Chapter 24

24.2 The square surface shown in Fig 24-25 measures 3.2 mm on each side. It is immersed ina uniform electric field with magnitude E = 1800 N/C. The field lines make an angle of 35 degrees with a normal to the surface as shown. Take the normal to be directed "outward" as though the surface were one face of a box. Calculate the electric flux through the surface.

The flux through this surface is

$$\begin{aligned}
\varphi &= E\,A\cos\theta \\
\theta &= 180^\circ - 35^\circ \\
\varphi &= (1800N/C)\cdot(.0032m)^2\cdot\cos(180^\circ - 35^\circ) \\
&= -1.51\times10^{-2}\,Nm^2/C
\end{aligned}$$

Note that the angle is 180-35. This makes the flux negative--which means the flow is into the box. A net flow into a closed surface is taken to be negative.

24.3. The cube in Fig 24-26 has edge length of 1.4 m and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the field (in N/C) is given by (a) $6.00\hat{i}$, (b) $-2.00\hat{j}$ and (c) $-3.00\hat{i}+4.00\hat{k}$.

The area vector for the right face is

$$\vec{A} = (1.4m)^2\hat{j}$$

We can now compute flux.

(a) $\vec{E}\cdot\vec{A} = 6.00\hat{i}\cdot(1.4m)^2\hat{j} = 0$

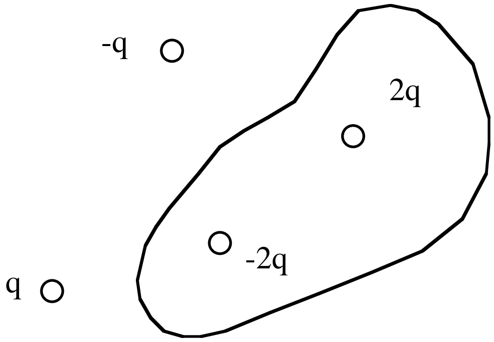
(b) $\vec{E}\cdot\vec{A} = -2.00\hat{j}\cdot(1.4m)^2\hat{j} = -2.00\cdot(1.4m)^2 = -3.92Nm^2/C$

(c) $\vec{E}\cdot\vec{A} = (-3.00\hat{i}+4.00\hat{k})\cdot(1.4m)^2\hat{j} = 0$

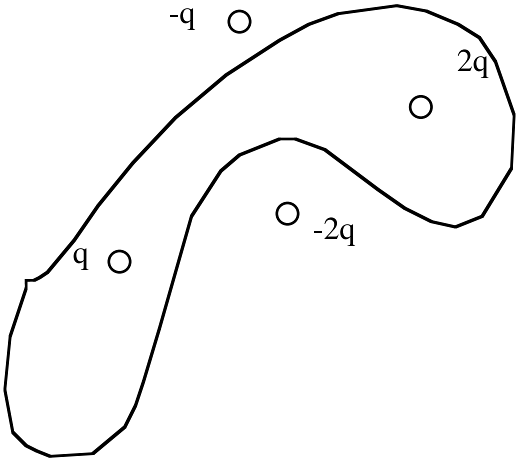
(d) The total flux through the cube is zero. A uniform field is present--every field line that enters onside of the cube leaves the other.

24.4 You have four point charges 2q,q,-q,-2a. If possible, describe how you would place a closes surface that enclose at least the charge 2q (and perhaps other charges) and through which the net electric flux is (a) 0, (b) $3q/\varepsilon_0$ and (c) $-2q/\varepsilon_0$

a)

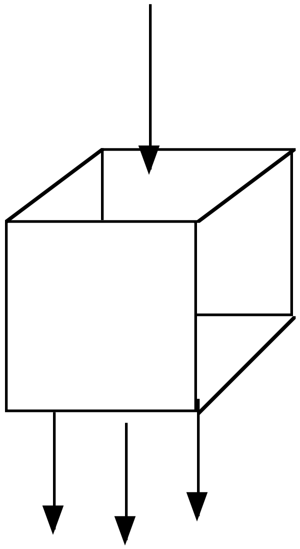


b)



c) Not possible.

24.9. It is found experimentally that the electric field i a certain region o Earth's atmosphere is directed vertically down. At an altitude of 300m, the field has magnitude 60N/C; at an altitude of 200m, the magnitude is 100N/C. Find the net amount of charge contained in a cube 100m on edge with horizontal faces at altitudes of 200 and 300 m. Neglect the curvature of the earth.



We calculate the net flux

$$\begin{aligned}
\varphi &= 60N/C\cdot(100m)^2\cos 180 + 100N/C\cdot(100m)^2\cos 0 \\
&= 100N/C\cdot(100m)^2 - 60N/C\cdot(100m)^2 \\
&= 40N/C\cdot(100m)^2 \\
&= 4\times 10^5 \\
q_{enc} &= \varepsilon_0\varphi \\
&= 8.85\times 10^{-12}\cdot 4\times 10^5 \\
&= 3.54\times 10^{-6}C
\end{aligned}$$

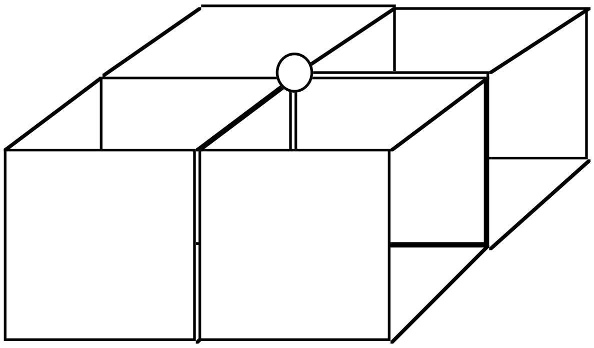
24.11 A point charge q is placed at one corner of a cube of edge a. What is the flux through each of the cube surfaces?

To capture 1/2 the total flux, we would need 4 cubes as shown in the figure (where q is in the corner of each as shown). These 4 cubes have 3 external square sides each. So 1/2 the total flux goes through 12 identical surfaces. The flux through one square on external surface of one cube is

$$\varphi = \frac{1}{12} \cdot \frac{\varphi_{closed}}{2}$$

$$\varphi_{closed} = \frac{q}{\varepsilon_0}$$

$$\varphi = \frac{1}{24} \cdot \frac{q}{\varepsilon_0}$$



24.14 Space vehicles traveling through Earth's radiation belt can intercept a significant number of electrons. The resulting charge buildup can damage electronic components and disrupt operations. Suppose a spherical metallic satellite 1.3 m in diameters accumulates 2.4 μC in one orbital revolutions. (a) Find the resulting surface charge density. (b) Calculate the magnitude of the electric field just outside the surface of the satellite due to the surface charge.

$$\sigma = \frac{2.4 \times 10^{-6} C}{4\pi \cdot (1.3m)^2} = 1.13 \times 10^{-7} C/m^2$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1.13 \times 10^{-7}}{8.85 \times 10^{-12}} = 1.28 \times 10^{4} N/C$$

24.15 An isolated conductor of arbitrary shape has a net charge of $10\times10^{-6}C$. Inside the conductor is a cavity within which is a point charge of $q=+3\times10^{-6}C$. What is the charge (a) on the cavity wall and (b) on the outer surface of the conductor?

a The charge on the cavity wall is $-3\times10^{-6}C$. The guarantees that the enclosed charge is always zero for all Gaussian surfaces inside the conductor. This is necessary since E= 0 in the conductor, so that the electric flux must be zero, so the enclosed charge must be zero.

b) The charge on the outer surface is $+13\times10^{-6}C$. If the total charge is $10\times10^{-6}C$ and $-3\times10^{-6}C$ is on the inner cavity, $+13\times10^{-6}C$ must be on the outer surface.

24.19 A very long conducting cylinder rod of length L with a a total charge +q surrounded by a conducting cylindrical shell also of length L with a total charge -2q, as shown in Fig. 24-29. Use Gauss' law to find (a) the electric field at points outside conducting shell, (b) the distribution of charge on the shell and (c) the electric field in the region between the shell and rod.

The net charge per unit length (as seen from outside) is

$$\lambda_{net}=\frac{(q+-2q)}{L}=\frac{-q}{L}$$

If we write out Gauss' law for a surface that is outside both cylinders

$$\oint\vec{E}\cdot d\vec{A}=\frac{q_{enc}}{\varepsilon_0}$$

$$\int_{ends}\vec{E}\cdot d\vec{A}+\int_{curve}\vec{E}\cdot d\vec{A}=\frac{q_{enc}}{\varepsilon_0}$$

$$0+\int_{curve}\vec{E}\cdot d\vec{A}=\frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curve}E\,dA=\frac{q_{enc}}{\varepsilon_0}$$

$$E\int_{curve}dA=\frac{q_{enc}}{\varepsilon_0}$$

$$E\cdot2\pi r\,L=\frac{\lambda_{net}L}{\varepsilon_0}$$

$$E=\frac{\lambda_{net}}{2\pi\,\varepsilon_0 r}$$

b) If we choose a Gaussian Surface in the conducting shell, we know that the E field and flux must be zero. If we solve for the charge per unit length on the inner surface of the shell, we find that it must be equal and opposite to the charge on the rod.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 = \frac{q_{enc}}{\varepsilon_0}$$

$$0 = \frac{\lambda_{rod} L + \lambda_{inner} L}{\varepsilon_0}$$

$$\lambda_{rod} L = -\lambda_{inner} L$$

$$\lambda_{inner} = -\lambda_{rod}$$

$$q_{inner} = -q_{rod}$$

If -q is on the inner surface of the shell and -2q is on the total shell, then -q must be on the outer surface of the shell.

c. In the gap....

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{ends} \vec{E} \cdot d\vec{A} + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\lambda_{rod} = \frac{q}{L}$$

$$\int_{curve} E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \int_{curve} dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r L = \frac{\lambda_{rod} L}{\varepsilon_0}$$

$$E = \frac{\lambda_{rod}}{2\pi \varepsilon_0 r}$$

24.25 Charge is distributed uniformly throughout the volume of an infinitely long cylinder of radius R. (a) Show that , at a distance from the cylinder (for r< R) is

$$E = \frac{\rho r}{2\varepsilon_0}$$

Write an expression for E when r>R.

a) For radii r<R,

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{ends} \vec{E}\cdot d\vec{A} + \int_{curve} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int_{curve} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curve} E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\int_{curve} dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\cdot 2\pi r\, L = \frac{\rho(\pi r^2 L)}{\varepsilon_0}$$

$$E = \frac{\rho\, r}{2\varepsilon_0}$$

b) For radii r>R

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{ends} \vec{E}\cdot d\vec{A} + \int_{curve} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int_{curve} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curve} E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\int_{curve} dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\cdot 2\pi r\, L = \frac{\rho(\pi R^2 L)}{\varepsilon_0}$$

$$E = \frac{\rho\, R^2}{2\varepsilon_0 r}$$

24.28 A large, flat, nonconducting surface has a uniform charge density σ. A small circular hole of radius R has been cut in the middle of the surface as shown in Fig. 24-32. Ignore fringing of the field lines around all edges and calculate the electric field at a point P a distance z from the center of the hole along its axis. (Hint: See Eq. 23-26 and use superposition)

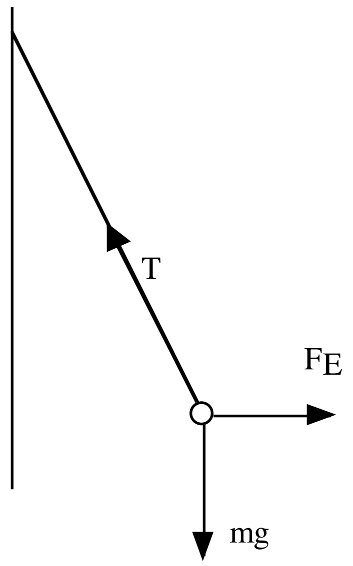
Superposition is the key to this problem Superposition is the principle that allows you to construct the field due to a number of charge distributions by adding the fields from each distribution together as vectors.

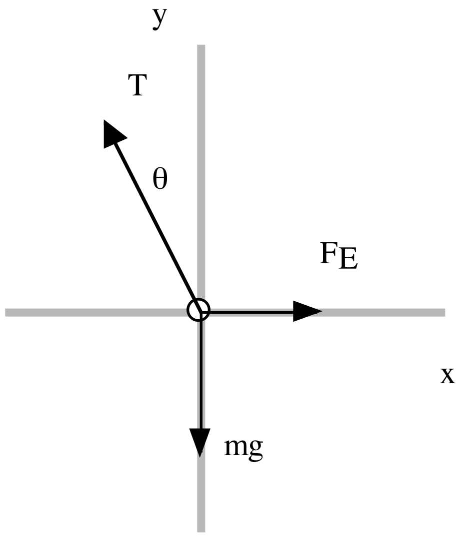
In this problem, the field at the point p will be the field due to an infinite plane - the field due to a disk. We think of this distribution as a plain and then we subtract the hole.

$$\begin{aligned} E &= E_{plane} - E_{disk} \\ &= \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \\ &= \frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{z^2 + R^2}}\right) \end{aligned}$$

24.29 In Fig. 24-33, a small nonconducting ball of mass $m = 1\,mg$ and charge $q = 2\times10^{-8}C$ (distributed uniformly through its volume hangs from an insulating thread that makes an angle $\theta = 30°$ with a vertical uniformly charged nonconducting sheet (shown in cross section). Considering the gravitational force on the ball and assuming that the sheet extends far vertically and into and out of the page, calculate the surface charge density σ of the sheet.

We begin by drawing a free body diagram. We know that the forces must ad to zero.





We solve for and eliminate the tension between the equations to find the electric force. We can then use the expression for the electric field for an infinite plane to solve for the charge density.

$x - direction$

$$0 = F_E - T\sin\theta$$

$y - direction$

$$0 = T\cos\theta - mg$$

$$T = \frac{mg}{\cos\theta}$$

$$F_E = T\sin\theta = \frac{mg}{\cos\theta}\cdot\sin\theta = mg\tan\theta$$

$$F_E = mg\tan\theta$$

$$F_E = qE = q\frac{\sigma}{2\varepsilon_0}$$

$$q\frac{\sigma}{2\varepsilon_0} = mg\tan\theta$$

$$\sigma = \frac{2\varepsilon_0 mg}{q}\tan\theta = 5.0\times10^{-6}C/m^2$$

24.31 An electron is shot directly toward the center of a large metal plate that has excess negative charge with surface charge density $2.0 \times 10^{-6} C/m^2$. If the initial kinetic energy of the electron is 100 eV and if the electron is to stop (owing to electrostatic repulsion from the plate) just as it reaches the plate, how far from the plate must it be shot.

We begin again with the electric field due to an infinite plane of charge. This allows us to find the field and force on the electron

$$F = qE = q\frac{\sigma}{2\varepsilon_0}$$

We can now find the acceleration that the electron experiences.

$$ma = q\frac{\sigma}{2\varepsilon_0}$$

$$a = \frac{q\sigma}{2m\varepsilon_0}$$

We can now us our constant acceleration equations to find the distance that the electron needs to stop. Note that the acceleration is negative since this electron is moving rightward and decelerating.

$$x_f - x_i = ?$$

$$v_i = v_0$$

$$v_f = 0$$

$$a = -\frac{q\sigma}{2m\varepsilon_0}$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

$$0 = v_0^2 - 2\frac{q\sigma}{2m\varepsilon_0}(x_f - x_i)$$

$$(x_f - x_i) = \frac{v_0^2}{\frac{q\sigma}{m\varepsilon_0}} = \frac{\varepsilon_0 m v_0^2}{q\sigma}$$

$$= \frac{2\varepsilon_0 \cdot \frac{1}{2} m v_0^2}{q\sigma}$$

$$= \frac{2\varepsilon_0 \cdot K}{q\sigma}$$

$$K = 100\,eV \cdot \frac{1.6 \times 10^{-19} J}{1\,eV} = 1.6 \times 10^{-17} J$$

$$(x_f - x_i) = \frac{2\varepsilon_0 \cdot K}{q\sigma}$$

$$= \frac{2 \cdot 8.85 \times 10^{-12} \cdot 1.6 \times 10^{-17} J}{1.6 \times 10^{-19} \cdot 2.0 \times 10^{-6}}$$

$$= 8.85 \times 10^{-4} m$$

24.37 In a 1911 paper, ernest Rutherford said: "In order to from some ideas of the forces required to deflect an alpha particle through a large angle, consider an atom [as] containing a point positive charge Ze at its center and surrounded by a distribution of negative electricity -Ze uniformly distributed within a s sphere of radius R. The electric filed E ... at a distance r from the center for a point inside the atom [is]

$$E = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

We choose a Gaussian surface with r < R. The charged contained will consist of the pointlike central positive charge and some part of the distribution of negative charge. We define the charge density of the negative charge.

$$\rho = \frac{-Ze}{\frac{4}{3}\pi R^3}$$

We now apply Gauss' Law, paying special attention to how to compute the enclosed charge

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{encl}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Ze + \rho \cdot \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$E = \frac{Ze + \rho \cdot \frac{4}{3}\pi r^3}{4\pi \varepsilon_0 r^2}$$

$$= \frac{Ze + \frac{-Ze}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3}{4\pi \varepsilon_0 r^2}$$

$$E = \frac{Ze}{4\pi \varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

24.44 Figure 24.35a shows a spherical shell of charge with uniform charge density ρ. Plot E due to the shell for distance from the shell for distances ranging from 0 to 30 cm. Assume that $\rho = 1.0 \times 10^{-6} C/m^3$, a=10 cm and b=20 cm

Begin with Gauss' Law.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Choose a spherical Gaussian surface that has radius r (r<a). For this surface, the integration simplifies because E and dA are parallel everywhere on the surface, and E is constant in magnitude over the surface

$$\oint E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

The charge enclosed is the charge in the Gaussian surface...

(r < a)

$$E \cdot 4\pi r^2 = \frac{0}{\varepsilon_0}$$

$$E = 0$$

(a < r < b)

$$E \cdot 4\pi r^2 = \frac{\rho \cdot \frac{4}{3}\pi (r^3 - a^3)}{\varepsilon_0}$$

$$E = \frac{\rho}{3\varepsilon_0}(r - \frac{a^3}{r^2})$$

(r > b)

$$E \cdot 4\pi r^2 = \frac{\rho \cdot \frac{4}{3}\pi (b^3 - a^3)}{\varepsilon_0}$$

$$E = \frac{\rho \cdot (b^3 - a^3)}{3\varepsilon_0 r^2}$$