Chapter 31

31.2 What is the capacitance of an oscillating LC circuit if the maximum charge on the capacitor is $1.60\mu C$ and the total energy is $140\mu J$

$$
\begin{aligned}
U &= \frac{q^2}{2C} \\
C &= \frac{q^2}{2U} \\
&= 9.14\times 10^{-9}F
\end{aligned}
$$

31.4 The frequency of oscillation of a certain LC circuit is 200 kHz. At time t=0, plate A of the capacitor has the maxiumum positive charge. At what earliest time t>0 will (a) plate A again have maximum positive charge, (b) the other plate of the capacitor have maximum positive charge, and (c) the inductor have maximum magnetic field.
The period of the oscillation is given by

$$T = \frac{1}{f} = 5\times 10^{-6}s$$

a) The time from max charge on plate A to max charge on plate A is one period.

$$t = T = 5\times 10^{-6}s$$

b) The time from max charge on plate A to max charge on plate B is T/2

$$t = T/2 = 2.5\times 10^{-6}s$$

c) The maximum field occurs when the current is maximum. This occurs at T/4.

$$t = T/4 = 1.25\times 10^{-6}s$$

31.5 An oscillating LC circuit consists of a 75.0 mH inductor and a $3.6\mu F$ capacitor. If the maximum charge on the capacitor is $2.90\mu C$, what are (a) the total energy in the circuit and (b) the maximum current.

$$U = \frac{q^2}{2C} = \frac{(2.9\times 10^{-6}C)^2}{2\cdot 3.6\times 10^{-6}F} = 1.17\times 10^{-6}J$$

We can find the maximum current from

$$
\begin{aligned}
q &= q_0\sin(\omega_0 t) \\
\frac{dq}{dt} &= q_0\omega_0\cos(\omega_0 t) \\
i_0 &= q_0\omega_0 = 2.9\times 10^{-6}C\cdot\sqrt{\frac{1}{75\times 10^{-3}\cdot 3.6\times 10^{-6}F}} = 5.58\times 10^{-3}
\end{aligned}
$$

31.9 In an oscillating LC circuit with $L = 50mH$ and $C = 4.0\mu F$ the current is initial a maximum. How long will it take before the capacitor is fully charged the first time.
The time from maximum current to maximum charge is only T/4. In this case

$$\omega = \sqrt{\frac{1}{LC}} = 2236.1s^{-1}$$
$$T = \frac{2\pi}{\omega} = 2.81 \times 10^{-3} s$$
$$t = T / 4 = 1.40 \times 10^{-3} s$$

31.25 What resistance R should be connected in series with an inductance $L = 220mH$ and capacitance $C = 12\mu F$ for the maximum charge on the capacitor to decay to 99% of its initial value in 50 cycles.

The question implies that the solution is oscillatory--which means that we have an underdamped solution. We need the natural oscillating frequency. After computing that, we need to write charge as a function of time. The charge is given by

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{220 \times 10^{-3} \cdot 12 \times 10^{-6} F}} = 615.5$$
$$q = q_0 e^{-\gamma t} \sin(\omega t - \varphi)$$

Now that we have the natural osc. frequency, we can compute the time for the decay.

$$T = \frac{2\pi}{\omega_0}$$
$$t = 50T = \frac{100\pi}{\omega_0}$$

The decay is controlled by the leading exponential term, so we only need to consider that term.

$$0.99 q_0 = q_0 e^{-\gamma \cdot \frac{100\pi}{\omega_0}}$$
$$e^{-\gamma \cdot \frac{100\pi}{\omega_0}} = 0.99$$
$$-\gamma \frac{100\pi}{\omega_0} = \ln(0.99)$$
$$\gamma = -\frac{\omega_0}{100\pi} \ln(0.99)$$
$$\gamma = \frac{R}{2L}$$
$$\frac{R}{2L} = -\frac{\omega_0}{100\pi} \ln(0.99)$$
$$R = -\frac{2L\omega_0}{100\pi} \ln(0.99) = 8.66 \times 10^{-3} \Omega$$

31.28 A 50mH inductor is connected as in Fig 31-8a to an ac generator with $\varepsilon_m = 30V$. What is the amplitude of the resulting alternating current if the frequency of the emf is (a)1.00kHz and (b) 8.00 kHz?

$$\varepsilon_m = i_m X_L$$

$$i_m = \frac{\varepsilon_m}{X_L}$$

$$X_L = (2\pi f)L$$

$$f = 1000\,Hz \Rightarrow X_L = 314.2\Omega \Rightarrow i_m = 9.55\times 10^{-2} A$$

$$f = 8000\,Hz \Rightarrow X_L = 2513.6\,\Omega \Rightarrow i_m = 1.19\times 10^{-2} A$$

31.29 (a) At what frequency would a $6.0\,mH$ inductor and a $10\,\mu F$ capacitor have the same reactance? What would the reactance be? (c) Show that this frequency would be the natural oscillating frequency?

$$X_L = (2\pi f)L$$

$$X_C = \frac{1}{(2\pi f)C}$$

$$X_L = X_C$$

$$(2\pi f)\,L = \frac{1}{(2\pi f)C}$$

$$f = \frac{1}{(2\pi)}\sqrt{\frac{1}{LC}}$$

31.52 What is the maximum value of an ac voltage whose rms value is 100V?

$$V_{RMS} = \frac{V_0}{\sqrt{2}}$$

$$V_0 = \sqrt{2}\,V_{RMS} = 141.4V$$

31.55 An air conditioner connected to a 120V rms ac line id equivalent to a 12Ω resistor and a 1.3 Ωinductive reactance in series. (a) Calculate the impedance of the air conditioner. (b) find the average rate at which energy is supplied to the appliance.

(a) We find the impedance Z first.

$$Z = \sqrt{X_L^{\ 2} + R^2} = 12.07\Omega$$

(b) The rate at which energy is supplied depends on R since the inductor is not an energy dissipating device. The current still depends on the impedance, however.

$$P_{avg} = i_{rms}^{\ 2} R = \left(\frac{V_{rms}}{Z}\right)^2 R = 1186W$$

31.63 A generate supplies 100V to the primary coil of a transformer of 50 turns. If the secondary coil has 500 turns, what is the secondary voltage?

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{500\,turns}{50\,turns} \cdot 100V = 1000V$$