Chapter 9

9.1 A 2.00 kg particle has the xy coordinates $(-1.20m,\ 0.500m)$, and a 4.00 kg particle has the xy coordinates $(0.600m,\ -0.750m)$. Both lie on a horizontal plane. At what (a) x and (b) y cooridatnes must you place a 3.00 kg particle such that the center of mass of the three particle system is $(-0.500m,\ -0.700m)$?

We begin by writing out the center of mass for x and y. We then solve for the unknown position of the 3kg mass.

$$
\begin{aligned}
x_{cm} &= \frac{2.00kg \cdot -1.20m + 4.00kg \cdot 0.600m + 3.00kg \cdot x}{2.00kg + 4.00kg + 3.00kg} \\
&= \frac{2.00kg \cdot -1.20m + 4.00kg \cdot 0.600m + 3.00kg \cdot x}{9.00kg} \\
&= \frac{3.00kg \cdot x}{9.00kg} \\
x &= 3x_{cm} = -1.5m
\end{aligned}
$$

$$
\begin{aligned}
y_{cm} &= \frac{2.00kg \cdot 0.500m + 4.00kg \cdot -0.750m + 3.00kg \cdot y}{2.00kg + 4.00kg + 3.00kg} \\
&= \frac{-2.00kgm + 3.00kg \cdot y}{9.00kg} \\
&= \frac{-2.00kgm}{9.00kg} + \frac{3.00kg \cdot y}{9.00kg} = \frac{-2.00}{9.00}m + \frac{1.00}{3.00}y \\
y &= 3(y_{cm} + \frac{2.00}{9.00}m) = 3(-0.700 + 0.222) \\
&= -1.43m
\end{aligned}
$$

9.2 Figure 9-36 shows a three particle system. What are (a) the x coordinate and (b) the y coordinate of the center of mass of the three particle system . (c) What happens to the center of mass as the mass of the m_3 particle is increased.

We can calculate the center of mass in the x and y directions.

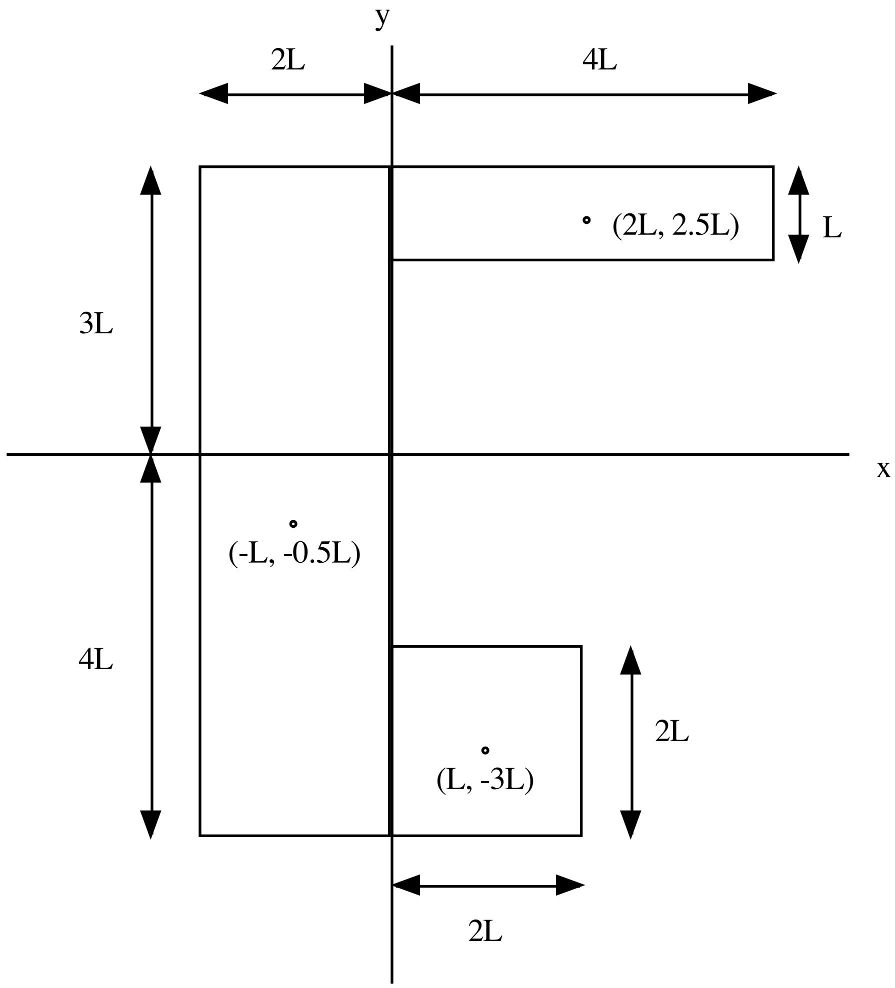
$$
\begin{aligned}
x_{cm} &= \frac{1}{(m_1 + m_2 + m_3)} \cdot (m_1 x_1 + m_2 x_2 + m_3 x_3) \\
&= \frac{1}{(3.0kg + 4.0kg + 8.0kg)} \cdot (3.0kg \cdot 0m + 4.0kg \cdot 2.0m + 8.0kg \cdot 1.0m) \\
&= \frac{1}{(15kg)} \cdot (16.0kgm) \\
&= \frac{16}{15} m
\end{aligned}
$$

$$
\begin{aligned}
y_{cm} &= \frac{1}{(m_1 + m_2 + m_3)} \cdot (m_1 y_1 + m_2 y_2 + m_3 y_3) \\
&= \frac{1}{(3.0kg + 4.0kg + 8.0kg)} \cdot (3.0kg \cdot 0m + 4.0kg \cdot 1.0m + 8.0kg \cdot 2.0m) \\
&= \frac{1}{(15kg)} \cdot (20.0kgm) \\
&= \frac{20}{15} m = \frac{4}{3} m
\end{aligned}
$$

As the topmost mass grows larger, the cm will move toward that mass.

9.3 What are (a) the x coordinate and (b) the y coordinate of the center of mass for the uniform plate shown in Fig 9-38

For continuous objects, we can often use symmetry to find the cm. In the drawing below, we have labeled the location of the center of each piece of the plate. We can now consider each piece of the plate as a point mass at the center. The mass of each plate is proportional to the area.



$$x_{cm} = \frac{1}{Total\ Area}\sum Area_i \cdot x_i$$
$$= \frac{(2L\cdot 7L)\cdot(-L) + (4L\cdot L)\cdot 2L + (2L\cdot 2L)\cdot L}{(2L\cdot 7L) + (4L\cdot L) + (2L\cdot 2L)}$$
$$= \frac{-2L^3}{22L^2}$$
$$= -\frac{1}{11}L$$
$$= -0.455cm$$

$$
\begin{aligned}
y_{cm} &= \frac{1}{Total\ Area}\sum Area_i \cdot y_i \\
&= \frac{(2L\cdot 7L)\cdot(-0.5L)+(4L\cdot L)\cdot 2.5L+(2L\cdot 2L)\cdot(-3L)}{(2L\cdot 7L)+(4L\cdot L)+(2L\cdot 2L)} \\
&= \frac{-9L^3}{22L^2} \\
&= -\frac{9}{22}L \\
&= -2.045cm
\end{aligned}
$$

9.7 The figure shows a slab with dimensions $d_1 = 11.0cm$, $d_2 = 2.80cm$, and $d_3 = 11.0\,cm$. Half the slab consists of aluminum (density $= 2.70\,g/cm^3$) and half consists of iron density $= 7.85g/cm^3$) . What are the coordinates of the center of mass.

We begin finding the coordinates of the center of mass of each slab (Aluminim and Iron).

$$
x_{Al} = -\frac{d_3}{2} = -6.50\,cm \qquad x_{Fe} = -\frac{d_3}{2} = -6.50cm
$$

$$
y_{Al} = d_1 + \frac{d_1}{2} = 16.5cm \qquad y_{Fe} = \frac{d_1}{2} = 5.50\,cm
$$

$$
z_{Al} = \frac{d_2}{2} = 1.40cm \qquad z_{Fe} = \frac{d_2}{2} = 1.40cm
$$

The volume of each slab is the same. We use the volume to find the mass of each slab.

$$
\begin{aligned}
V &= d_1\, d_2\, d_3 \\
m_{Al} &= \rho_{Al} V \\
m_{Fe} &= \rho_{Fe} V
\end{aligned}
$$

$$x_{cm} = \frac{m_{Al}x_{Al} + m_{Fe}x_{Fe}}{m_{Al} + m_{Fe}}$$

$$= \frac{\rho_{Al} V\, x_{Al} + \rho_{Fe} V\, x_{Fe}}{\rho_{Al} V + \rho_{Fe} V}$$

$$= \frac{\rho_{Al}\ x_{Al} + \rho_{Fe}\ x_{Fe}}{\rho_{Al} + \rho_{Fe}} = \frac{2.70 \cdot -6.50 + 7.85\ \cdot -6.50}{2.70 + 7.85}$$

$$= -6.50$$

$$y_{cm} = \frac{m_{Al}y_{Al} + m_{Fe}y_{Fe}}{m_{Al} + m_{Fe}}$$

$$= \frac{\rho_{Al}\ y_{Al} + \rho_{Fe}\ y_{Fe}}{\rho_{Al} + \rho_{Fe}} = \frac{2.70 \cdot 16.5 + 7.85\ \cdot 5.50}{2.70 + 7.85}$$

$$= 8.32\,cm$$

$$z_{cm} = \frac{m_{Al}z_{Al} + m_{Fe}a_{Fe}}{m_{Al} + m_{Fe}}$$

$$= \frac{\rho_{Al}\ z_{Al} + \rho_{Fe}\ z_{Fe}}{\rho_{Al} + \rho_{Fe}} = \frac{2.70 \cdot 1.40 + 7.85\ \cdot 1.40}{2.70 + 7.85}$$

$$= 1.40\,cm$$

9.10 Two skaters , one with mass 65 kg and the other with mass 40 kg stand on an ice rink hold a ole with a length of 10m and negligible mass. Starting from the ends of the pole, the skaters pull themselves along the pole until they meet. How far will the 40 kg skater move?

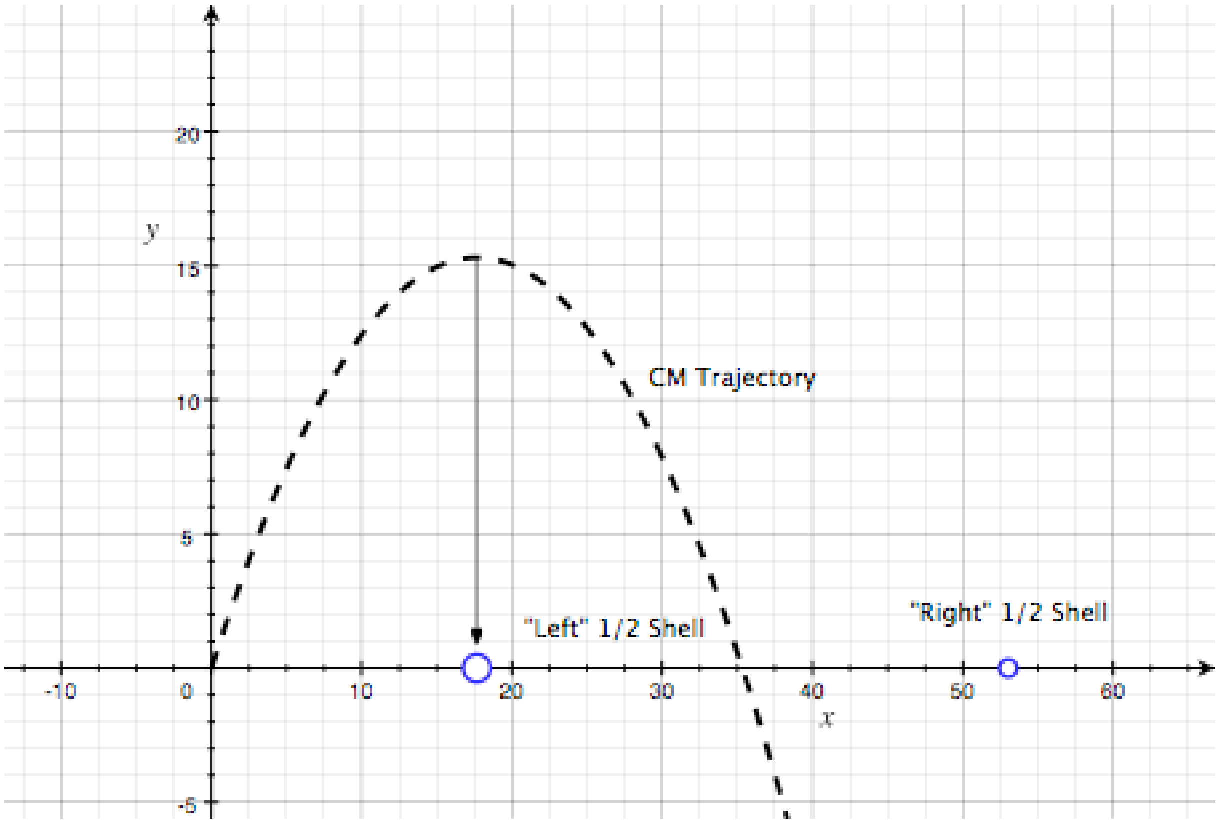
Assume that the 65 kg is at 10 and the 40 kg is at 0m. The cm does not move, so both skaters end up at the cm. The 40 kg skater moves from 0 to the cm.

$$x_{cm} = \frac{40 \cdot 0 + 65 \cdot 10}{105} = 6.19\,m$$

The 40 kg skater moves 6.19 m.

9.15 A shell is fired from a gun with a muzzle velocity of 20m/s at an angle of 60˚. At the top of the trajectory, the shell explodes into two equal mass fragments. One fragment, whose speed immediately after the explosion is zero falls vertically. How far from the gun does the other fragment land, assuming that the terrain is level and that the air drag is negligible.

Let’s sketch what happens first



Examine how long it took to reach the explosion point.

$$v_{fy} = 0$$
$$v_{iy} = 20m/s \cdot \sin 60 = 17.32m/s$$
$$v_{ix} = 20m/s \cdot \cos 60 = 10m/s$$
$$a = -g$$
$$v_{fy} = v_{iy} - gt$$
$$t = \frac{v_{iy}}{g} = 1.767s$$

Next we find the distance traveled before the explosion.

$$x_L = v_{ix}t = 17.67m$$

The "left" half-shell lands at $x_L = 17.67m$. If the shell had not exploded, we know that it would land at $x_{cm} = 2 \cdot 17.67m = 35.34m$ Since the forces involved in the explosion were entirely internal, the center of mass of the two shells still lands at exactly this point. Knowing where the cm is allows us to find where the second half shell will land.

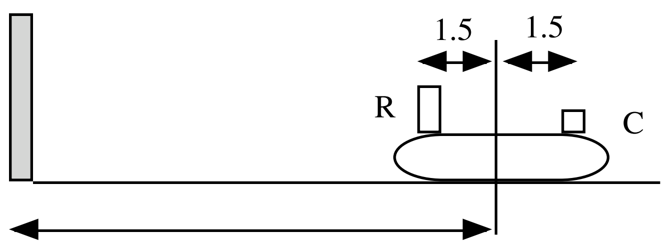
$$x_{cm} = \frac{\frac{1}{2}m \cdot x_L + \frac{1}{2}m \cdot x_R}{m}$$

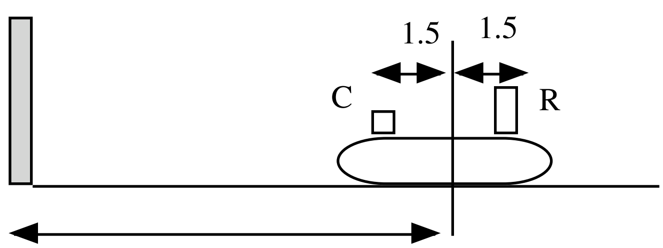
$$x_{cm} = \frac{1}{2}x_L + \frac{1}{2}x_R$$

$$x_R = 2x_{cm} - x_L$$
$$= 2 \cdot 35.34 - 17.67$$
$$= 53.01m$$

9.16 Ricardo, of mass 80kg, and carmelita, who is lighter, are enjoying Lake Merced at dusk in a 30.0 kg canoe. When the canoe is at rest in the placid waqter, they exchange seats, which are 3.0 me apart and symmetrically located with respecte to the canoe's center. If the canoe moves 40cm horizontally relative to a pier post, what is Carmelita's mass.

The picture is absolutely essential for this picture. The picture in the book is actually for problem 17. The correct picture is shown below. The top picture is before Roberto and Carmelita switch places. The bottom picture is after the switch, showing that the boat has moved.





The physics of this problems is that the center of mass cannot move. All of the forces are "internal forces" so the CM of the system must remain in the same place. We can now write the position of the cm before the switch and after the switch, and then set them equal.

$$x_{cm} = \frac{m_R(d-1.5) + m_b d + m_C(d+1.5)}{m_R + m_b + m_R} \quad (before)$$

$$x_{cm} = \frac{m_R(d-0.4+1.5) + m_b(d-0.4) + m_C(d-0.4-1.5)}{m_R + m_b + m_R} \quad (after)$$

$$\frac{m_R(d-1.5)+m_b d+m_C(d+1.5)}{m_R+m_b+m_R}=\frac{m_R(d-0.4+1.5)+m_b(d-0.4)+m_C(d-0.4-1.5)}{m_R+m_b+m_R}$$

$$m_R(d-1.5)+m_b d+m_R(d+1.5)=m_R(d-0.4+1.5)+m_b(d-0.4)+m_C(d-0.4-1.5)$$

$$m_R(-1.5)+m_C(+1.5)=m_R(+1.1)+m_b(-0.4)+m_C(-1.9)$$

$$m_C(+1.5+1.9)=m_R(+1.1+1.5)+m_b(-0.4)$$

$$m_C=\frac{m_R(+1.1+1.5)+m_b(-0.4)}{(+1.5+1.9)}=\frac{80kg(2.6)+30kg(-0.4)}{3.4}$$

$$m_C=57.6kg$$

(provided that I haven't made a typo!)

9.18 A 0.70 kg ball is moving horizontally with a speed of 5.0 m/s when it strikes a vertical wall. The ball rebounds with a speed of 2.0 m/s. What is the magnitude of the change in the linear momentum of the ball

$$\begin{aligned}\Delta\vec{p}&=\vec{p}_f-\vec{p}_i\\&=0.7kg\cdot 5m/s-0.7kg\cdot(-2m/s)\\&=4.9kgm/s\end{aligned}$$

9.26 In February of 1955, a paratrouper fell 370 m from an airplane without being able to open his chute but happened to land in snow, suffering only minor injuries. Assume that his speed at impact was 56 m/s (terminal speed), that his mass (including gear) was 85 kg, and that the magnitude of the force on him from the snow was at the survivable limit of $1.2\times10^5 N$. What are (a) the minimum depth of snow that would have stopped him safely and (b) the magnitude of the impulse.

The snow does work in stopping this very lucky paratrouper. The work that the snow does is equal to the change in kinetic energy

$$\begin{aligned}\Delta KE&=KE_f-KE_i\\&=0-\frac{1}{2}mv_i^2\\&=-1.333\times10^5\end{aligned}$$

Since we know the force, we can write the work done by the snow in terms of the depth.

$$\Delta KE=W$$

$$\Delta KE=Fd\cos180°=-Fd$$

$$\begin{aligned}d&=-\frac{\Delta KE}{F}=-\frac{-1.333\times10^5 J}{1.2\times10^5 N}\\&=1.111m\end{aligned}$$

The impulse is the change in momentum.

$$\begin{aligned}\Delta\vec{p} &= \vec{p}_f - \vec{p}_i \\ &= 0 - 85kg \cdot -56m/s \\ &= 4760kgm/s\end{aligned}$$

9.33

9.36

9.39 A 91 kg man lying on a surface of negligible friction shoves a 68 g stone away from himself, giving it a speed of 4 m/s. What speed does he acquire as a result?

This is a momentum conservation problem. The total momentum is zero (man and stone at rest at the beginning). The total momentum must remain zero. In the final state

$$\begin{aligned}\vec{p}_f &= 0 = \vec{p}_m + \vec{p}_s \\ \vec{p}_m &= -\vec{p}_s \\ m_m\vec{v}_m &= -m_s\vec{v}_s \\ \vec{v}_m &= -\frac{m_s}{m_m}\vec{v}_s \\ &= -\frac{0.068kg}{91.0kg}\cdot 4m/s \\ &= 0.002989m/s\end{aligned}$$

9.49 A bullet of mass 10 g strikes a ballistic pendulum of mass 2.0 kg. The center of mass of the pendulum rises a vertical distance of 12 cm. Assuming that the bullet remains embedded in the pendulum, calculate the bullet's initial speed.

This is both a momentum conservation problem and an energy conservation problem. The energy conservation part is the motion of the pendulum. The total energy of the pendulum at the top of the swing is equal to the total energy of the pendulum after it has been struck by a bullet.

$$E_f = (M+m)g\,y$$
$$E_i = \frac{1}{2}(M+m)v^2$$
$$E_i = E_f$$
$$\frac{1}{2}(M+m)v^2 = (M+m)g\,y$$
$$v = \sqrt{2g\,y}$$
$$= \sqrt{2\cdot 9.8\cdot 0.12}$$
$$= 1.534\ m/s$$

Now that we know the velocity of the bullet-pendulum immediately after the bullet strikes the pendulum, we can use momentum conservation to find the bullet's velocity

$$\vec{p}_i = \vec{p}_f$$
$$mv_b = (M+m)v$$
$$v_b = \frac{(M+m)}{m}v$$
$$= \frac{(2.0kg + 0.01kg)}{0.01kg}\cdot 1.534m/s$$
$$= 308.3m/s$$

9.50

9.54 In the figure, a 10 g bullet moving directly upward at 1000m/s strikes and passes through the center of mass of a 5.00 kg block initially at rest. The bullet emerges from he block moving directly upward at 400 m/s. Tow what maximum height does the block then rise above its initial position.

As in the ballistic pendulum case, this problem uses both momentum conservation and energy conservation. We begin with momentum conservation in the collision.

$$\vec{p}_i = m_b\vec{v}_b + M_B\vec{v}_B$$
$$\vec{p}_i = 10.0\times 10^{-3}kg\cdot 1000m/s + 5.0\,kg\cdot 0m/s$$
$$= 10\,kgm/s$$

$$\vec{p}_f = m_b\vec{v}_b' + M_B\vec{v}_B'$$
$$= m_b\vec{v}_b' + M_B\vec{v}_B'$$
$$= 10\times 10^{-3}kg\cdot 400m/s + 2.0\,kg\cdot \vec{v}_B'$$

$$\vec{p}_f = \vec{p}_i$$
$$10\times 10^{-3}kg\cdot 400m/s + 2.0kg\cdot \vec{v}_B' = 10.0\,kg\,m/s$$
$$\vec{v}_B' = \frac{10.0\,kg\,m/s - 10.0\times 10^{-3}kg\cdot 400m/s}{2.0kg}$$
$$= 3m/s$$

Now we know the velocity of the block after the collision. We can use conservation of energy to find the maximum height.

$$\frac{1}{2}Mv^2 = Mgh$$

$$h = \frac{v^2}{2g} = \frac{(3m/s)^2}{2 \cdot 9.8m/s^2}$$

$$= 0.459m$$

Note that the conservation of energy is for the block, changing kinetic into potential. The collision itself is not energy conserving.

9.61.

9.78