Exam 2
Physics 130

Short Answer Section. Please answer all of the questions.

1. A 50 kg mass moves with a constant speed of 10 m/s in a circle of radius 10m. What *centripetal force* must be acting on it?

$$F_c = \frac{mv^2}{r} = \frac{50kg \cdot (10m/s)^2}{10m} = 500N$$

2. A bucket with a mass of 20 kg hangs from a single cable and is accelerating *downward* at $0.2m/s^2$. What is the tension in the cable?

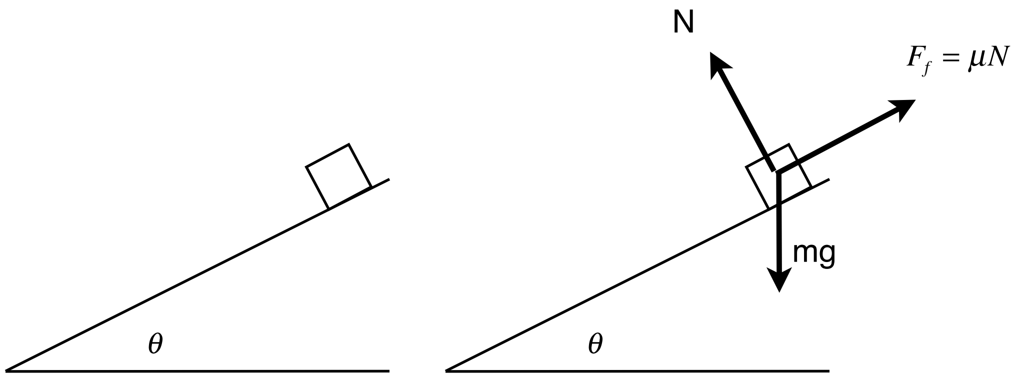
$$a = -0.2m/s^2$$
$$ma = T - mg$$
$$T = mg + ma = 20kg \cdot 9.8m/s^2 + 20kg \cdot (-0.2m/s^2) = 192N$$

3. A pumpkin in a "pumpkin chucking contest" is fired with a speed of 60 m/s at an angle of 30 degrees with respect to horizontal. How long does it fly before it lands back on level ground? How far does it go horizontally?

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$
$$0 = 0 + v_{iy}t - \frac{1}{2}gt^2$$
$$0 = (v_{iy} - \frac{1}{2}gt)t$$
$$t = \frac{2v_{iy}}{g} = \frac{2 \cdot 60 \sin 30°}{9.8} = 6.12s$$

$$\begin{aligned} x_f &= x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ &= 0 + v_{ix}t + 0 \\ &= 60\cos 30° \cdot 6.12s \\ &= 318m \end{aligned}$$

4. A mass accelerates *down* an incline with angle θ. Friction is present. Draw ALL the forces on the mass and compute the acceleration down the incline? Assume that the kinetic coefficient of friction is μ_k



$$
\begin{aligned}
ma_x &= \mu N - mg\sin\theta \\
&= \mu mg\cos\theta - mg\sin\theta \\
a_x &= \mu g\cos\theta - g\sin\theta
\end{aligned}
$$

5. A mass of 1000 kg is acted on by three forces: $\vec{F}_1 = 50\hat{i} + 100\hat{j} - 200\hat{k}$, $\vec{F}_2 = 250\hat{i} + 100\hat{j} - 200\hat{k}$ and $\vec{F}_3 = 200\hat{i} + 150\hat{j} + 400\hat{k}$. Find the net force and the acceleration in i, j, k components.

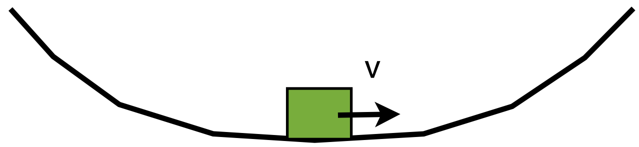
$$
\begin{aligned}
\vec{F}_{net} &= (50 + 250 + 200)\hat{i} + (100 + 100 + 150)\hat{j} + (-200 - 200 + 400)\hat{k} \\
&= (500)\hat{i} + (350)\hat{j} + (0)\hat{k} \\
\vec{a} &= \frac{(500)\hat{i} + (350)\hat{j} + (0)\hat{k}}{1000}
\end{aligned}
$$

6. A projectile is fired *perfectly horizontally* from a table top of height 1.5m. How long does it take to land? How far does it go horizontally if it had an initial velocity of 5 m/s.

$$
\begin{aligned}
y_f &= y_i + v_{iy}t + \frac{1}{2}a_y t^2 \\
0 &= y_i + 0 - \frac{1}{2}gt^2 \\
t &= \sqrt{\frac{2y_i}{g}} = \sqrt{\frac{2 \cdot 1.5m}{9.8m/s}} = 0.553s
\end{aligned}
$$

$$
\begin{aligned}
x_f &= x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\
&= 0 + v_{ix}t + 0 \\
&= 5m/s \cdot 0.553s \\
&= 2.765m
\end{aligned}
$$

7. A sled slides on the bottom of an icy ***circular*** valley of radius 64m. The mass of the sled is 5 kg and its speed is 12 m/s. Draw the forces and compute the normal force on the sled. You may assume that this is a uniform circular motion problem. There is no friction.

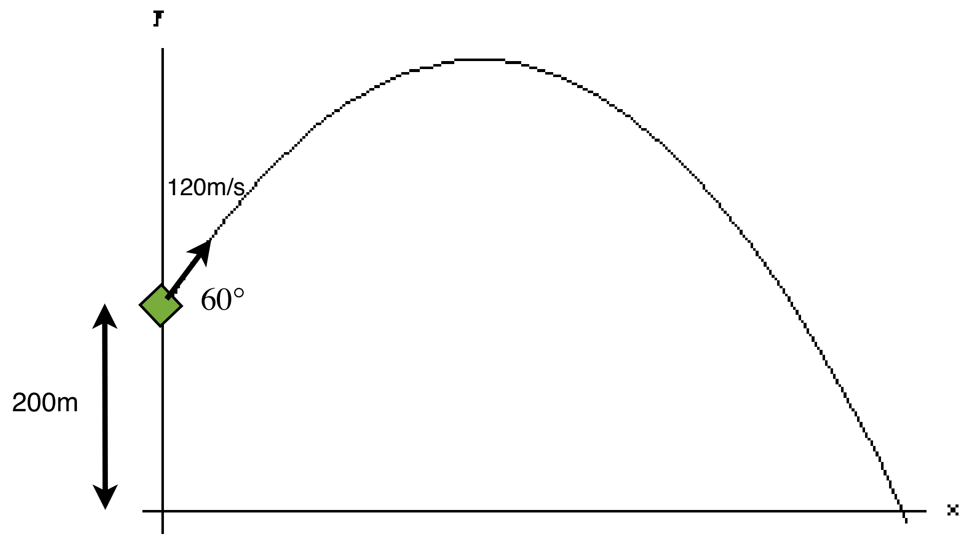


$$\frac{mv^2}{r} = N - mg$$

$$N = \frac{mv^2}{r} + mg = 60.25N$$

Problems. Please work two of three.

1. A rescue aircraft needs to drop a care package to a sinking ship. The aircraft performs this by "lobbing" the package into a harbor as sketched below. When it releases the package, the aircraft is at an altitude of 200 m and it is climbing with a velocity of 120m/s at an angle of 60 degrees with respect to horizontal.



a) Write the initial position and velocity of the package at release? (Write the $x_i, y_i, v_{ix}, and\ v_{iy}$).

$$x_i = 0$$
$$y_i = 200m$$
$$v_{ix} = 120\cos 60 = 60.0m/s$$
$$v_{iy} = 120\sin 60 = 103.9m/s$$

b) How long does it take for the package to reach its maximum height?

$$v_{fy} = v_{iy} + a_y t$$
$$0 = v_{iy} - gt$$
$$t = \frac{v_{iy}}{g} = 10.6s$$

c) What is its maximum height?

$$
\begin{aligned}
y_f &= y_i + v_{iy}t + \frac{1}{2}a_y t^2 \\
&= 200m + 103.9m/s \cdot 10.6s - \frac{1}{2}g(10.6s)^2 \\
&= 750.8m
\end{aligned}
$$

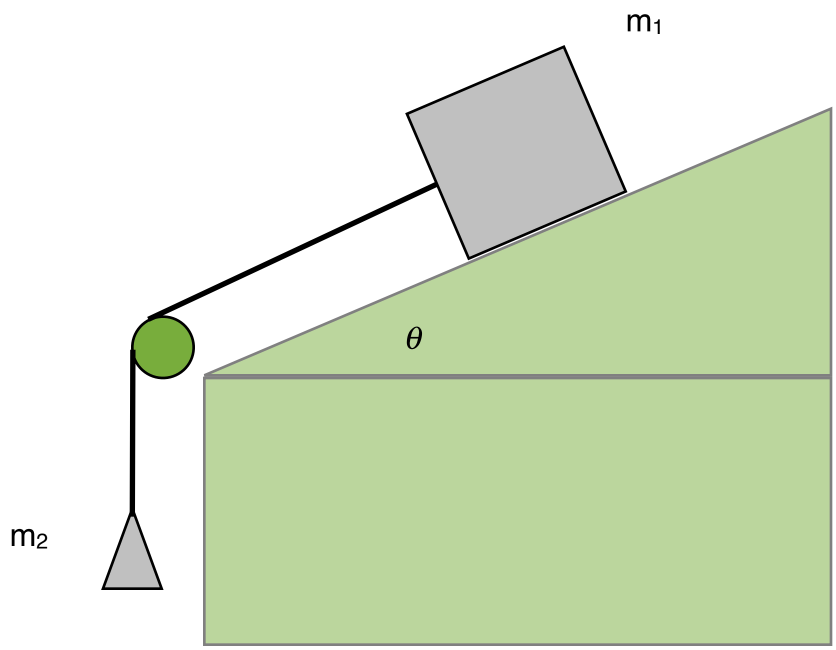
d) How long does it take to fall from its maximum height to the ground What is the total time of flight?

$$
\begin{aligned}
&y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \\
&0 = y_i + 0 - \frac{1}{2}gt^2 \\
&t = \sqrt{\frac{2y_i}{g}} = 12.4s \\
&t_{total} = 10.6s + 12.4s = 23.0s
\end{aligned}
$$

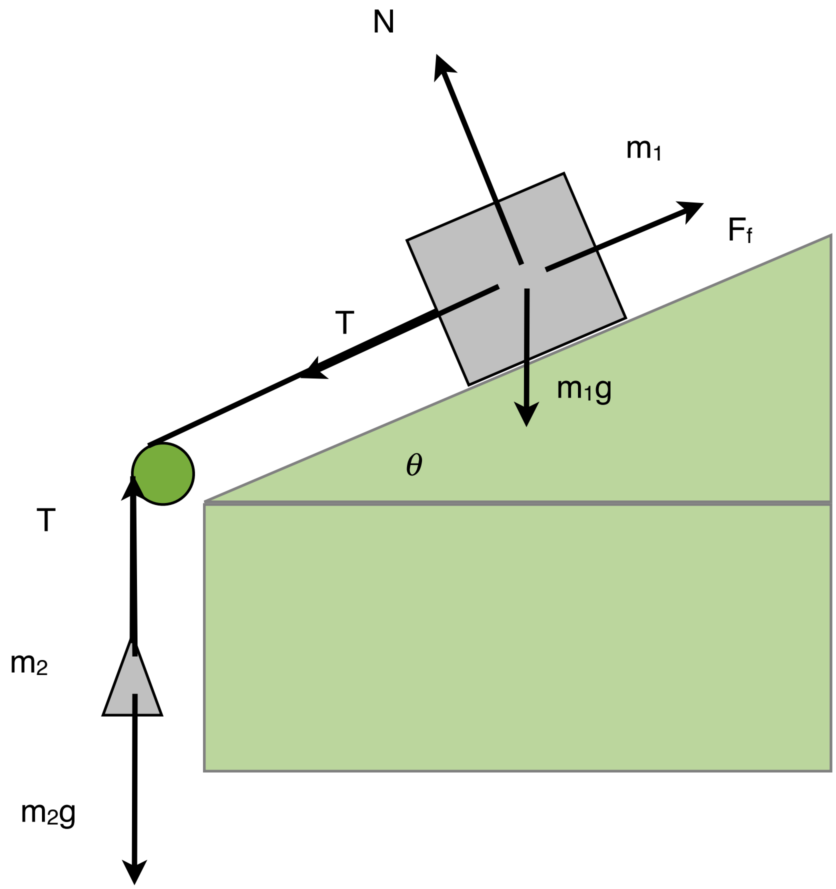
e) Where does it land?

$$
\begin{aligned}
x_f &= x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\
&= 0 + v_{ix}t + 0 \\
&= 60m/s \cdot 23.0s \\
&= 1380.0m
\end{aligned}
$$

2. Consider the masses below. Friction is present with coefficient μ .



a) Draw the forces on each block and draw a free body diagram for each block.



b) Write the net force(s) on each block.

$$
\begin{aligned}
0 &= N - m_1 g\cos\theta \\
N &= m_1 g\cos\theta \\
m_1 a_x &= \mu m_1 g\cos\theta - m_1 g\sin\theta - T
\end{aligned}
\qquad\qquad m_2 a_x = T - m_2 g
$$

c) What acceleration do the blocks experience.

$$
\begin{aligned}
T &= m_2 a_x + m_2 g \\
m_1 a_x &= \mu m_1 g\cos\theta - m_1 g\sin\theta - T \\
&= \mu m_1 g\cos\theta - m_1 g\sin\theta - (m_2 a_x + m_2 g) \\
m_1 a_x + m_2 a_x &= \mu m_1 g\cos\theta - m_1 g\sin\theta - m_2 g \\
a_x &= \frac{\mu m_1 g\cos\theta - m_1 g\sin\theta - m_2 g}{m_1 + m_2}
\end{aligned}
$$

d) What coefficient of friction is necessary for the blocks to move at constant speed?

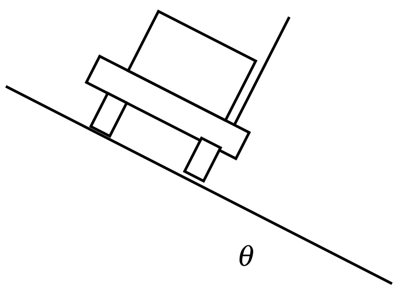
$$a_x = \frac{\mu m_1 g \cos\theta - m_1 g \sin\theta - m_2 g}{m_1 + m_2}$$

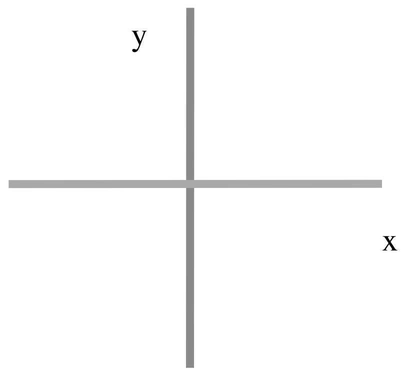
$$0 = \mu m_1 g \cos\theta - m_1 g \sin\theta - m_2 g$$

$$\mu m_1 g \cos\theta = m_1 g \sin\theta + m_2 g$$

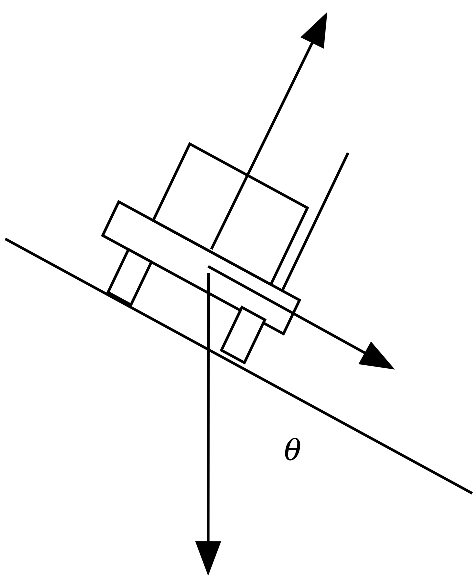
$$\mu = \frac{m_1 g \sin\theta + m_2 g}{m_1 g \cos\theta} = \frac{m_1 \sin\theta + m_2}{m_1 \cos\theta}$$

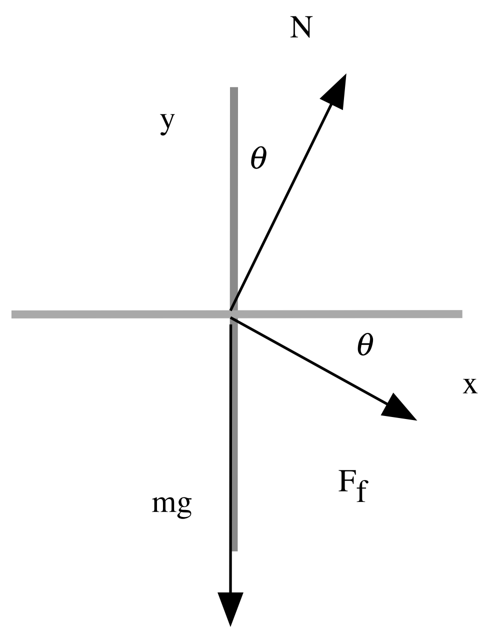
3. A car travels around a banked circular track with radius r as shown below. The car is traveling at the ***maximum*** velocity that it can. If it goes any faster, it will slide up and off the top of the bank. Friction keeps it from sliding off the curve. Take the coefficient of friction to be μ.





a) Draw the free body diagram on a coordinate system above.





b) Write the net force in the vertical and inward direction?

vertical

$$ma = 0 = N\cos\theta - F_f\sin\theta - mg$$

inward

$$\frac{mv^2}{r} = N\sin\theta + F_f\cos\theta$$

c) What is the car's velocity?

$$\begin{aligned} 0 &= N\cos\theta - F_f\sin\theta - mg \\ &= N\cos\theta - \mu N\sin\theta - mg \\ N &= \frac{mg}{\cos\theta - \mu\sin\theta} \end{aligned}$$

$$\begin{aligned} \frac{mv^2}{r} &= N\sin\theta + F_f\cos\theta \\ &= N\sin\theta + \mu N\cos\theta \\ &= \frac{mg(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta} \\ v &= \sqrt{rg\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}} \end{aligned}$$

d) What is the one velocity that the car can have if the friction goes to zero?

$$v = \sqrt{rg\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}}$$

$$\mu = 0$$

$$v = \sqrt{rg\frac{\sin\theta}{\cos\theta}} = \sqrt{rg\tan\theta}$$