work done is the area under the curve. A=20N*100m=2000J.

b. How much work was done on the mass as it moved from 100 to 150m?

The work done is the area under the curve. A=(1/2)*(20N)*50m=500J

c. How much work was done on the mass as it moved from 150 to 250 m?

The work done is the area under the curve. A=(-10N)*100=-1000J

d. What total work was done?

W=2000J+500J-1000J=1500J

e. If the mass was 50 kg and it started from rest, what was its final velocity?

$$\Delta K = K_f - K_i = W$$

$$\frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2} = W$$

$$\frac{1}{2}mv_f^{\,2} - 0 = W$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \cdot 1500J}{50kg}} = 7.75m/s$$