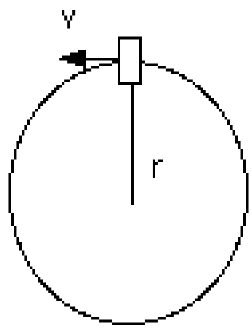
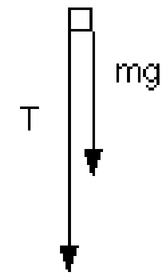
Sample Exam 2

Short answer.

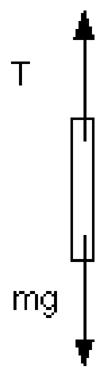
1. You whirl a rock on a string in a vertical circle. What is the minimum speed that the rock needs to have at the top so that the string does not become limp? Note: You do not need the mass of the rock.

$$\frac{mv^2}{r} = T + mg$$
$$T = 0$$
$$\frac{mv^2}{r} = mg$$
$$v = \sqrt{rg}$$





2. You are rescued by rope. The rope pulls you upward with a constant acceleration of 0.5 m/s2. Your mass 80 kg. What is the tension in the rope.

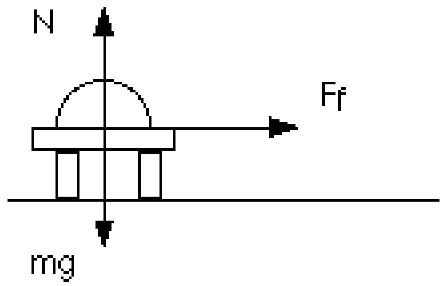


$$ma = T - mg$$
$$T = ma + mg = 80kg \cdot 0.5m/s^2 + 80kg \cdot 9.8m/s^2$$
$$= 824N$$

3. A mass of 10 kg is acted on by two forces: F1 = 2 i + 4 j and F2 = 5i -3 j. Find the net force and the acceleration in i, j, k components.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$
$$= 7\hat{i} + 1\hat{j}$$
$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{7}{10}\hat{i} + \frac{1}{10}\hat{j}$$

4. A frictional force with coefficient m acts on a car's tires. Derive an expression for the minimum radius curve the car can negotiate if it has velocity v_0 and there is no banking.



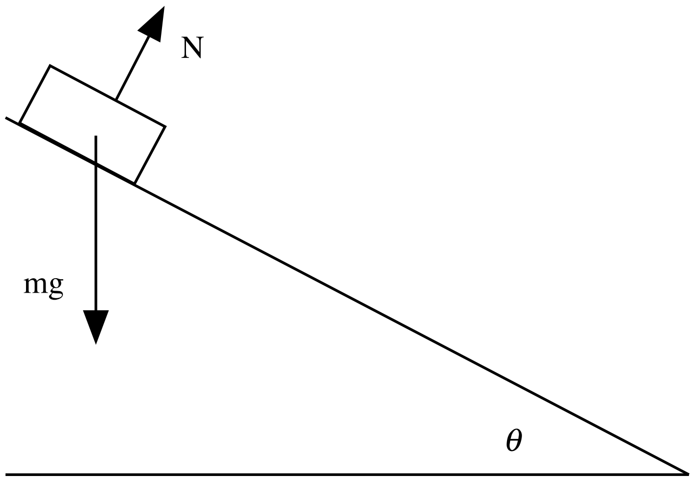
$$\frac{mv^2}{r} = \mu N$$
$$N = mg$$
$$\frac{mv^2}{r} = \mu mg$$
$$r = \frac{v^2}{\mu g}$$

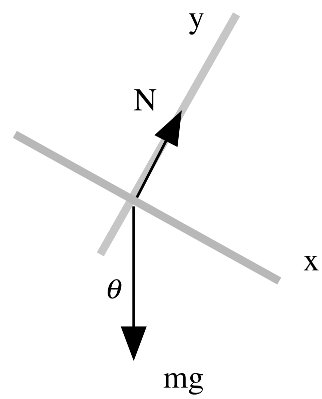
5. A mass falls through air with constant velocity. If the m=100 kg, what must be the frictional force acting on it?

If the mass falls with constant velocity, the frictional force must balance against the weight to produce zero net force.

$$F_f = mg = 100kg \cdot 9.8m/s^2 = 980N$$

6. A 50kg mass slides down an incline. The incline makes an angle of 30 degrees with horizontal and the mass slides a distance of 100 m. What is its velocity at the bottom of the incline?





We begin by computing the acceleration due to the forces that we have drawm.

$$0 = N - mg\cos\theta$$
$$N = mg\cos\theta$$

$$ma = mg\sin\theta$$
$$ma = mg\sin\theta$$
$$= mg\sin\theta$$
$$= mg\sin\theta$$
$$a = g\sin\theta$$

$$x_i = 0$$
$$x_f = d$$
$$v_i = 0$$
$$v_f = ?$$
$$a = g\sin\theta$$

$$v_f^{\,2} = v_i^{\,2} + 2a(x_f - x_i)$$
$$v_f^{\,2} = 0 + 2\cdot g\sin\theta\cdot(d-0)$$
$$v_f = \sqrt{2\cdot g\sin\theta\cdot d}$$
$$= 31.3\,m/s$$

7. A projectile is fired at an angle of 60 degrees and it rises to a maximum height of 100 m. What was the initial magnitude of the velocity of the projectile? When will it land? Where will it land?

First we solve for the initial velocity...

$$y_f = 100m$$
$$y_i = 0$$
$$v_{iy} = v_0\sin 60$$
$$v_{fy} = 0$$
$$a_y = -g$$

$$v_{fy}^{\,2} = v_{iy}^{\,2} + 2a(y_f - y_i)$$
$$0 = v_{iy}^{\,2} - 2g(y_f - 0)$$
$$v_{iy}^{\,2} = 2gy_f$$
$$v_{iy} = \sqrt{2gy_f} = \sqrt{2\cdot 9.8m/s^2\cdot 100m} = 44.3m/s$$
$$v_0 = \frac{v_{iy}}{\sin 60} = 51.1\,m/s$$

Note: This is another way to do it--different from the way that I did it in class.

Now compute the total time of flight to land. (Note: This is now the entire flight)

$$y_f = y_i + v_{iy}t + \frac{1}{2}at^2$$

$y_f = 0m$

$y_i = 0$

$v_{iy} = 51.1\sin 60$

$v_{fy} = 0$

$a_y = -g$

$$0 = 0 + v_{iy}t - \frac{1}{2}gt^2$$

$$= t(v_{iy} - \frac{1}{2}gt)$$

$$t = \frac{2v_{iy}}{g} = \frac{2 \cdot 51.1\sin 60}{9.8} = 9.03s$$

Now compute where it lands.

$x_f = ?$

$x_i = 0$

$v_{ix} = 51.1\cos 60$

$v_{fy} = v_{ix}$

$a_x = 0$

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$

$$x_f = + v_{ix}t + 0$$

$$= 51.1\cos 60\, t$$

$$= 51.1\cos 60 \cdot 9.03s$$

$$= 230.7m$$

8. A car travels around a curve with a radius of 100 m with a speed of 20 m/s. What centripetal acceleration does it experience?

$$a_c = \frac{v^2}{r} = \frac{(20m/s)^2}{100m} = 4m/s^2$$

9. A rifle fires a bullet perfectly horizontally at the center of a target that is 100 m away. If the speed of the bullet is 1000m/s, how long does the bullet take to hit the target, and how far below the center does it hit.

First we compute how long it takes to get to the target.

$x_i = 0$

$x_f = 100m$

$v_{ix} = 1000m/s$

$v_{fx} = 1000m/s$

$a_x = 0$

$t = ?$

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$

$$x_f = 0 + v_{ix}t + 0$$

$$t = \frac{x_f}{v_{ix}} = \frac{100m}{1000m/s} = 0.1s$$

Now that we know the time, we can compute how far it falls.

$$y_i = 0 \qquad y_f = ? \qquad v_{iy} = 0m/s \qquad v_{fy} = ? \qquad a_y = -g \qquad t = 0.1s$$

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$

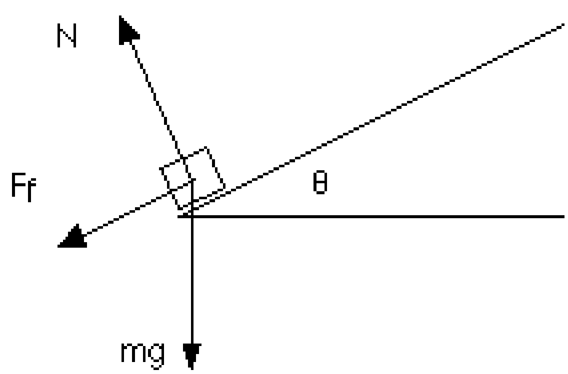
$$y_f = 0 + 0 - \frac{1}{2}g t^2$$

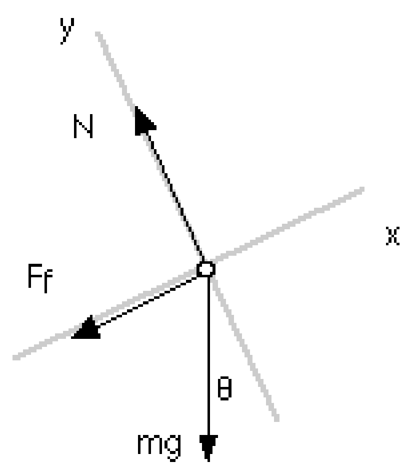
$$y_f = -\frac{1}{2}\cdot 9.8\frac{m}{s^2}\cdot(0.1s)^2 = -0.049m$$

Problems (Note: There are four practice problems here, but the test will still be pick two of three)

1. A mass is projected up an incline and friction is present.. The initial velocity of the mass was v0, the angle of the incline is θ and the distance that the mass travels up the incline is d.

a) Draw the free body diagram of the forces on the mass as it slides up the incline.





b) Derive an expression for the coefficient of friction m that must be present.

We begin by computing the acceleration due to the forces that we have drawm.

$$ma = -mg\sin\theta - F_f$$

$$0 = N - mg\cos\theta$$
$$N = mg\cos\theta$$

$$\begin{aligned} ma &= -mg\sin\theta - F_f \\ &= -mg\sin\theta - \mu N \\ &= -mg\sin\theta - \mu mg\cos\theta \\ a &= -g\sin\theta - \mu g\cos\theta \end{aligned}$$

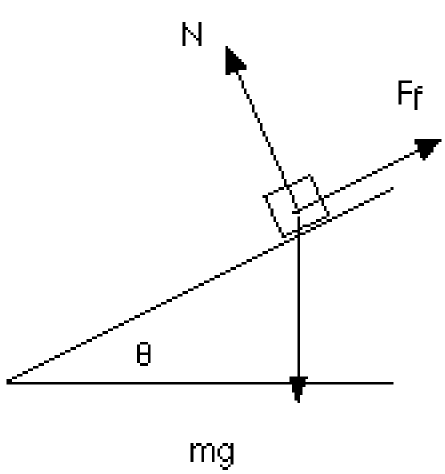
We can also compute the acceleration from the givens

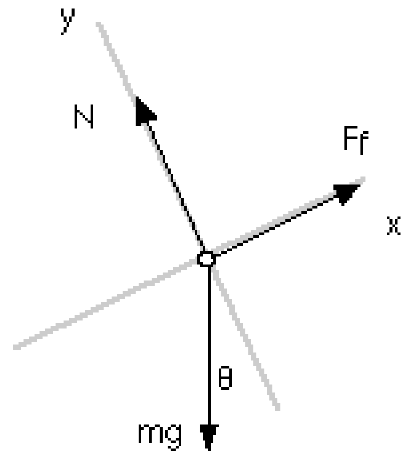
$$\begin{aligned} & x_f - x_i = d \\ & v_i = v_0 \\ & v_f = 0 \\ & v_f^{\,2} = v_i^{\,2} + 2a(x_f - x_i) \\ & a = \frac{v_f^{\,2} - v_i^{\,2}}{2d} = \frac{-v_i^{\,2}}{2d} \end{aligned}$$

These accelerations must be equal.

$$\begin{aligned} a &= -g\sin\theta - \mu g\cos\theta \\ a &= \frac{-v_i^{\,2}}{2d} \\ \frac{-v_i^{\,2}}{2d} &= -g\sin\theta - \mu g\cos\theta \\ \mu &= \frac{\frac{v_i^{\,2}}{2d} - g\sin\theta}{g\cos\theta} \end{aligned}$$

c) Now consider the mass sliding down. Draw the free body diagram. Use μ as the coefficient of friction (now known from part b).



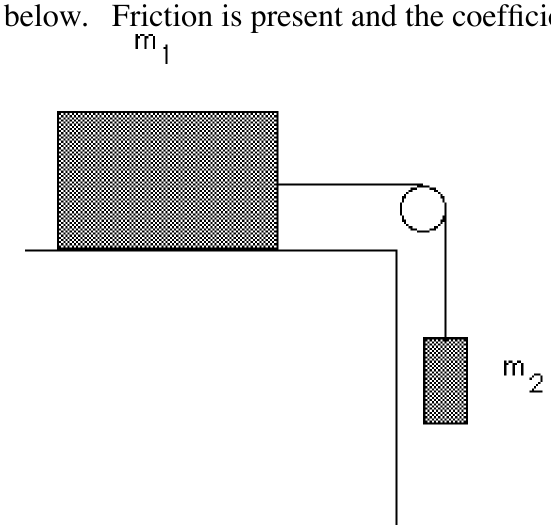


d) Derive an expression for the acceleration that the mass feels as it slides back down the incline.

$$0 = N - mg\cos\theta$$
$$N = mg\cos\theta$$

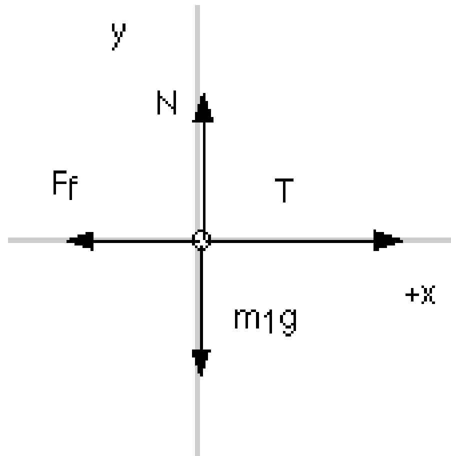
$$F_{net} = ma = -mg\sin\theta + F_f$$
$$m\,a = -mg\sin\theta + F_f$$
$$= -mg\sin\theta + \mu N$$
$$= -mg\sin\theta + \mu mg\cos\theta$$
$$a = -g\sin\theta + \mu g\cos\theta$$

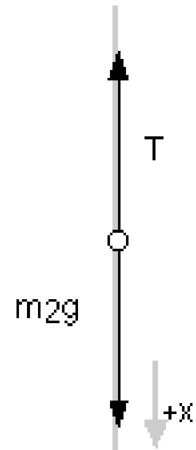
2. Consider the masses arranged below. Friction is present and the coefficient of friction is 0.1.



Take m1 = 25 kg and m2 = 5 kg.

a. Draw the free body diagram for each mass, and write out the equations for the net force on each mass.





b. What acceleration do the masses experience

We begin by writing out all of the forces.

$$0 = N - m_1 g \qquad N = m_1 g$$

$$\begin{aligned} m_1 a_x &= T - F_f \\ &= T - \mu N \\ &= T - \mu m_1 g \end{aligned}$$

$$m_2 a_x = m_2 g - T$$

We now solve for T in the right most equation, and plug it into the center equation

$$\begin{aligned} T &= m_2 g - m_2 a_x \\ m_1 a_x &= T - \mu m_1 g \\ m_1 a_x &= m_2 g - m_2 a_x - \mu m_1 g \\ m_1 a_x + m_2 a_x &= m_2 g - \mu m_1 g \\ a_x &= \frac{m_2 g - \mu m_1 g}{m_1 + m_2} \end{aligned}$$

c. What is the tension in the rope?

We can now substitute in for acceleration to get the tension

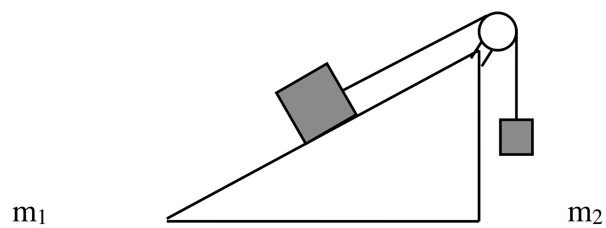
$$\begin{aligned} T &= m_2 g - m_2 a_x \\ &= m_2 g - m_2 \cdot \frac{m_2 g - \mu m_1 g}{m_1 + m_2} \end{aligned}$$

d. What frictional force would be necessary for the system to remain at rest or move with constant speed.

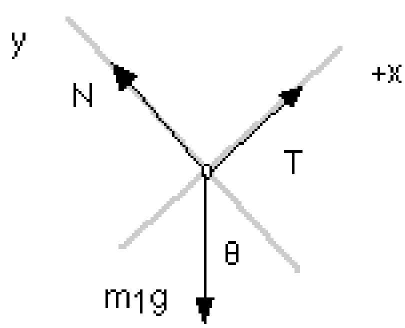
Constant speed means the acceleration is zero

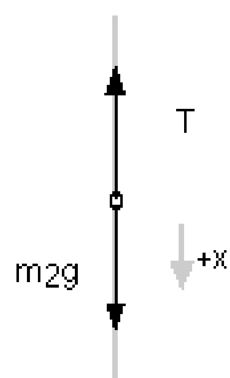
$$a_x = \frac{m_2 g - \mu m_1 g}{m_1 + m_2}$$
$$a_x = 0$$
$$0 = \frac{m_2 g - \mu m_1 g}{m_1 + m_2}$$
$$0 = m_2 g - \mu m_1 g$$
$$\mu m_1 g = m_2 g$$
$$\mu m_1 g = F_f$$
$$F_f = m_2 g$$

3. Consider the arrangement of masses below. Assume initially that no friction is present.



a. Draw a free body diagram for each mass.





b. Compute the tension and acceleration of the masses.

$$0 = N - m_1 g \cos\theta$$
$$N = m_1 g \cos\theta \qquad m_1 a_x = T - m_1 g \sin\theta \qquad m_2 a_x = m_2 g - T$$

We again solve the right most equation for T and plug into the middle equation.

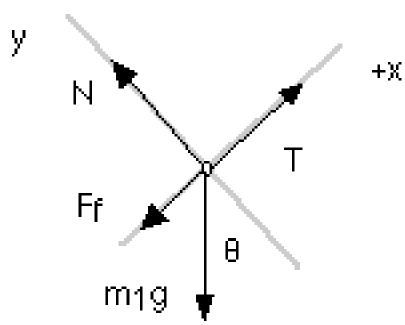
$$T = m_2 g - m_2 a_x$$
$$m_1 a_x = T - m_1 g \sin\theta$$
$$m_1 a_x = m_2 g - m_2 a_x - m_1 g \sin\theta$$
$$m_1 a_x + m_2 a_x = m_2 g - m_1 g \sin\theta$$
$$a_x = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2}$$

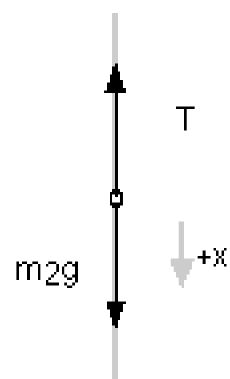
We find the tension by plugging back in for acceleration.

$$\begin{aligned} T &= m_2 g - m_2 a_x \\ &= m_2 g - m_2 \cdot \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2} \end{aligned}$$

c. Now assume that a frictional force is present. Assume that m1 slides up the incline. Find the acceleration of the masses and the tension.

We begin with a drawing that includes the frictional force.





We write the new equations that include the frictional force

$$\begin{aligned} 0 &= N - m_1 g \cos\theta \\ N &= m_1 g \cos\theta \end{aligned}$$

$$\begin{aligned} m_1 a_x &= T - m_1 g \sin\theta - F_f \\ &= T - m_1 g \sin\theta - \mu N \\ &= T - m_1 g \sin\theta - \mu m_1 g \cos\theta \end{aligned}$$

$$m_2 a_x = m_2 g - T$$

We now solve as before.

$$
\begin{aligned}
T &= m_2 g - m_2 a_x \\
m_1 a_x &= T - m_1 g\sin\theta - \mu m_1 g\cos\theta \\
m_1 a_x &= m_2 g - m_2 a_x - m_1 g\sin\theta - \mu m_1 g\cos\theta \\
m_1 a_x + m_2 a_x &= m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta \\
a_x &= \frac{m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta}{m_1 + m_2}
\end{aligned}
$$

We compute the tension by plugging in the new acceleration

$$
\begin{aligned}
T &= m_2 g - m_2 a_x \\
&= m_2 g - m_2 \cdot \frac{m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta}{m_1 + m_2}
\end{aligned}
$$

d. What friction coefficient is necessary for the masses to remain at rest?

The acceleration is zero, so we substitute in and solve.

$$
\begin{aligned}
a_x &= \frac{m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta}{m_1 + m_2} \\
0 &= \frac{m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta}{m_1 + m_2} \\
0 &= m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta \\
\mu &= \frac{m_2 g - m_1 g\sin\theta}{m_1 g\cos\theta}
\end{aligned}
$$

4. A parachutist jumps off a 1500 m cliff with a horizontal velocity of 3 m/s. Three seconds after her jump, she releases the parachute.

a. Write her initial position and velocity vectors in i, j, k notation.

$$
\begin{aligned}
\bar{r} &= 0\hat{i} + 1500\hat{j} + 0\hat{k} \\
\bar{v} &= 3\frac{m}{s}\hat{i} + 0\hat{j} + 0\hat{k}
\end{aligned}
$$

b. What velocity does she have when she releases the chute? Express your result in both i, j, k notation and in magnitude and direction.

$$
\begin{aligned}
v_{iy} &= 0.0\, m/s \\
v_{ix} &= 3.0 m/s \\
a_y &= -g \\
a_x &= 0 m/s^2 \\
t &= 3s \\
v_{fy} &= v_{iy} + a_y t \\
&= 0 m/s - (9.8 m/s^2)\cdot(3s) = -29.4\, m/s \\
v_{fx} &= v_{ix} + a_x t \\
v_{fx} &= v_{ix} = 3.0 m/s
\end{aligned}
$$

Now we can write this in i, j, k and magnitude and direction...

$$
\begin{aligned}
&\vec{v}_f = 3.0\hat{i} + (-29.4)\,\hat{j}\ m/s \\
&\left|\vec{v}_f\right| = \sqrt{3.0^2 + (-29.4)^2} = 29.55 m/s \\
&\tan\theta = \frac{-29.4}{3.0} \Rightarrow \theta = -84.2^\circ
\end{aligned}
$$

c. What is her height above the ground and how far is she from the cliff in the horizontal direction?

$$
\begin{aligned}
y_f &= y_i + v_{iy}t + \frac{1}{2}a_y t^2 \\
y_f &= y_i + v_{iy}t - \frac{1}{2}gt^2 \\
&= 1500m + (0m/s)\cdot 3s - \frac{1}{2}\cdot(9.8m/s^2)\cdot(3s)^2 \\
&= 1455.9m \\
x_f &= x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\
x_f &= 0 + v_{ix}t + 0 \\
&= (3.0m/s)\cdot 3s \\
&= 9m
\end{aligned}
$$

The parachute slows her decent to a **constant** 3 m/s in the vertical direction in an instant at this height. Her horizontal velocity goes to zero.

d. What is her total time to land from the jump?

$$y_f = 0$$
$$y_i = 1455.9m$$
$$a_y = 0$$
$$v_{iy} = -3m/s$$
$$v_{fy} = -3m/s$$
$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$
$$0 = y_i + v_{iy}t + 0$$
$$t = -\frac{y_i}{v_{iy}} = -\frac{1455.9m}{-3m/s} = 485.3s$$

The total time of flight is $488.3s$.