Chapter 23 Problems

23.1 The square surface shown in Fig 23-26 measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude E = 1800 N/C. The field lines make an angle of 35 degrees with a normal to the surface as shown. Take the normal to be directed "outward" as though the surface were one face of a box. Calculate the electric flux through the surface.

The flux through this surface is

$$\varphi = E\, A\cos\theta$$
$$\theta = 180° - 35°$$
$$\varphi = (1800 N/C)\cdot(.0032 m)^2 \cdot \cos(180° - 35°)$$
$$= -1.51\times 10^{-2}\, Nm^2/C$$

Note that the angle is 180-35. This makes the flux negative--which means the flow is into the box. A net flow into a closed surface is taken to be negative.

23.2. The cube in Fig 23-27 has edge length of 1.4 m and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the field (in N/C) is given by (a) $6.00\hat{i}$, (b) $-2.00\hat{j}$ and (c) $-3.00\hat{i} + 4.00\hat{k}$.

The area vector for the right face is

$$\vec{A} = (1.4m)^2\hat{j}$$

We can now compute flux.

(a) $\vec{E}\cdot\vec{A} = 6.00\hat{i}\cdot(1.4m)^2\hat{j} = 0$

(b) $\vec{E}\cdot\vec{A} = -2.00\hat{j}\cdot(1.4m)^2\hat{j} = -2.00\cdot(1.4m)^2 = -3.92 Nm^2/C$

(c) $\vec{E}\cdot\vec{A} = (-3.00\hat{i} + 4.00\hat{k})\cdot(1.4m)^2\hat{j} = 0$

(d) The total flux through the cube is zero. A uniform field is present--every field line that enters onside of the cube leaves the other.

23.4 In Fig. 23-28 a butterfly net is in a uniform electric field of magnitude $E = 3.0 mN/C$. The rim, a circle of radius $a = 11.0cm$ is aligned perpendicular to the filed. The net contains no net charge. Find the electric flux through the netting

The netting plus the circle make a closed surface. We can use this surface and Gauss' Law to find the flux through the netting. We begin with Gauss

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{netting} \vec{E}\cdot d\vec{A} + \int_{circle} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{netting} \vec{E}\cdot d\vec{A} + \int_{circle} \vec{E}\cdot d\vec{A} = 0$$

$$\int_{netting} \vec{E}\cdot d\vec{A} = -\int_{circle} \vec{E}\cdot d\vec{A}$$

$$\int_{netting} \vec{E}\cdot d\vec{A} = -3mN/C\cdot\pi(0.11m)^2$$

$$= -0.114\frac{mN}{C}\cdot m^2$$

23.7 In Fig. 23-29, a proton is a distance d/2 directly above the center of a square of side d. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge d).

If we think of the charge as enclosed by a cube with the charge at the center, we can use Gauss' Law to find the flux through the cube

$$\varphi_{cube} = \frac{q_{enc}}{\varepsilon_0} = 1.81\times10^{-8}Cm^2$$

$$\varphi_{side} = \frac{1}{6}\varphi_{cube} = 3.01\times10^{-9}\frac{N}{C}m^2$$

23.8 When a shower is turned on in a closed bathroom, the splashing of the water on the bare tube can fill the room's air with negatively charged ions and produce an electric field in the air as great as 1000 N/C. Consider a bathroom with dimension 2.5m x 3.0 m x 2.0m . Along the ceiling floor, and four walls, approximate the electric field in the air as being directed perpendicular to the surface and as having a uniform magnitude of 600 N/C. Also, treat those surfaces as forming a closed Gaussian surface around the room's air. What are (a) the volume charge density ρ and (b)the number of excess elementary charges e per cubic meter.

If we consider the walls, floor, and ceiling as forming a Gaussian surface, we can compute the total flux and find the charge contained in the bathroom. This will allow us to find the volume charge density and the number of excess elementary charges e per cubic meter.

The flux through each surface can be written $\varphi = \vec{E}\cdot\vec{A}$. In this case, the flux will be negative since the field points in. The total flux will be the sum of all the fluxes. Note that we've written a 2 in from of each surface calculation, representing floor and ceiling, left and right wall, and front and back wall.

$$\varphi = -2 \cdot 600\frac{N}{C} \cdot 2.5m \cdot 3.0m - 2 \cdot 600\frac{N}{C} \cdot 2.5m \cdot 2.0m - 2 \cdot 600\frac{N}{C} \cdot 2.0m \cdot 3.0m$$
$$= -2.22 \times 10^4 \frac{N}{C} m^2$$

We now use this result to find the total charge and charge density,.

$$\varphi = \frac{q_{enc}}{\varepsilon_0}$$
$$q_{enc} = \varepsilon_0 \varphi = 1.96 \times 10^{-7} C$$

Now we can compute the charge density and electron number density

$$\rho = \frac{q}{V} = \frac{1.96 \times 10^{-7} C}{2.5m \cdot 2.0m \cdot 3.0m} = 1.31 \times 10^{-8} \frac{C}{m^3}$$
$$\rho = \left(\#\, of\ electrons\ per\, m^3\right) \cdot \left(chg / electron\right)$$
$$\left(\#\, of\ electrons\ per\, m^3\right) = \frac{\rho}{\left(chg / electron\right)} = \frac{1.31 \times 10^{-8} C / m^3}{1.6 \times 10^{-19} C}$$
$$= 8.19 \times 10^{10} e / m^3$$

23.12 *Flux and nonconducting shells* A charged particle is suspended at the center of two concentric spherical shells that are very tin and made of non conducting material. Figure 23-31a showed a cross section. Figure 32-31b gives the net flux φ through a Gaussian sphere centered on the particle as a function of the radius r of the sphere. (a) What is the charge on the central particle? What are the net charges of (b) shell A and (c) shell B?

If we know that flux through the Gaussian surface, we can find the charge enclosed.

For r inside the inner shell A, only the suspended point charge is enclosed.

$$\varphi = \frac{q_{enc}}{\varepsilon_0}$$
$$q_{enc} = \varphi \varepsilon_0 = 2 \times 10^5 \cdot 8.85 \times 10^{-12} = 1.77 \times 10^{-6} C$$
$$q_{enc} = q_{center} = 1.77 \times 10^{-6} C$$

For r between shells A and B, the Gaussian surface encloses both the central point charge and the charge on shell A.

$$\varphi = \frac{q_{enc}}{\varepsilon_0}$$
$$q_{enc} = \varphi \varepsilon_0 = -4 \times 10^5 \cdot 8.85 \times 10^{-12} = -3.54 \times 10^{-6} C$$
$$q_{enc} = q_{center} + q_A = -3.54 \times 10^{-6} C$$
$$q_A = -3.54 \times 10^{-6} C - q_{center} = -5.31 \times 10^{-6} C$$

A sphere outside B encloses all the charge.

$$\varphi = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = \varphi\varepsilon_0 = 6\times10^5 \cdot 8.85\times10^{-12} = 5.31\times10^{-6}\,C$$

$$q_{enc} = q_{center} + q_A + q_B = 5.31\times10^{-6}C$$

$$q_B = 5.31\times10^{-6}C - q_{center} - q_A = 8.85\times10^{-6}\,C$$

23.15 A uniformly charged conducting sphere of 1.2m diameter has a surface charge density of $8.1\times10^{-6}C/m^2$ (a) Find the net charge on the sphere. What is the total electric flux leaving the surface of the sphere.

$$\text{(a)}\ \ q = \sigma\cdot 4\pi r^2 = 3.66\times10^{-5}\,C$$

$$\text{(b)}\ \ \varphi = \frac{q_{enc}}{\varepsilon_0} = \frac{3.66\times10^{-5}}{8.85\times10^{-12}}\frac{Nm^2}{C} = 4.14\times10^6\frac{Nm^2}{C}$$

23.20 Figure 23-34 shows a section of a long, thin walled metal tube of radius $R = 3.00cm$, with a charge per unit length $\lambda = 2.00\times10^{-8}\,C/m$ What is the magnitude E of the electric field at radial distance (a) $r = R/2.00$ and (b) $r = 2.00R$

Let's begin by considering this tube to be infinitely long. We can draw a cylindrical surface centered on the axis at some distance from the center.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{ends}\vec{E}\cdot d\vec{A} + \int_{curve}\vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int_{curve}\vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curve} E\,dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\int_{curve} dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\cdot 2\pi r\,L = \frac{\lambda_{net}L}{\varepsilon_0}$$

$$E = \frac{\lambda_{net}}{2\pi\,\varepsilon_0 r}$$

When our Gaussian Surface is inside the tube, there is NO net charge enclosed, so for r<R,

$$E = \frac{\lambda_{net}}{2\pi\varepsilon_0 r}$$
$$\lambda_{net} = 0$$
$$E = 0$$

Outside the tube at r = 2.0 R,

$$E = \frac{\lambda_{net}}{2\pi\varepsilon_0 r}$$
$$\lambda_{net} = 2.0\times10^{-8} C/m$$
$$r = 2.0\cdot R = 0.06m$$
$$E = \frac{2.0\times10^{-8} C/m}{2\pi\,\varepsilon_0 \cdot 0.06m} = 6\times10^{3} N/C$$

23.21 An infinite line of charge produces a field of magnitude $4.5\times10^{4} N/C$ at a distance of 2m. Calculate the linear charge density.

This problem is very similar to the previous problem. We will use the same approach to derive the expression for the field and then solve for the linear charge density.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$
$$\int_{ends} \vec{E}\cdot d\vec{A} + \int_{curve} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$
$$0 + \int_{curve} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$
$$\int_{curve} E\, dA = \frac{q_{enc}}{\varepsilon_0}$$
$$E\int_{curve} dA = \frac{q_{enc}}{\varepsilon_0}$$
$$E\cdot 2\pi\, r L = \frac{\lambda_{net} L}{\varepsilon_0}$$
$$E = \frac{\lambda_{net}}{2\pi\varepsilon_0 r}$$
$$\lambda_{net} = 2\pi\varepsilon_0 r\, E$$
$$r = 2.0m$$
$$E = 4.5\times10^{4} N/C$$
$$\lambda_{net} = 5\times10^{-6} C/m$$

23.27 Figure 23-37 is a section of a conduction rod of radius $R_1 = 1.30\,mm$ and length $L = 11.0m$ inside a thin-walled coaxial conducting cylindrical shell of radius $R_2 = 10.0R_1$ and the (same) length L. The net charge on the rod is $Q_1 = +3.40 \times 10^{-12}C$; that on the shell is $Q_2 = -2Q_1$ What are (a) the magnitude E and (b) direction (radially inward or outward) of electric field at a radial distance $r = 2R_2$? What are (c) E and (d) the direction at $r = 5R_1$ What is the charge on the (e)interior and (f) exterior surface of the shell.

For parts (a) and (b), we consider the field outside the outer shell. If we write out Gauss' law for a surface that is outside both the rod and shell

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{ends} \vec{E} \cdot d\vec{A} + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curve} E\,dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\int_{curve} dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r\,L = \frac{\lambda_{net} L}{\varepsilon_0}$$

$$E = \frac{\lambda_{net}}{2\pi\,\varepsilon_0 r}$$

The net charge per unit length is the net charge divided by the length. The net charge inside the enclosing Gaussian surface is the total charge on the rod and the shell.

$$q_{net} = Q_1 + Q_2 = -3.40 \times 10^{-12}C$$

$$\lambda_{net} = \frac{q_{net}}{L} = \frac{-3.40 \times 10^{-12}C}{11.0m} = -3.09 \times 10^{-13}C/m$$

$$r = 2.00R_2 = 0.026m$$

$$E = \frac{\lambda_{net}}{2\pi\,\varepsilon_0 r} = \frac{-3.09 \times 10^{-13}C/m}{2\pi\,\varepsilon_0 \cdot 0.026m} = -0.214N/C$$

The minus sign indicates that the field points inward.

For parts c and d, we choose a Gaussian surface in the gap since $r = 5R_1$ is in the gap. The Gauss' Law calculation is exactly the same, except the charge per unit length enclosed will be different since we only enclose the rod.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{ends} \vec{E} \cdot d\vec{A} + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curve} E \, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \int_{curve} dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r \, L = \frac{\lambda_{net} L}{\varepsilon_0}$$

$$E = \frac{\lambda_{net}}{2\pi \varepsilon_0 r}$$

Now we use the charge on the rod only.

$$q_{net} = Q_1 = 3.40 \times 10^{-12} C$$

$$\lambda_{net} = \frac{q_{net}}{L} = \frac{3.40 \times 10^{-12} C}{11.0 m} = 3.09 \times 10^{-13} C / m$$

$$r = 5.00 R_1 = 0.0065 m$$

$$E = \frac{\lambda_{net}}{2\pi \varepsilon_0 r} = \frac{3.09 \times 10^{-13} C / m}{2\pi \varepsilon_0 \cdot 0.0065 m} = 0.855 N / C$$

The field points radially outward.

To find the charge on the inner surface of the shell, we choose a Gaussian Surface inside the shell. (in the conductor itself). If we choose a Gaussian Surface inside the conducting shell, we know that the E field and flux must be zero. If we solve for the charge on the inner surface of the shell, we find that it must be equal and opposite to the charge on the rod, since the charge enclosed must be zero.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 = \frac{q_{enc}}{\varepsilon_0}$$

$$0 = \frac{q_{inner} + q_{rod}}{\varepsilon_0}$$

$$q_{inner} = -q_{rod} = -Q_1$$

$$= -3.40 \times 10^{-12} C$$

If $-Q_1$ is on the inner surface of the shell and $Q_2 = -2Q_1$ is on the total shell, then $-Q_1 = -3.4\times10^{-12}C$ must be on the outer surface of the shell.

23.34 A large, flat, nonconducting surface has a uniform charge density σ. A small circular hole of radius R has been cut in the middle of the surface as shown in Fig. 24-32. Ignore fringing of the field lines around all edges and calculate the electric field at a point P a distance z from the center of the hole along its axis. (Hint: See Eq. 23-26 and use superposition)

Superposition is the key to this problem Superposition is the principle that allows you to construct the field due to a number of charge distributions by adding the fields from each distribution together as vectors.

In this problem, the field at the point p will be the field due to an infinite plane - the field due to a disk. We think of this distribution as a plane and then we subtract the hole.

$$\begin{aligned} E &= E_{plane} - E_{disk} \\ &= \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \\ &= \frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{z^2+R^2}}\right) \end{aligned}$$

23.47 In Fig. 23-48, a nonconducting spherical shell of radius $a = 2.00cm$ and outer radius $b = 2.40cm$ has (within its thickness) a positive volume charge density $\rho = \frac{A}{r}$ where A is a constant and r is the distance from the center of the shell. In addition a small ball of charge $q = 45.0\,fC$ is located at that center. What value should A have if the electric field in the shell $(a \le r \le b)$ is to be uniform.

We need to arrange the charge so that the field in the shell is constant in magnitude. We begin with a spherical Gaussian surface in the shell $(a \le r \le b)$ For this spherical geometry, the electric field is parallel to the dA vector everywhere on the surface, so the dot product becomes a simple multiplication because the angle between the E vector and the dA vector is zero everywhere. The magnitude of E is the same everywhere by symmetry, so we can pull E out of the integration. We are left with integrating dA over our spherical Gaussian surface.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint E\,dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

We need to carefully calculate the charge enclosed. It consists of the inner ball of charge and a portion of the shell's charge. Since the charge density of the shell is not uniform, we need to find the shell portion of the charge by integrating.

$$\begin{aligned} q_{enc} &= q_{ball} + \int_a^r \rho\,dV \\ &= q_{ball} + \int_a^r \frac{A}{r} \cdot 4\pi r^2\,dr \\ &= q_{ball} + 4\pi A \int_a^r r\,dr \\ &= q_{ball} + 4\pi A\left(\frac{r^2}{2} - \frac{a^2}{2}\right) \\ &= q_{ball} + 2\pi A r^2 - 2\pi A a^2 \\ E \cdot 4\pi r^2 &= \frac{q_{enc}}{\varepsilon_0} \\ E \cdot 4\pi r^2 &= \frac{q_{ball} + 2\pi A r^2 - 2\pi A a^2}{\varepsilon_0} \end{aligned}$$

For E to be a constant, the left hand side of this equation must match the right hand side. Since the right hand side has only an r^2 term, the right hand side may only have the same r dependence. We can choose A so that this happens.

$$E \cdot 4\pi r^2 = \frac{q_{ball} + 2\pi A r^2 - 2\pi A a^2}{\varepsilon_0}$$

$$q_{ball} - 2\pi A a^2 = 0$$

$$A = \frac{q_{ball}}{2\pi a^2} = \frac{45.0 \times 10^{-15}\,C}{2\pi \cdot (0.02)^2} = 1.79 \times 10^{-11}\,C/m^2$$

23.49 In Fig. 23-50, a solid sphere of radius $a = 2.00cm$ is concentric with a spherical conducting shell of inner radius $b = 2.00a$ and outer radius $c = 2.40a$. The sphere has a net uniform charge $q_1 = +5.00\,fC$; The shell has a net charge $q_2 = -q_1$. What is the magnitude of the electric field at radial distances (a) $r = 0$, (b) $r = a/2.00$, (c) $r = a$, (d) $r = 1.5a$, (e) $r = 2.3a$, and (f) $r = 3.5a$. What is the net charge on the (g) inner and (h) outer surface of the shell.

We begin by considering a spherical Gaussian Surface. For this spherical geometry, the electric field is parallel to the dA vector everywhere on the surface, so the dot product becomes a simple multiplication because the angle between the E vector and the dA vector is zero everywhere. The magnitude of E is the same everywhere by symmetry, so we can pull E out of the integration. We are left with integrating dA over our spherical Gaussian surface.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

The task for each part of the problem is to find the charge enclosed. For part (a), since our Gaussian surface is at zero radius, the charge enclosed is zero.

$$r = 0$$
$$E = 0$$

(b) When we put our Gaussian surface at $r = a/2.00$, we enclose some portion of the inner sphere.

$$q_{enc} = \rho \cdot V_{enc} = \rho \cdot \frac{4}{3}\pi r^3$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$E = \frac{\rho r}{3\varepsilon_0}$$

$$\rho = \frac{5.0 \times 10^{-15} C}{\frac{4}{3}\pi (0.02m)^3} = 1.49 \times 10^{-10} C / m^3$$

$$r = a / 2 = 0.01m$$

$$E = \frac{1.49 \times 10^{-10} \cdot 0.01}{3\varepsilon_0}$$

$$= 0.562 N / C$$

We can use this same expression to find the field at $r = a$

$$E = \frac{\rho r}{3\varepsilon_0}$$

$$\rho = 1.49 \times 10^{-10} C / m^3$$

$$r = a / 2 = 0.02m$$

$$E = \frac{1.49 \times 10^{-10} \cdot 0.02}{3\varepsilon_0}$$

$$= 0.112 N / C$$

When we consider the field in the gap, $r = 1.5a$, we need to reexamine the charge enclosed. The charge enclosed is the total charge on the inner sphere.

$$q_{enc} = 5.0 \times 10^{-15} C$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{q_{enc}}{4\pi \varepsilon_0 r^2}$$

$$r = 1.5a = 0.03m$$

$$E = \frac{5.0 \times 10^{-15}}{4\pi \varepsilon_0 (0.03)^2}$$

$$= 0.04995 N / C$$

At $r = 2.3a$, our Gaussian surface is in the conductor. We know that the electric field must be zero.

$$r = 2.3a$$
$$E = 0$$

At $r = 3.5a$, our Gaussian surface encloses both the inner sphere and the spherical shell.

$$q_{enc} = q_1 + q_2 = q_1 - q_1 = 0$$
$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$
$$E = 0$$

We can use our result from inside the conducting sphere to find the charges on the inner and outer surface of the shell. Since we know that the E=0 in a conductor, we know that the charge enclosed is zero.

$$\begin{aligned} E \cdot 4\pi r^2 &= \frac{q_{enc}}{\varepsilon_0} \\ E &= 0 \\ q_{enc} &= 0 = q_1 + q_{inner} \\ q_{inner} &= -q_1 = -5.0 \times 10^{-15} C \\ q_{shell} &= q_{inner} + q_{outer} \\ q_{outer} &= q_{shell} - q_{inner} = q_2 - q_{inner} = -q_1 - (-q_1) \\ &= 0 \end{aligned}$$

23.50 Figure 23-51 shows a spherical shell with uniform volume charge density $\rho = 1.84 \times 10^{-9} C/m^3$, inner radius $a = 10.0cm$ and outer radius $b = 2.00a$. What is the magnitude of the electric field at radial distances (a) $r = 0$, (b) $r = a/2.0$, (c) $r = a$, (d) $r = 1.5a$, (e) $r = b$, (f) $r = 3.00b$

We begin by considering a spherical Gaussian Surface. For this spherical geometry, the electric field is parallel to the dA vector everywhere on the surface, so the dot product becomes a simple multiplication because the angle between the E vector and the dA vector is zero everywhere. The magnitude of E is the same everywhere by symmetry, so we can pull E out of the integration. We are left with integrating dA over our spherical Gaussian surface.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

For (a) $r = 0$, (b) $r = a/2.0$, and (c) $r = a$, the charge enclosed is zero so $E = 0$. For d) $r = 1.5a$, and (e) $r = b$, we need to calculate the charge enclosed.

$$q_{enc} = \rho \cdot V_{enc} = \rho \cdot \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3 \right)$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\rho \cdot \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3 \right)}{\varepsilon_0}$$

$$E = \frac{\rho (r^3 - a^3)}{3\varepsilon_0 r^2}$$

$$= \frac{1.84 \times 10^{-9} C/m^3 (r^3 - a^3)}{3\varepsilon_0 r^2}$$

$$r = 1.5a = 0.15m$$

$$E = 7.32 N/C$$

$$r = b = 0.20m$$

$$E = 12.13 N/C$$

For the last section (f), we enclose the entire shell so we use the entire charge on the shell.

$$q_{enc} = \rho \cdot V_{enc} = \rho \cdot \left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 \right)$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\rho \cdot \left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 \right)}{\varepsilon_0}$$

$$E = \frac{\rho (b^3 - a^3)}{3\varepsilon_0 r^2}$$

$$= \frac{1.84 \times 10^{-9}\, C/m^3 (b^3 - a^3)}{3\varepsilon_0 r^2}$$

$$r = 3b = 0.60\, m$$

$$E = 1.35\, N/C$$