Chapter 11

11.1 An automobile traveling at 80.0 km/h has tires of 75.0 cm diameter. (a) What is the angular speed of the tires aobut their axles? (b) if the car is brought to a stop uniformly in 30.0 complete turns of the tires (without skidding), what is the magniude of the angular accleration for the wheels? (c) How far does the car move during the braking?

$$v = \frac{80km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = 22.22m/s$$

$$\omega = \frac{v}{r} = \frac{22.22}{(0.75m/2)} = 59.26 rad/s$$

$$x = 30rev \cdot \frac{2\pi r}{1rev} = 70.69m$$

$$\theta_i = 0 rad$$

$$\theta_f = 30rev \cdot \frac{2\pi rad}{1rev} = 188.5 rad$$

$$\omega_i = 59.26 rad/s$$

$$\omega_f = 0 rad/s$$

$$\alpha = ?$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\theta_f - \theta_i)} = \frac{0^2 - 59.26^2}{2(188.5 - 0)}$$

$$= -9.31 rad/s^2$$

11.7 A solid cylinder of radius 10cm and mass 12kg starts from rest and rolls without slipping a distance of 6m down a house roof that is inclined at 30 degrees (a) What is the angular speed of the cylinder about its center as it leaves the house roof? (b) The roof's edge is 5.0m. How far horizontally from the roof's edge does the cylinder hit the level ground.

This problem is very similar to the previous problem. We begin with energy conservation. Take the ground as zero height.

$$H = 5m + 6m\sin 30 = 8m$$

$$E_i = mgH$$

$$h = 5m$$

$$E_f = mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$E_f = E_i$$

$$mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$$

$$mgh + \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}mR^2)\frac{v^2}{R^2} = mgH$$

$$\frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}m)v^2 = mg(H-h)$$

$$\frac{1}{2}v^2 + \frac{1}{4}v^2 = g(H-h)$$

$$\frac{3}{4}v^2 = g(H-h)$$

$$v = \sqrt{\frac{4}{3}g(H-h)} = \sqrt{\frac{4}{3}\cdot 9.8m/s^2(8m-5m)}$$

$$= 6.26m/s$$

$$\omega = \frac{v}{r} = \frac{6.26m/s}{0.1m} = 62.6rad/s$$

This problem differs from the previous problem in that the cylinder is not launched horizontally. It has an initial velocity in the y direction as well as in the x direction...

$$x_i = 0$$
$$x_f = ?$$
$$v_{ix} = 6.26\cos 30 = 5.42m/s$$
$$v_{fx} = v_{ix}$$
$$a_x = 0$$

$$y_i = h$$
$$y_f = 0$$
$$v_{iy} = -6.26\sin 30 = -3.13m/s$$
$$v_{fy} = ?$$
$$a_y = -g$$
$$t = ?$$

Again we find the time

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$

$$0 = h + v_{iy}t - \frac{1}{2}g t^2$$

$$t = \frac{-v_{iy} \pm \sqrt{v_{iy}{}^2 - 4\cdot(-\frac{1}{2}g)\cdot h}}{2\cdot(-\frac{1}{2}g)} = \frac{-(-3.13) \pm \sqrt{(-3.13)^2 - 4\cdot(-\frac{1}{2}\cdot 9.8)\cdot 5}}{2\cdot(-\frac{1}{2}\cdot 9.8)}$$

$$= 0.74s$$

Again we find the horizontal distance

$$
\begin{aligned}
x_f &= x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\
&= 0 + v_{ix}t + 0 \\
x_f &= 5.42m/s \cdot 0.74s \\
&= 4.01m
\end{aligned}
$$

This problem is considerably more complicated because the projectile is not launched horizontally.

11.9 A solid ball starts from rest at the upper end of the track shown in Fig 12-31 and rolls without slipping until it rolls off the right hand end. If H=6.0m and h = 2.0 m and the track is horizontal at the right-hand end, how far horizontally from point A does the ball land on the floor.

If we know the linear velocity the ball has when it leaves the track, we can compute where it will land (as a projectile problem). First we need to find that velocity. We use energy conservation.

$$E_i = mgH \qquad E_f = mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$E_f = E_i$$

$$mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$$

$$mgh + \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}mR^2)\frac{v^2}{R^2} = mgH$$

$$\frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}m)v^2 = mg(H-h)$$

$$\frac{1}{2}v^2 + \frac{1}{5}v^2 = g(H-h)$$

$$\frac{7}{10}v^2 = g(H-h)$$

$$
\begin{aligned}
v &= \sqrt{\frac{10}{7}g(H-h)} = \sqrt{\frac{10}{7}\cdot 9.8m/s^2(6m-2m)} \\
&= 7.48m/s
\end{aligned}
$$

Now we have a projectile problem where the initial velocity is 7.48 m/s in the horizontal direction

$$
\begin{aligned}
x_i &= 0 \\
x_f &= ? \\
v_{ix} &= 7.48m/s \\
v_{fx} &= v_{ix} \\
a_x &= 0
\end{aligned}
\qquad
\begin{aligned}
y_i &= h \\
y_f &= 0 \\
v_{iy} &= 0 \\
v_{fy} &= ? \\
a_y &= -g \\
t &= ?
\end{aligned}
$$

First find the time to fall.

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$
$$0 = h + 0 - \frac{1}{2}gt^2$$
$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 2m}{9.8m/s^2}} = 0.6389s$$

Now find the horizontal distance traveled in this time.

$$\begin{aligned} x_f &= x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ &= 0 + v_{ix}t + 0 \\ x_f &= 7.48m/s \cdot 0.6389s \\ &= 4.78m \end{aligned}$$

11.18 **I**n unit-vecotr notation, what is the torque about the origin on a particle located at coordinates (0, -4.0, 3.0) if tha ttorque i due to (a) a force $\vec{F}_1$ with components $F_{1x} = 2.0\,N,\ F_{1y} = F_{1z} = 0$ (b) (a) a force $\vec{F}_2$ with components $F_{2x} = 0$, $F_{2y} = 2.0\,N,\ F_{1z} = 4.0N$

For these problems, we need to calculate $\vec{r}$. We recall that $\vec{r}$ can be calculated using "endpoint minus starting point" Here the starting point is the origin, and the endpoint is the location of the particle.

$$\begin{aligned} \vec{r} &= (0-0)\ \hat{i} + (-4.0-0)\ \hat{j} + (3.0-0.0)\ \hat{k} \\ &= -4.0\ \hat{j} + 3.0\ \hat{k} \end{aligned}$$

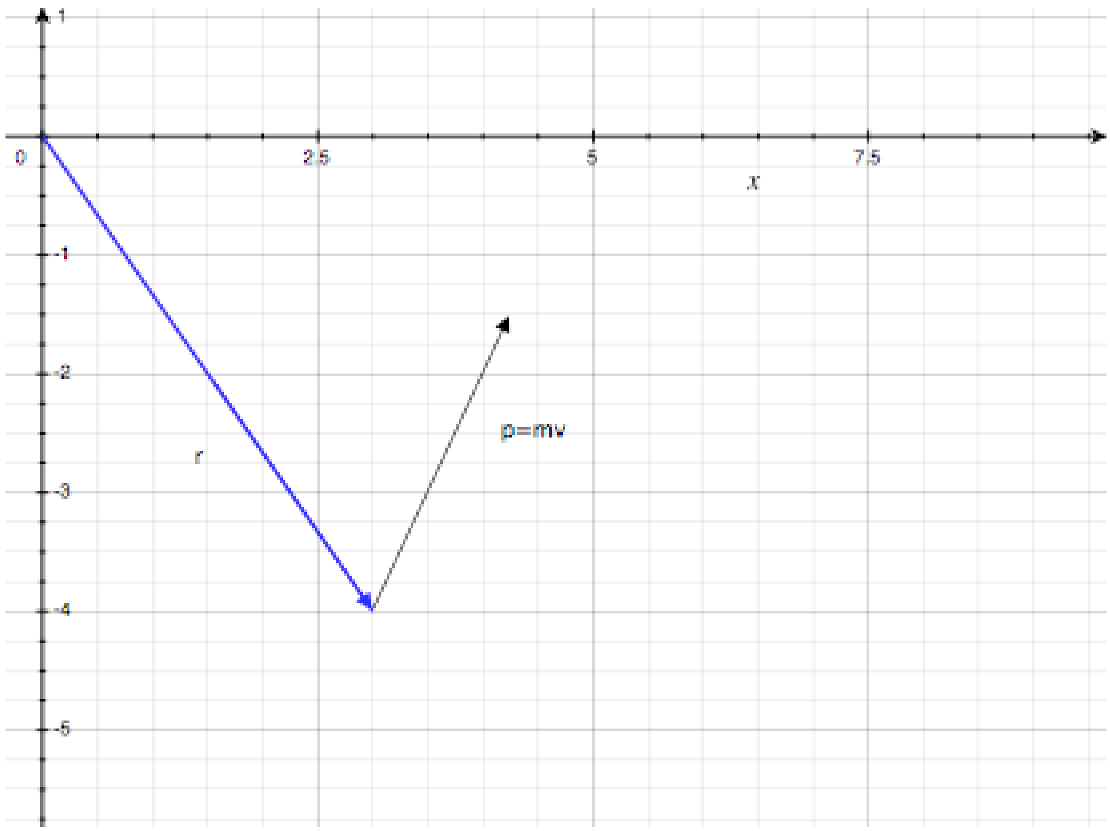
Now consider each force.

$$\begin{aligned} \vec{\tau} &= \vec{r} \times \vec{F}_1 \\ &= \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.0 & -4.0 & 3.0 \\ 2.0 & 0.0 & 0.0 \end{bmatrix} \\ &= \hat{i}\,(-4.0 \cdot 0.0 - 3.0 \cdot 0.0) \\ &\quad - \hat{j}(0.0 \cdot 0.0 - 3.0 \cdot 2.0) \\ &\quad + \hat{k}(0.0 \cdot 0.0 - (-4.0) \cdot 2.0) \\ &= 0.0\hat{i} + 6.0\,\hat{j} + 8.0\ \hat{k} \end{aligned}$$

$$\begin{aligned} \vec{\tau} &= \vec{r} \times \vec{F}_2 \\ &= \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.0 & -4.0 & 3.0 \\ 0.0 & 2.0 & 4.0 \end{bmatrix} \\ &= \hat{i}\,(-4.0 \cdot 4.0 - 3.0 \cdot 2.0) \\ &\quad - \hat{j}(0.0 \cdot 4.0 - 3.0 \cdot 0.0) \\ &\quad + \hat{k}(0.0 \cdot 2.0 - (-4.0) \cdot 0.0) \\ &= -22.0\hat{i} + 0.0\,\hat{j} + 0.0\ \hat{k} \end{aligned}$$

11.26 A 2.0 kg particle-like object moves in a plane with a velocity components $v_x = 30m/s$ and $v_y = 60m/s$ as it passes through the point with (x,y) coordinates of (3.0, -4.0) m. Jus then, in unit-vector notatin, what is the angular momentum and (b) the point (-2.0, -2.0) m?

We can draw what is happening in each case. The r is defined as coming from the axis and ending where the force is applied. In the first case

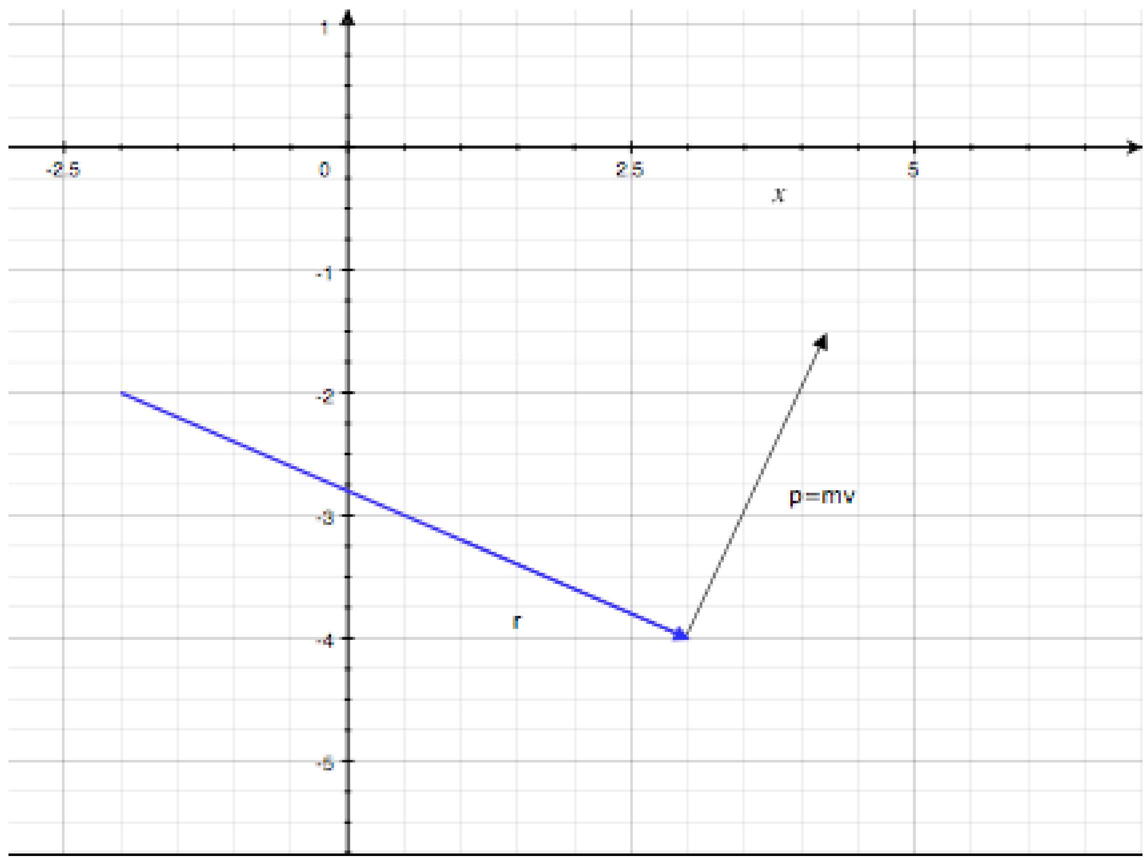


$$\vec{r} = (3.0 - 0.0)\hat{i} + (-4.0 - 0.0)\hat{j} = 3.0m\,\hat{i} - 4.0m\,\hat{j}$$

$$\vec{p} = m\vec{v} = 60kg\,m/s\,\hat{i} + 120kg\,m/s\,\hat{j}$$

$$L = \vec{r} \times \vec{p} = \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 0 \\ 60 & 120 & 0 \end{bmatrix} = (3 \cdot 120 - (-4) \cdot 60)\hat{k} = 600kgm^2/s\,\hat{k}$$

In the second case, the axis is at (-2,2), so we need to recalculate r.



$$\vec{r} = (3.0-(-2.0))\hat{i} + (-4.0-(-2.0))\hat{j} = 5.0m\ \hat{i} - 2.0m\ \hat{j}$$

$$\vec{p} = m\vec{v} = 60kg\,m/s\ \hat{i} + 120kg\,m/s\ \hat{j}$$

$$L = \vec{r}\times\vec{p} = \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -2 & 0 \\ 60 & 120 & 0 \end{bmatrix} = (5\cdot 120 - (-2)\cdot 60)\hat{k} = 720\,kgm^2/s\,\hat{k}$$

11.31 A 3.0 kg particle with velocity $\vec{v} = (5.0m/s)\hat{i} - (6.0m/s)\hat{j}$ is at x=3.0m , y=8.0m It is pulled by a 7.0 N force in the -x direction. About the origin, what are (a) the particles angular momentum (b)(the torque acting on the particle and (c) the rate at whiche the angular momentum is changing?

This problem is rather similar to problem 18. We begin by writing the $\vec{r}$ vector

$$\begin{aligned}\vec{r} &= (3.0-0)\ \hat{i} + (8.0-0)\ \hat{j} + (0.0-0.0)\ \hat{k} \\ &= 3.0\ \hat{i} + 8.0\ \hat{j}\end{aligned}$$

Now we compute angular momentum and torque.

$$\vec{L} = \vec{r} \times \vec{p}$$
$$= \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.0 & 8.0 & 0.0 \\ 3kg \cdot 5.0 & 3kg \cdot -6.0 & 0.0 \end{bmatrix}$$
$$= \hat{i}\,(8.0 \cdot 0.0 - 0.0 \cdot 3kg \cdot -6.0)$$
$$- \hat{j}(3.0 \cdot 0.0 - 0.0 \cdot 3kg \cdot 5.0)$$
$$+ \hat{k}\,(3.0 \cdot 3kg \cdot -6.0 - 8.0 \cdot 3kg \cdot 5.0)$$
$$= -174.0\,kg\,m^2 / s \;\hat{k}$$

$$\vec{\tau} = \vec{r} \times \vec{F}_1$$
$$= \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.0 & 8.0 & 0.0 \\ 7.0 & 0.0 & 0.0 \end{bmatrix}$$
$$= \hat{i}\,(8.0 \cdot 0.0 - 0.0 \cdot 0.0)$$
$$- \hat{j}(3.0 \cdot 0.0 - 0.0 \cdot 7.0)$$
$$+ \hat{k}\,(3.0 \cdot 0.0 - 8.0 \cdot 7.0)$$
$$= -56.0\,N\,m \;\hat{k}$$

The rate of change of the angular momentum IS the torque: $\vec{\tau} = \frac{d\vec{L}}{dt}$

11.46 The rotational inertial of a collapsing spinning star changes to 1/3 its initial value. What is the ratio of the new rotational kinetic energy to the initial kinetic energy?

$$I_f\,\omega_f = I_i\,\omega_i$$
$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{I_i}{(I_i/3)}\omega_i$$
$$\omega_f = 3\omega_i$$

$$R = \frac{\cancel{\tfrac{1}{2}} I_f\,\omega_f^{\,2}}{\cancel{\tfrac{1}{2}} I_i\,\omega_i^{\,2}} = \frac{\cancel{\tfrac{1}{2}}(I_i/3)(3\omega_i)^2}{\cancel{\tfrac{1}{2}} I_i\,\omega_i^{\,2}} = 3$$