Chapter 10

10.10 Starting from rest, a disk rotates about its central axis with constant angular acceleration. In 5.0 s, it rotates 25 rad. During that time, what are the magnitudes of (a) the angular acceleration and (b) the average angular velocity? (c) What is the instantaneous angular velocity of the disk at the end of the 5.0 s? (d) With the angular acceleration unchanged, through what additional angle will the disk turn during the next 5s.

This problem is the angular analog of many of the constant linear acceleration problems that we did.
Parts (a) and (b) ... the angular acceleration and average angular velocity.

$$\theta_i = 0 \qquad \theta_f = 25rad \qquad \omega_i = 0\ rad/s \qquad \omega_f = ? \qquad \alpha = ? \qquad t = 5s$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\theta_f = 0 + 0 + \frac{1}{2}\alpha t^2$$

$$\alpha = \frac{2\theta_f}{t^2} = \frac{2 \cdot 25rad}{(5s)^2} = 2rad/s^2$$

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{25rad - 0rad}{5s - 0s} = 5rad/s$$

We can compute the instantaneous angular velocity...part (c)

$$\begin{aligned} \omega_f &= \omega_i + \alpha t \\ &= 0rad/s + 2rad/s^2 \cdot 5s \\ &= 10rad/s \end{aligned}$$

Through what additional angle will the disk turn through. We'll consider our starting angle at t=5s as zero so that we can easily compute how far the disk turns in the next 5s. We need to remember that the disk has an initial angular velocity for the second 5s.

$$\theta_i = 0 \qquad \theta_f = ? \qquad \omega_i = 10\ rad/s \qquad \omega_f = ? \qquad \alpha = 2rad/s^2 \qquad t = 5s$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\begin{aligned} \theta_f &= 0 + 10rad/s \cdot 5s + \frac{1}{2} \cdot 2rad/s^2 \cdot (5s)^2 \\ &= 75rad \end{aligned}$$

10.11 A disk, initially rotating at 120 rad/s is slowed down with a constant angular acceleration of magnitude of $4.0 rad/s^2$ (a) How much time does the disk take to stop? (b) Through what angle does the disk rotate during that time?

$$\theta_i = 0$$
$$\theta_f = ?$$
$$\omega_i = 120 \ rad/s$$
$$\omega_f = 0$$
$$\alpha = -4 \ rad/s^2$$
$$t = ?$$

$$\omega_f = \omega_i + \alpha t$$
$$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0\,rad/s - 120\,rad/s}{-4.0\,rad/s^2}$$
$$= 30s$$
$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$
$$\theta_f = 0 + 120 rad/s \cdot 30s + \frac{1}{2} \cdot -4 rad/s^2 \cdot (30s)^2$$
$$= 1800\,rad$$

10.33 Calculate the rotational inertia of a wheel that has a kinetic energy of 24,400J when rotating at 602 rev/min.

$$\omega = \frac{602 rev}{\min} \cdot \frac{2\pi\,rad}{1 rev} \cdot \frac{1\min}{60s} = 63.04 rad/s$$
$$KE_R = \frac{1}{2} I\omega^2$$
$$I = \frac{2KE_R}{\omega^2} = \frac{2 \cdot 24{,}400J}{(63.04 rad/s)^2} = 12.28 kgm^2$$

1**0.50** If a 32.0 Nm torque on a wheel causes angular acceleration of $\alpha = 25 rad/s^2$, what is the wheel's rotational inertia?

$$\tau = I\alpha$$
$$I = \frac{\tau}{\alpha} = \frac{32Nm}{25 rad/s^2} = 1.28 kgm^2$$

10.66 A uniform spherical shell of mass M and radius R rotates about a vertical axis on frictionless bearings (Fig 11-45). A massless cord passes around the equator of the shell over a pulley of rotational inertia I and radius r and is attached to a small object of mass m. There is no friction on the pulley's axle; the cord does not slip on the pulley. What is the speed of the object after it falls a distance h form rest. use energy considerations.

This is an energy conservation problem

$$E_i = mgh \qquad\qquad E_f = \frac{1}{2}mv^2 + \frac{1}{2}I_{sph}\,\omega_{sph}{}^2 + \frac{1}{2}I_p\,\omega_p{}^2$$

Now set the final energy equal to the initial energy...

$$E_f = E_i$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I_{sph}\,\omega_{sph}{}^2 + \frac{1}{2}I_p\,\omega_p{}^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{3}MR^2)\frac{v}{R^2}{}^2 + \frac{1}{2}I_p\frac{v}{r^2}{}^2 = mgh$$

$$v^2(m + \frac{2}{3}M + \frac{I_p}{r^2}) = 2mgh$$

$$v = \sqrt{\frac{2mgh}{(m + \frac{2}{3}M + \frac{I_p}{r^2})}}$$