Linear Restoring Forces

One of the most common and useful motions that we observe in nature is vibration, or oscillation. We refer to this motion as "Simple Harmonic Motion." An object moves back and forth about an equilibrium point. While we can find lots of systems that behave like this (springs, pendula, atoms in a crystal lattice, protons in a nucleus, etc.), the forces that cause this motion have some common characteristics in the region where simple harmonic motion occurs. We refer to forces that behave like this as "Linear Restoring Forces."

In this experiment we will investigate the properties of linear restoring forces in two systems and we will investigate the properties of motion that result from those forces. In particular, we wish to determine:

Is the force is linear?
What strength it has as a function of position?
What frequency/period of motion results?
What determines the frequency of motion?
What common features do linear restoring forces and the motions that result share?
Where does a force become "nonlinear"?

Properties of the Force.

Force is defined by Newton's second Law

$$\vec{F} = m\,\vec{a}$$

If m is expressed in grams and acceleration is in cm/s^2, force will be in dynes.

All forces that cause simple harmonic motion share the following properties (at least in approximation).

1. The force depends on position. At the equilibrium point the force is zero.
2. When the object is displaced from equilibrium, the force acts to push it back toward equilibrium.
3. Some constant can be used to characterize how strong the force is.

When we investigate further, we find that there is a specific type of relationship that all of these forces share. We find that the force is linearly related to how far from equilibrium an object is displaced. For a spring, we write this relationship as

$$F = -kx$$

Force is a vector, but we consider motion of the spring in only one dimension. This equation indicates a linear relationship between force and position. Here k is the spring constant the measures the strength of the force and x is how far from equilibrium the mass is displaced. The minus sign indicates the direction--when an object is displaced in the +x direction, the force pushes in the - direction back toward equilibrium. When an object is pushed in the -x direction, the force pushes in the + direction.

Properties of the Motion from a Linear Restoring Force.

We expect that the motion that systems undergo due to linear restoring forces will show certain common characteristics. Qualitatively, we see that the motion of an oscillating system has the following properties.

1. The object oscillates back and forth about equilibrium. Its "turning points" are the places at the end of its trajectory where it stops and turns around.
2. The object has its maximum speed when it passes through equilibrium. Its speed changes direction as it oscillates back and forth.
3. The object oscillates with a well defined frequency or period of oscillation.

When we write this motion quantitatively, we find that

$$x(t) = A\sin\omega\, t$$

Here A is the amplitude of the motion as measured from equilibrium to a turning point. ω is the angular frequency.

$$\omega = 2\pi\ f = \frac{2\pi}{T}$$

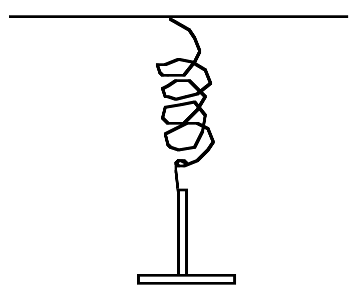
Here f is frequency in Hz and T is period in seconds. Using these relationships, we will investigate the motion due to linear restoring forces in two systems.

Procedure.

The two systems we will study are the mass spring system and the simple pendulum. The mass-spring system is the classic example of a simple harmonic oscillator due to a linear restoring force. The simple pendulum is also a simple harmonic oscillator as long as the amplitude of the motion remains small.

In the mass spring system, we will use the fact that the mass is in equilibrium to measure the force that spring exerts. Since the mass is not accelerating, we know that the force of the spring upward is equal to the weight of the mass. We can now measure the spring force as a function of position.

I. Mass-Spring system.



Equilibrium point x_0 -->

A. Testing Hooke's Law- Is the Force Linear?

Construct a data table with columns that looks like:

mass(g)	position x(cm)	$x - x_0$(cm)	weight(=mg in dynes)

1. Hang your spring vertically and place the mass-holder on the spring. This will serve as your "zero mass" state. The 50 gram mass holder serves to stretch the spring slightly.
2. Measure the position of the mass holder. This will be your "equilibrium position"
3. Place 20 grams on the holder and measure the position that the mass holder sinks to.
4. Repeat with 40 g, 60g, 80 g, 100g, 120g, 140g, 160g, 180g, and 200 g and record the position for each mass.
5. Compute the difference in position from equilibrium for each mass and record.
6. Compute the weight for each mass and record.
7. Plot the force that is exerted by the spring (which equals the weight) on the y axis verses displacement from equilibrium ($x - x_0$) on the x axis and find the slope. The slope is the spring constant k.

B Investigating Period.

In each case you should find the time for 20 oscillations and then divide by 20 to determine the period.

Dependence on Amplitude

The amplitude is the maximum displacement from equilibrium.

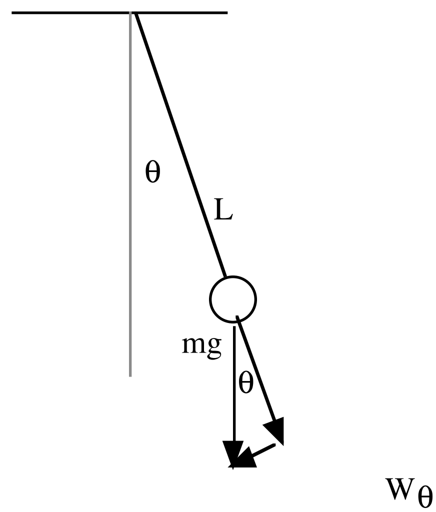
1. Place 50 g on the mass holder. Displace the mass holder by 1 cm and measure the period?
2. Repeat for displacements of 1cm, 5 cm, and 10 cm.
3. Plot period vs. displacement. Does period depend on amplitude?

Dependence on mass

1. Place 20 g on the mass holder and displace the mass holder by 10 cm. Measure the period and record.
2. Repeat with 40g, 60g, 80 g and 100 g.
3.* Does period depend on mass . Plot T^2 vs m and determine the slope. NOTE: You must include the mass of the holder here. How is the slope related to k? What k does your data predict?

II. Simple Pendulum--Investigating Period.

Set up a simple pendulum. As in the earlier section, you should determine the period by measuring the time for 20 oscillations and dividing by 20.



$$W_\theta = -mg\sin\theta$$

Dependence on Amplitude.

1. Place 50 g on the end of the string of length 20 cm. and displace the mass by an angle of 10 degrees. Measure the period.
2. Repeat this measurement with the same mass but with initial displacements of 15, 20, 25 and 30 degrees.
3. Plot the period vs. Amplitude. Does period depend on amplitude?

Dependence on Length.

1. Place 50 g on the end of the string of length 10 cm. Displace the mass by and angle of 20 degrees and measure the period.
2. Repeat for lengths of 30, 40, 50, 60 and 70 cm. You can use the 20 cm data from the earlier section.
3.* Does period depend on length? Plot T^2 vs L to determine relationship. How is the slope related to g?

Testing Linearity*

1. Make a plot of the "Theta" component of the gravitational force W_θ for amplitude angles 0 through 90 degrees. Be sure to make your plot as Force verses angle in radians.
2. For small angles (< 30 degrees), is the function roughly linear? What is the slope?
3. Can you determine what the physical interpretation of the slope is. Hint: Recall that the mass that you used was 50 g. What is the "k" for this linear restoring force?
4. What happens at larger angles? Does the force remain linear in angle? Do you expect that our analogy with the spring will break down.

Questions

1. Show that the x motion for the harmonic oscillator satisfies

$$F = -kx$$

if you choose ω correctly. Do this by writing

$$x(t) = A\sin\omega t$$

and taking derivatives to substitute in to

$$m\frac{d^2x}{dt^2} = -kx$$

Solve for what ω must be for this to work.

2. In this experiment you found that the period of the motion for a mass-spring system depended on the mass. This is because the acceleration depended on mass.

$$ma = -kx$$
$$|a| = \frac{k}{m}$$

Given what you know about gravitational acceleration, do you expect that the period of a pendulum will depend on mass? How could you investigate this? Hint: For small oscillations...

$$W_\theta = -mg\sin\theta$$
$$ma_\theta = -mg\sin\theta$$
$$|a_\theta| = g\sin\theta \approx g\theta$$

3. You have tested the properties of two linear restoring forces. What general conclusion might you draw about the dependence of period on amplitude?

4. What general conclusions might you draw about what properties of the force the period depends upon?