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A Conceptual Approach to Limiting-Reagent Problems

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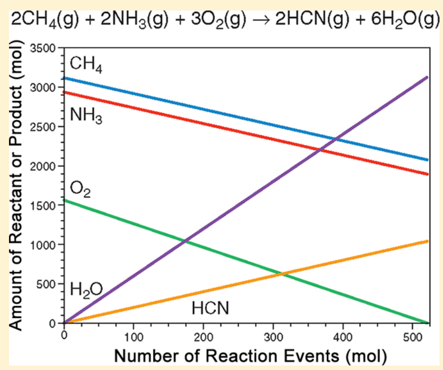
ABSTRACT: A solid foundation of chemistry principles is only gained through a true comprehension of the material as opposed to pure memorization. One of the most fundamental concepts in chemistry is that of determining the amount of product formed in a chemical reaction when one of the reactants is limiting. To increase students' comprehension of this important concept, a conceptual approach is presented that is tangible for limiting-reagent problems on the molecular level and extends directly to typical examples that would be encountered in a classroom or laboratory setting. To accommodate visual learners, a graphical methodology is incorporated that determines the limiting reagent as the reactant that is exhausted first while simultaneously finding the amounts of the excess reagents and products.



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Comprehending the notion of a limiting reagent is a difficult task for many students taking introductory chemistry courses. Many of the common mistakes made by students when determining the amount of product formed involve issues with stoichiometry[1−5] and include the following: recognizing the reactant that is smaller in quantity to be the limiting reagent;[1] calculating the amount of product as a summation of the amounts of reactants thereby not appreciating that atoms in molecules rearrange when reactions take place;[2] solving the problem directly from grams and not converting to moles;[3] not appreciating that there will be some excess reactant at the end of the reaction;[4] and not assigning the coefficients to the reactants they describe.[1] A lot of these errors are a byproduct of students memorizing algorithms, such as amounts tables, and for these reasons, the students do not gain a deeper understanding of the concept.[1]

The most common way in which students determine which reactant is limiting is through the method of minimal product. By calculating how much product forms for each of the reactants, the limiting reagent is determined to be the reactant that produces the least amount of product.[3] However, it is also important for students to recognize the role the excess reactant(s) take in the reaction.[1,3,4] To help students comprehend the concept and avoid memorization, the limiting reagent can be explained as the reactant that requires more moles to react than are available in the experiment.[2] Unfortunately, many students continue to memorize equations and steps in the form of flowcharts and perform well despite not fully comprehending the importance of the concept. An integrated method that offers an explanation that is both quantitative and visual would be beneficial for potentially minimizing memorization and increasing comprehension.[6]

Herein, we present a conceptual approach to limiting-reagent problems that is based on the following definition: the limiting reagent of a reaction is the reactant that is exhausted first. To accommodate visual learners, a graphical methodology is incorporated that determines the limiting reagent while simultaneously finding the amounts of the excess reagents and products. By graphing the chemical components of the reaction together, students can grasp both the quantitative and conceptual understanding of limiting reagents to further develop valuable critical-thinking and quantitative-reasoning skills. This approach is tangible for limiting-reagent problems on the molecular level and extends directly to typical examples that would be encountered in a classroom or laboratory setting. For simplicity sake, we begin with an example involving the number of molecules produced and consumed in a reaction. This calculation is then scaled up to moles, the more common unit for expressing the quantity of a chemical substance.

■ MOLECULAR EXAMPLE

A simple example used to illustrate this method is the Haber−Bosch process of mixing hydrogen gas with nitrogen gas to form ammonia. From the balanced chemical equation

$$3H_2(g) + N_2(g) \rightarrow 2NH_3(g) \quad (1)$$

a reaction event is defined to be 3 molecules of hydrogen reacting with 1 molecule of nitrogen to produce 2 molecules of ammonia.[7] A very small example with 12 molecules of hydrogen and 5 molecules of nitrogen initially present is first considered. After one reaction event, there will be 9 molecules of hydrogen and 4 molecules of nitrogen remaining with 2

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molecules of ammonia produced. The reaction continues as shown in Table 1. After the fourth reaction event, all of the hydrogen molecules are exhausted making hydrogen the limiting reagent even though there were more molecules of hydrogen at the start of the reaction. At the conclusion of the reaction, there is 1 excess molecule of nitrogen remaining and 8 molecules of ammonia produced.

Table 1. The Number of Molecules Unreacted (H_2 and N_2) and Produced (NH_3) versus the Number of Reaction Events

Number of Reaction Events	Number of Molecules of H_2	Number of Molecules of N_2	Number of Molecules of NH_3
0	12	5	0
1	9	4	2
2	6	3	4
3	3	2	6
4	0	1	8

When the initial number of reactant molecules is scaled up by a factor of 10, hydrogen is still the limiting reagent even though it takes 10 times the number of reaction events to run to completion (Table 2). Although the number of molecules in this case is too small to be useful in most applications, a graph may be a more appropriate way to look at the number of reactant and product molecules. Figure 1 depicts a linear relationship between the number of reactant molecules with the number of reaction events r.

Table 2. The Number of Molecules Unreacted (H_2 and N_2) and Produced (NH_3) versus the Number of Reaction Events

Number of Reaction Events	Number of Molecules of H_2	Number of Molecules of N_2	Number of Molecules of NH_3
0	120	50	0
10	90	40	20
20	60	30	40
30	30	20	60
40	0	10	80

$$\text{number of molecules of } H_2 = 120 - 3r \tag{2}$$

$$\text{number of molecules of } N_2 = 50 - r \tag{3}$$

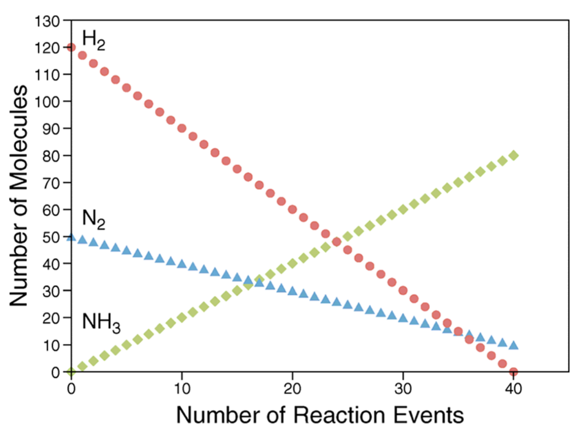

Figure 1. The number of molecules unreacted (H_2 and N_2) and produced (NH_3) versus the number of reaction events. Initially the system has 120 molecules of H_2 and 50 molecules of N_2. The graph indicates that H_2 is the limiting reagent due to the smallest x-intercept.

Each line has a y-intercept equal to the initial number of molecules and a slope that is the negative of the stoichiometric coefficient from eq 1.

Figure 1 displays geometrically that the limiting reagent is the reactant whose line has the smallest x-intercept. Conceptually then, the limiting reagent is the reactant requiring the fewest number of reaction events before running out. By calculating the number of allowed reaction events,

$$\frac{120 \text{ molecules of } H_2}{3 \text{ molecules of } H_2 \text{ per reaction event}} = 40 \text{ reaction events} \tag{4}$$

$$\frac{50 \text{ molecules of } N_2}{1 \text{ molecule of } N_2 \text{ per reaction event}} = 50 \text{ reaction events} \tag{5}$$

it is apparent that hydrogen is the limiting reagent in this example as it can only undergo 40 reaction events. Using the linear relation in eq 3, it is apparent that after 40 reaction events there will be 10 excess molecules of nitrogen remaining.

In a similar fashion, the product ammonia also has a linear relation with the number of reaction events with a positive slope as the stoichiometric coefficient from eq 1 and no initial ammonia.

$$\text{number of molecules of } NH_3 = 2r \tag{6}$$

When the hydrogen runs out after 40 reaction events, there will be 80 molecules of ammonia produced.

■ TYPICAL EXAMPLE

Turning to a larger example commonly seen in introductory chemistry courses, this method can be used to find the limiting reagent when 4.26 mol of hydrogen gas react with 1.78 mol of nitrogen gas. Because a mole of hydrogen is 6.02×10^{23} molecules of hydrogen, the appropriate scaling factor to shift from molecules to moles is Avogadro's number (N_A). Noting that 4.26 mol of H_2 is the same as $4.26 \times N_A$ molecules of H_2, the number of allowed reaction events is calculated for each reagent.

$$\frac{4.26 \text{ mol of } H_2}{3 \text{ molecules of } H_2 \text{ per reaction event}} = 1.42 \times N_A \text{ reaction events} \tag{7}$$

$$\frac{1.78 \text{ mol of } N_2}{1 \text{ molecule of } N_2 \text{ per reaction event}} = 1.78 \times N_A \text{ reaction events} \tag{8}$$

Hydrogen is the limiting reagent in this example with the reaction completing in 1.42 mol of reaction events where a "mole of reaction events" is explicitly defined to be N_A reaction events.[7] It is equivalent to say that the maximum extent of reaction is 1.42 mol.[7,8] With this convention, the amount of each reactant or product is a linear function of the number of moles of reaction events r.[7,9]

$$\text{number of moles of } H_2 = 4.26 - 3r \tag{9}$$

$$\text{number of moles of } N_2 = 1.78 - r \tag{10}$$

$$\text{number of moles of } NH_3 = 2r \tag{11}$$

When the reaction concludes, the amount of excess nitrogen and produced ammonia can be estimated graphically from Figure 2 or calculated directly from eqs 10 and 11. When the hydrogen becomes limiting after 1.42 mol of reaction events, there will be 0.36 mol of excess nitrogen and 2.84 mol of ammonia produced.

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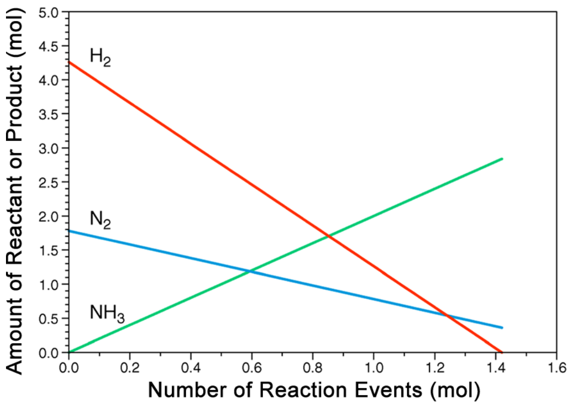

Figure 2. The number of moles unreacted (H_2 and N_2) and produced (NH_3) versus the number of reaction events. Initially, the system has 1.78 mol of N_2 and 4.26 mol of H_2. The graph indicates that H_2 is the limiting reagent due to the smallest x-intercept.

THREE REACTANT EXAMPLE

The methods presented here for solving problems that involve a limiting reagent easily generalize to other types of reactions, including those with more than two reactants. As a final example, consider the industrial process for the production of hydrogen cyanide

$$2CH_4(g) + 2NH_3(g) + 3O_2(g) \rightarrow 2HCN(g) + 6H_2O(g) \quad (12)$$

with initially 50.00 kg of each of the reactants ammonia, oxygen, and methane. Because this method for solving limiting-reagent problems keeps track of the number of molecules in the reaction, it is necessary to convert each of the reactants to the units of moles. Using the molar masses, there are 3.117×10^3 mol of methane, 2.936×10^3 mol of ammonia, and 1.563×10^3 mol of oxygen. The possible number of reaction events for each reactant can now be calculated from the initial amount and the stoichiometric coefficient.

$$\frac{3.117 \times 10^3 \text{ moles of } CH_4}{2 \text{ molecules of } CH_4 \text{ per reaction event}} = 1.559 \times 10^3 \text{ moles of reaction events} \quad (13)$$

$$\frac{2.936 \times 10^3 \text{ moles of } NH_3}{2 \text{ molecules of } NH_3 \text{ per reaction event}} = 1.468 \times 10^3 \text{ moles of reaction events} \quad (14)$$

$$\frac{1.563 \times 10^3 \text{ moles of } O_2}{3 \text{ molecules of } O_2 \text{ per reaction event}} = 0.5210 \times 10^3 \text{ moles of reaction events} \quad (15)$$

Oxygen is the limiting reagent in this example with the reaction completing in 0.5210×10^3 mol of reaction events. Up until the point the reaction concludes, the amount of each reactant or product is a linear function of the number of moles of reaction events r.

$$\text{number of moles of } CH_4 = 3117 - 2r \quad (16)$$

$$\text{number of moles of } NH_3 = 2936 - 2r \quad (17)$$

$$\text{number of moles of } O_2 = 1563 - 3r \quad (18)$$

$$\text{number of moles of HCN} = 2r \quad (19)$$

$$\text{number of moles of } H_2O = 6r \quad (20)$$

When the reaction concludes, the amount of methane and ammonia in excess and amount of hydrogen cyanide and water that was produced can be estimated graphically from Figure 3 or calculated directly from eqs 16−20. When the oxygen

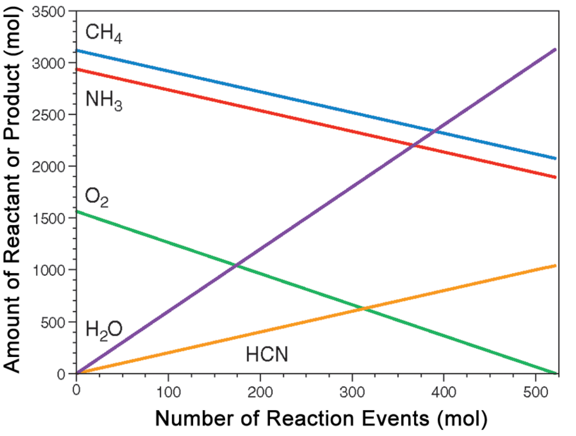

Figure 3. The number of moles unreacted (CH_4, NH_3, and O_2) and produced (HCN and H_2O) versus the number of reaction events. Initially, the system has 3.117×10^3 mol of CH_4, 2.936×10^3 mol of NH_3, and 1.563×10^3 mol of O_2. The graph indicates that O_2 is the limiting reagent due to the smallest x-intercept.

becomes limiting after 0.5210×10^3 mol of reaction events, there will be an excess of 2.073×10^3 mol of methane and 1.892×10^3 mol of ammonia while producing 1.044×10^3 mol of hydrogen cyanide and 3.132×10^3 mol of water.

DISCUSSION

This new pedagogical method of solving limiting-reagent problems can be presented to students as a classroom activity or in parallel to a lab experiment that they are performing. It is important to note that before starting this procedure, the student should be working from a balanced chemical equation and should have each of the constituents converted to moles (or molecules for introductory examples). For an in-class activity, students can work through an introductory problem similar to the first example by creating a table by hand or with the aid of a spreadsheet program to discover which reactant is limiting. As seen when this approach was presented to a second-semester introductory chemistry class, having students sketch the amount of each reactant or product as a function of the number of reaction events gave the students a qualitative feel for limiting reagents and aided in their conceptual understanding. While working with the number of reaction events, students began questioning the concept of time and the rate at which each chemical constituent is produced or runs out. This type of critical thinking that was generated by the students could now easily result in a conversation on chemical kinetics.

Equally important would be the use of this method as a lab experiment workup. The students could be presented with a common limiting-excess reagent lab such as the formation of carbon dioxide from the reaction of baking soda and vinegar.[10,11] In the lab, the students can measure the amount of excess baking soda and also the amount of carbon dioxide formed. Postlab, the students could graphically compare their experimental and theoretical results.

This conceptual approach may take more instruction time to develop as it would ideally be presented as a hands-on activity in the classroom or laboratory. In our assessment, the students were open to learning the method and felt that this approach to solving limiting-reagent problems was easily retained. The

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students quickly recognized the conceptual nature of this method. Many times throughout the period, students commented on the helpfulness of seeing the amounts of reactants and products as opposed to just calculating numbers. It is important to note that the introduction of an additional pedagogical strategy to those commonly used in introductory chemistry textbooks may lead to students combining strategies. However, it is our assessment that the addition of this visual approach has great potential for increasing student comprehension of this important chemistry topic.

■ SUMMARY

The traditional methods that students enrolled in an introductory chemistry course rely on for solving limiting-reagent problems involve memorizing, mnemonics, and algorithms. Although these methods may enable some students to do well, they do not provide an avenue for students to develop critical-thinking or quantitative-reasoning skills. The pedagogical method for teaching limiting reagents described in this article is an integrated learning approach that utilizes chemistry concepts and mathematical reasoning to increase retention of material learned and to build a foundation for future courses.

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Notes

The authors declare no competing financial interest.

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