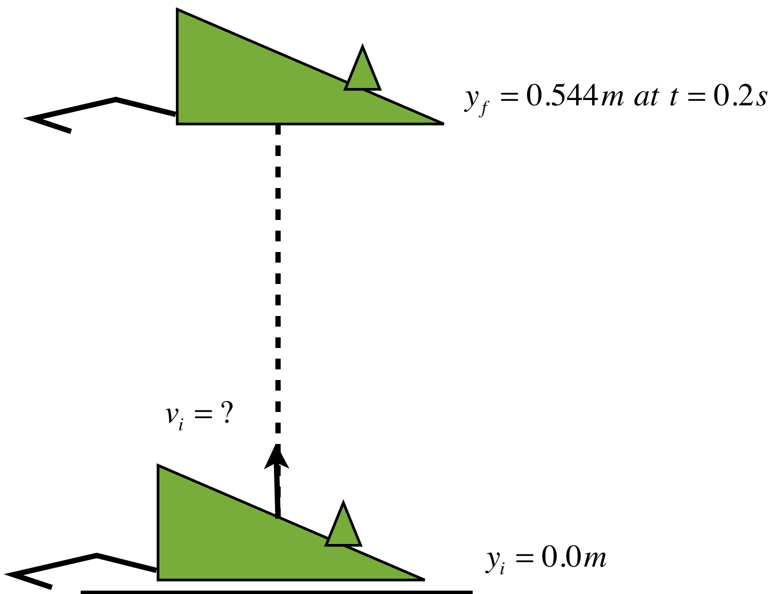
Chapter 2 Problems

2.44 When startled, and armadillo will leap upward. Suppose it rised 0.544 m in the first 0.2 s. (a) What is is initial speed as it leaves the ground? (b) What is its speed at the height of 0.544 m? (c) How much higher does it go?



First we consider the armadillo's jump to 0.544m. We can compute his initial velocity

$$\begin{aligned} y_i &= 0m \\ y_f &= 0.544m \\ v_i &= ? \\ v_f &= ?m/s \\ t &= 0.2s \\ a &= -g \end{aligned}$$

$$y_f = y_i + v_i t + \frac{1}{2} a t^2$$

$$y_f = 0 + v_i t - \frac{1}{2} g t^2$$

$$v_i = \frac{y_f + \frac{1}{2} g t^2}{t} = 3.7m/s$$

Now that we know the initial velocity, we can find his velocity at 0.544 m.

$$\begin{aligned} v_f &= v_i + at \\ v_f &= v_i - gt \\ &= 1.74m/s \end{aligned}$$

Two compute how much higher he goes, we can compute the maximum height he reaches We can do this because we know that he stops at the top.

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$
$$0 = v_i^2 - 2g(y_f - 0)$$
$$y_f = \frac{v_i^2}{2g} = 0.698$$
$$\Delta y = 0.698 - 0.544 = 0.154m$$

He rises an additional 0.154m

2.45 (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50m. (b) How long will it be in the air. Sketch y, v, a, vs t.

$$y_i = 0m$$
$$y_f = 50m$$
$$v_i = ?$$
$$v_f = 0m/s$$
$$t = ?$$
$$a = -g$$

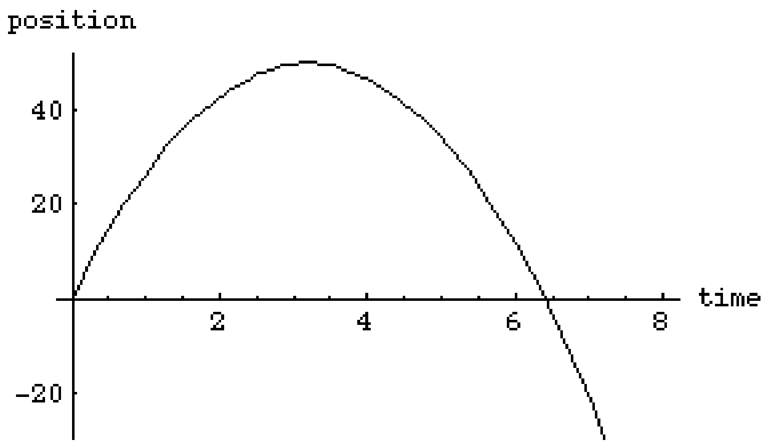
(a). We compute the initial velocity first.

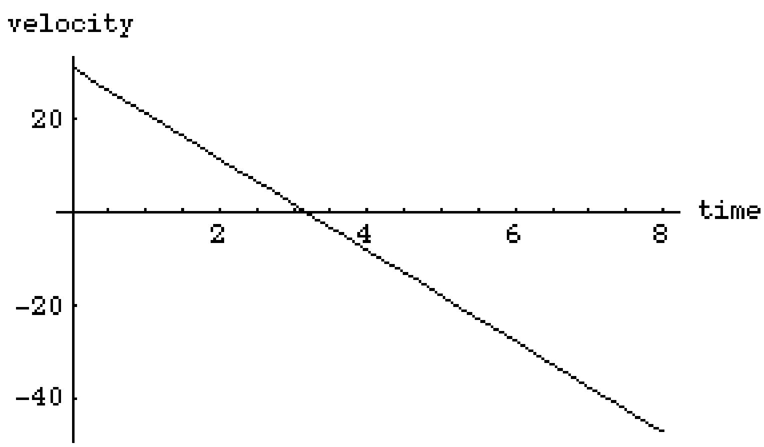
$$v_f^{\,2} = v_i^{\,2} + 2a(y_f - y_i)$$
$$0 = v_i^{\,2} - 2g(y_f - 0)$$
$$v_i = \sqrt{2g(50m)} = 31.3m/s$$

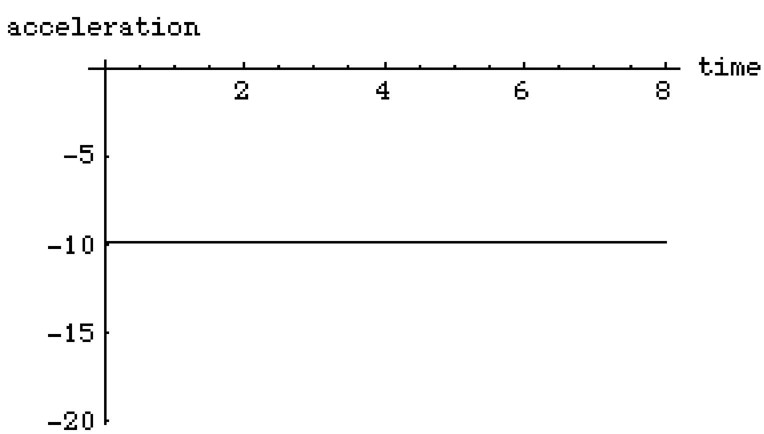
(b) Now that we know initial velocity, we can find the time to reach the highest point.

$$v_f = v_i + at$$
$$0 = v_i - gt$$
$$t = \frac{v_i}{g} = \frac{31.3m/s}{9.8m/s^2} = 3.19s$$

The path is symmetric, so the entire time of flight is 6.38s.







2.46 Raindrops fall to Earth from a cloud 1700m above the Earth's surface. If they were not slowed by air resistance, how fast would the drops be moving when they struck the ground? Would it be safe to walk outside during a rainstorm?

$$v_f{}^2 = v_i^2 + 2a(y_f - y_i)$$
$$v_f{}^2 = 0^2 - 2\cdot 9.8m/s\cdot(-1700m)$$
$$v_f = 182.5m/s!$$

It's not safe.

2.48 A hoodlum throws a stone vertically downward with an initial speed of 12m/s from the roof of a building, 30 m above the ground. How long does it take the stone to reach the ground? (b) What is the speed of the stone at impact.

$$y_i = 30m$$
$$y_f = 0m$$
$$v_i = -12m/s$$
$$v_f = ?$$
$$t = ?$$
$$a = -g$$

(a). We compute the time. We use the solution to a quadratic equation and choose the positive sign to get the positive time.

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$
$$0 = 30 - 12t - \frac{1}{2}\cdot 9.8\cdot t^2$$
$$t = \frac{12 \pm \sqrt{12^2 - 4(\frac{-9.8}{2})(30)}}{2\cdot(\frac{-9.8}{2})}$$
$$t = 1.54s$$

(b) Now that we know the time, we can find the velocity. The direction is downward and the speed would be the absolute value of the velocity for this one dimensional problem.

$$v_f = v_i + at$$
$$= -12m/s - 9.8m/s^2\cdot 1.54s$$
$$= -27.1m/s$$

2.51

2.69 How far does the runner whose velocity-time graph is shown in Fig 2036 travel in 16 s? The figure's vertical scaling is et by v_s=8.0m/s

The runner's displacement is the area under the curve. We can compute the area geometrically

$$A = \frac{1}{2} \cdot 2s \cdot 8m/s + 8s \cdot 8m/s + \frac{1}{2} \cdot 2s \cdot 4m/s + 2s \cdot 4m/s + 4s \cdot 4m/s = 100m$$

Chapter 3 Problems

3.1 What are (a) the x component and (b) the y component of a vector a in the xy plane if its directions is 250 degrees counterclockwise from the positive directions of the x axis nd its magnitude is 7.3m.

(a) The x component is

$$a_x = 7.3 \cdot \cos 250° = -2.497$$

(b) The y component is

$$a_y = 7.3 \cdot \sin 250° = -6.86$$

3.3 The x component of a certain vector is -25.0 units and the y component is +40 units. (a) What is the magnitude and direction of the vector? (b) What is the angle between n the direction of the vector and the positive direction of x.

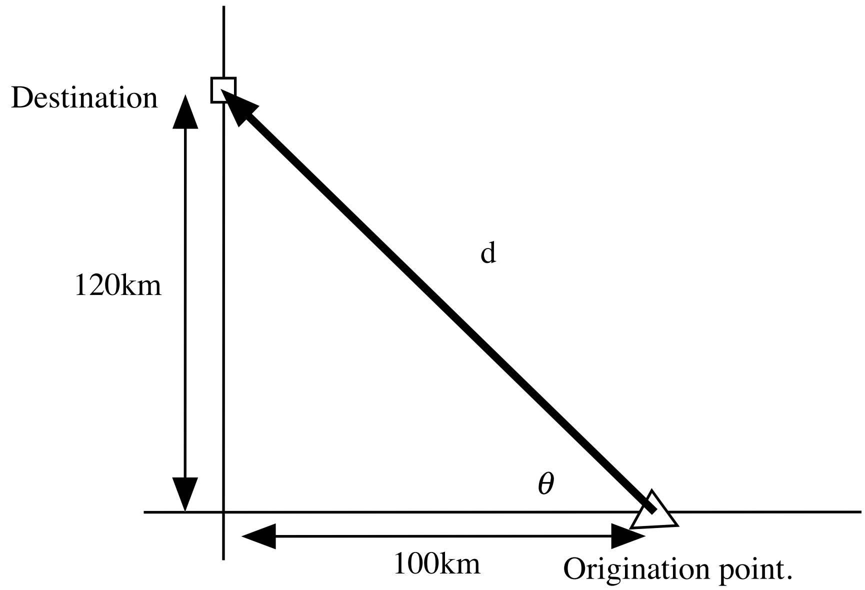
(a). The magnitude is

$$|\vec{v}| = \sqrt{(-25)^2 + (40)^2} = 47.17\, units$$

(b) The direction with respect to +x is:

$$\tan\theta = \frac{40}{-25}$$
$$\theta = 122°$$

3.5 A ship sets out to sail to a point 120 km due north. An unexpected storm blows the ship to a point 100km due east of its starting point. (a) How far and (b) in what direction must it now sail to reach its original destination.



The distance and angle that we need to sail is

$$d = \sqrt{(100km)^2 + (120km)^2} = 156.2m$$

$$\tan\theta = \frac{120}{100} \Rightarrow \theta = \tan^{-1}(\frac{120}{100})$$
$$\theta = 50.19° N\ of\ W$$

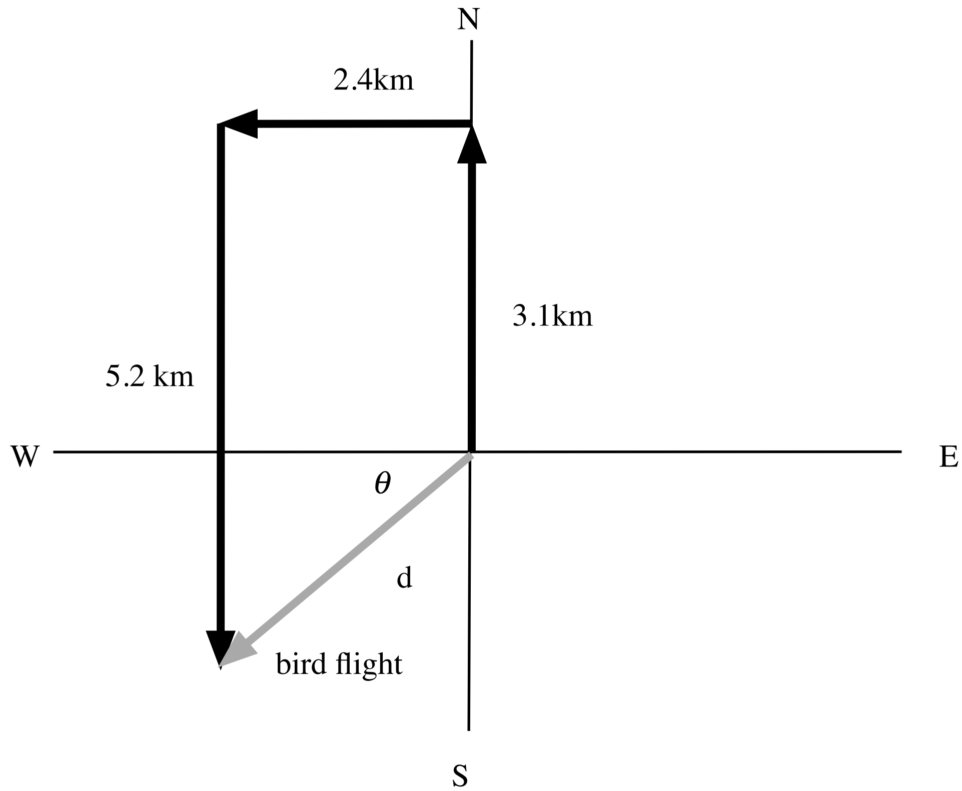
Note: The angle that I have calculated is the complement of the angle in answer section.

3.8 A person walks in the following pattern: 3.1 km north, then 2.4 km west, and finally 5.2 km south. (a) Sketch the vector diagram that represents theis motioin. (b) Howw far and (c) in what direction would a bird fly in a straight line from the same starting point to the same final point.

We can see from the picture that we can find the bird's flight by adding the vectors together. This is most easily done using vector notation.

$$\vec{d}_1 = 0\ \hat{i} + 3.1\ \hat{j}$$
$$\vec{d}_2 = -2.4\ \hat{i} + 0\ \hat{j}$$
$$\vec{d}_3 = 0\ \hat{i} + -5.2\ \hat{j}$$
$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$$
$$d = -2.4\ \hat{i} - 2.1\ \hat{j}$$

$$d = \sqrt{2.4^2 + 2.1^2} = 3.19m$$
$$\theta = \tan^{-1}\left(\frac{2.1}{2.4}\right)$$
$$= 41.12^\circ\ S\ of\ W$$



3.9 Two vectors are given by

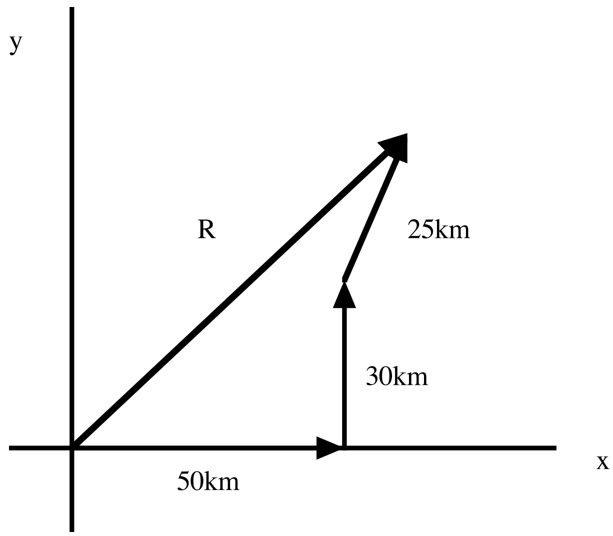
$$\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$$
$$\vec{b} = -\hat{i} + \hat{j} + 4\hat{k}$$

Find $\vec{a} + \vec{b}$, $\vec{a} - \vec{b}$ and a vector $\vec{c}$ such that $\vec{a} - \vec{b} + \vec{c} = 0$

$$\vec{a} + \vec{b} = 3\hat{i} - 2\hat{j} + 5\hat{k}$$
$$\vec{a} - \vec{b} = 5\hat{i} - 4\hat{j} - 3\hat{k}$$

$$
\begin{aligned}
0 &= \vec{a} - \vec{b} + \vec{c} \\
\vec{c} &= -(\vec{a} - \vec{b}) \\
&= -(5\hat{i} - 4\hat{j} - 3\hat{k}) \\
&= -5\hat{i} + 4\hat{j} + 3\hat{k}
\end{aligned}
$$

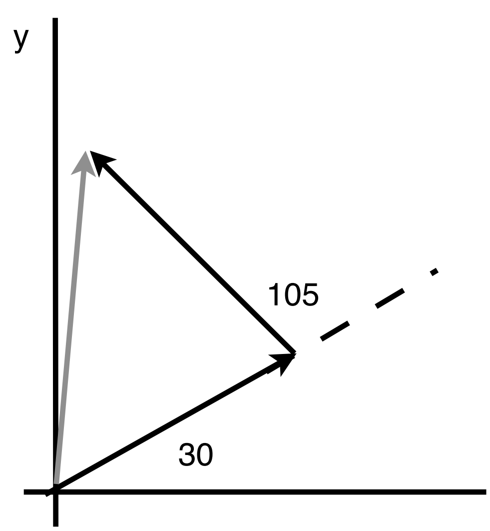
3.12 A car is driven east for a distance of 50 km, then north for 30 km, and then in a directions 30 degrees east of north for 25 km. Sketch the vector diagram and determine (a) the magnitude and (b)the angle of the car's total displacement from its starting point.



(a) We begin by writing the three vectors in component form and proceed to add them and compute the sum in component, magnitude and direction form.

$$
\begin{aligned}
&\vec{a} = 50km\,\hat{i} + 0\,\hat{j} \\
&\vec{b} = 0\,\hat{i} + 30\,km\,\hat{j} \\
&\vec{c} = 25\sin 30°km\,\hat{i} + 25\cos 30°km\,\hat{j} \\
&\vec{R} = (50 + 25\sin 30°)km\,\hat{i} + (30 + 25\cos 30°)km\,\hat{j} \\
&\vec{R} = 62.5\,km\,\hat{i} + 51.65\,km\,\hat{j} \\
&\left|\vec{R}\right| = \sqrt{62.5^2 + 51.65^2} = 81.08\,km \\
&\tan\theta = \frac{51.65}{62.5} \\
&\theta = \tan^{-1}(\frac{51.65}{62.5}) = 39.57°
\end{aligned}
$$

3.15 The two vectors $\vec{a}$ and $\vec{b}$ in Fig 3-30 have equal magnitudes of 10.0 m and the angles are $\theta_1 = 30^\circ$ and $\theta_2 = 105^\circ$. Find the a) x and (b) y components of their vector sum $\vec{r}$ (c) the magnitude of $\vec{r}$, and (d) the angle $\vec{r}$ makes with the positive direction of the x axis.



We need to write out the two vectors in component notation. To do this, we need to remember that we need to write each angle with respect to the positive x axis

$$\vec{a} = 10\cos 30\,\hat{i} + 10\sin 30\,\hat{j} = 8.66\,\hat{i} + 5\,\hat{j}$$
$$\vec{b} = 10\cos 135\,\hat{i} + 10\sin 135\,\hat{j} = -7.07\,\hat{i} + 7.07\,\hat{j}$$
$$\vec{r} = \vec{a} + \vec{b} = (8.66 - 7.07)\,\hat{i} + (5 + 7.07)\,\hat{j} = 1.59\,\hat{i} + 12.07\,\hat{j}$$
$$|\vec{r}| = \sqrt{(1.59)^2 + (12.07)^2} = 12.17$$
$$\theta = \tan^{-1}\left(\frac{12.07}{1.59}\right) = 82.5^\circ$$

3.18

3.22 What is the sum of the following four vectors in unit-vector notation? For that sum, what are (b) the magnitude, (c) the angle in degrees, and (d) the angle in radians?

$$\vec{E} : 6.00\,m\ at\ +0.900\,rad \qquad \vec{F} : 5.00\,m\ at\ +-75.0^\circ$$
$$\vec{G} : 4.00\,m\ at\ +1.200\,rad \qquad \vec{H} : 6.00\,m\ at\ -210.0^\circ$$

We can write out each of these vectors in vector notation.

$$\vec{E}: 6.00\,m\ at\ +0.900\,rad \Rightarrow \vec{E} = 6\cos(0.9rad)\,\hat{i} + 6\sin(0.9rad)\,\hat{j} = 3.73\ \hat{i} + 4.70\ \hat{j}$$
$$\vec{G}: 4.00\,m\ at\ +1.200\,rad \Rightarrow \vec{G} = 4\cos(1.2rad)\,\hat{i} + 4\sin(1.2rad)\,\hat{j} = 1.45\ \hat{i} + 3.73\ \hat{j}$$
$$\vec{F}: 5.00\,m\ at\ +-75.0° \Rightarrow \vec{F} = 5\cos(-75.0°)\,\hat{i} + 5\sin(-75.0°)\,\hat{j} = 1.29\ \hat{i} + -4.83\ \hat{j}$$
$$\vec{H}: 6.00\,m\ at\ -210.0° \Rightarrow \vec{H} = 6\cos(-210.0°)\,\hat{i} + 6\sin(-210.0°)\,\hat{j} = -5.19\ \hat{i} + 3.00\ \hat{j}$$

$$\begin{aligned}
\vec{R} &= (3.73 + 1.45 + 1.29 - 5.19)\hat{i} + (4.70 + 3.73 - 4.83 + 3.00)\hat{j} \\
&= 1.28\hat{i} + 6.60\hat{j} \\
\left|\vec{R}\right| &= \sqrt{1.28^2 + 6.60^2} = 6.73m \\
\theta &= \tan^{-1}\left(\frac{6.60}{1.28}\right) = 79.02° \\
&= 1.379\,rad
\end{aligned}$$

3.23 If $\vec{B}$ is added to $\vec{C} = 3\,\hat{i} + 4\,\hat{j}$, the result is a vector in the positive direction of y axis with a magnitude equal to that of $\vec{C}$. What is the magnitude of $\vec{B}$.

We know that the magnitude of resultant vector is 5 because we are told that it has the magnitude of $\vec{C}$, which is 5. We are also told that the resultant is in the y direction. So

$$\begin{aligned}
\vec{B} &= B_x\,\hat{i} + B_y\,\hat{j} \\
\vec{C} &= 3\,\hat{i} + 4\,\hat{j} \\
\vec{R} &= 0\,\hat{i} + 5\,\hat{j}
\end{aligned}$$

We can see that to get this resultant, $\vec{B} = -\,3\hat{i} + 1\,\hat{j}$. The magnitude of B is 3.2

3.26 What is the sum of the following four vectors in (a) unit-vector notation and as (b) magnitude and (c) angle.

$$\vec{A} = 2.00\hat{i} + 3.00\hat{j} \qquad \vec{B}: 4.00\ at 65°$$
$$\vec{C} = -4.00\hat{i} - 6.00\hat{j} \qquad \vec{D}: 5.00\,at - 235°$$

$$\vec{B} = 4\cos 65\hat{i} + 4\sin 65\hat{j} = 1.69i + 3.63j$$
$$\vec{D} = 5\cos(-235)\hat{i} + 5\sin(-235)\hat{j} = -2.87\hat{i} + 4.096\hat{j}$$
$$\vec{A} + \vec{B} + \vec{C} + \vec{D} = -3.18\hat{i} + 4.73\hat{j}$$
$$\vec{A} + \vec{B} + \vec{C} + \vec{D}\text{: } \sqrt{3.18^2 + 4.73^2} = 5.7m$$
$$\theta = \tan^{-1}\left(\frac{4.73}{-3.18}\right) = 123.9°$$

3.33 For the vectors in Fig. 3-32, with a=4, b=3, and c=5, find the magnitude and direction of (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \times \vec{c}$, and (c) $\vec{b} \times \vec{c}$.

In class and in lab, I solved this problem by moving vectors and using the right hand rule.

Another way of solving this problem is to write the three vectors in unit vector notation.

$$\vec{a} = 4.0\hat{i} + 0.0\hat{j}$$
$$\vec{b} = 0.0\hat{i} + 3.0\hat{j}$$
$$\vec{c} = -4.0\hat{i} + -3.0\hat{j}$$

Notice that we can write the components of the c vector since we can see the lengths of the sides of the triangle that it is the hypotenuse of.... but we need to remember that the directions of both the x and y components are negative.

Now we can compute the three cross products

$$\vec{a} \times \vec{b} = \det\left\{ \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 4.0 & 0.0 & 0.0 \\ 0.0 & 3.0 & 0.0 \end{array} \right\} = 12.0\hat{k}$$

$$\vec{a} \times \vec{c} = \det\left\{ \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 4.0 & 0.0 & 0.0 \\ -4.0 & -3.0 & 0.0 \end{array} \right\} = -12.0\hat{k}$$

$$\vec{b} \times \vec{c} = \det\left\{ \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 0.0 & 3.0 & 0.0 \\ -4.0 & -3.0 & 0.0 \end{array} \right\} = +12.0\hat{k}$$

3.34 Two vectors are given by $\vec{a} = 3.0\hat{i} + 5.0\hat{j}$ and $\vec{b} = 2.0\hat{i} + 4.0\hat{j}$ Find (a) , $\vec{a}\times\vec{b}$ (b) $\vec{a}\cdot\vec{b}$, and (c) $(\vec{a}+\vec{b})\cdot\vec{b}$ and (d) The component of $\vec{a}$ along $\vec{b}$,

(a) First we find the cross product by finding the determinant.

$$\vec{a}\times\vec{b} = \det\begin{Bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.0 & 5.0 & 0.0 \\ 2.0 & 4.0 & 0.0 \end{Bmatrix}$$

$$= \hat{i}(5\cdot 0 - 0\cdot 4) - \hat{j}(3\cdot 0 - 0\cdot 2) + \hat{k}(3\cdot 4 - 5\cdot 2)$$

$$= 2\hat{k}$$

(b) The dot-product.

$$\vec{a}\cdot\vec{b} = (3\cdot 2) + (5\cdot 4) = 26$$

(c) The combination of summing and dot product.

$$(\vec{a}+\vec{b})\cdot\vec{b} = ((3.0\hat{i} + 5.0\hat{j}) + (2.0\hat{i} + 4.0\hat{j}))\cdot(2.0\hat{i} + 4.0\hat{j})$$

$$= (5.0\hat{i} + 9.0\hat{j})\cdot(2.0\hat{i} + 4.0\hat{j})$$

$$= (5.0\cdot 2.0 + 9.0\cdot 4.0)$$

$$= 46$$

(d) We can find the component of $\vec{a}$ along $\vec{b}$,

$$\textit{component of } a \textit{ along } b = |\vec{a}|\cos\theta$$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$|\vec{a}|\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

$$= \frac{26}{\sqrt{2^2+4^2}}$$

$$= 5.82$$

3.37

3.38 For the following three vectors, what is $3\vec{C}\cdot(2\vec{A}\times\vec{B})$

$$\vec{A} = 2.00\hat{i} + 3.00\hat{j} - 4.00\hat{k}$$

$$\vec{B} = -3.00\hat{i} + 4.00\hat{j} + 2.00\hat{k}$$

$$\vec{C} = 7.00\hat{i} - 8.00\hat{j}$$

$$2\vec{A} = 4.00\,\hat{i} + 6.00\,\hat{j} - 8.00\,\hat{k}$$

$$3\vec{C} = 21.00\,\hat{i} - 24.00\,\hat{j}$$

$$2\vec{A}\times\vec{B} = \det\begin{Bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & -8 \\ -3 & 4 & 2 \end{Bmatrix}$$

$$= \hat{i}\,(6\cdot 2-(-8)\cdot 4) - \hat{j}\,(4\cdot 2-(-8)\cdot(-3)) + \hat{k}\,(4\cdot 4-6\cdot(-3))$$

$$= 44\,\hat{i} + 16\,\hat{j} + 34\,\hat{k}$$

$$3\vec{C}\cdot(2\vec{A}\times\vec{B}) = (21)\cdot 44 + (-24)\cdot 16 = 54$$