Physics 130
Sample Exam 4

Short Answer section. Please answer all of the short answer questions.

1. Compute the total angular momentum and kinetic energy that the earth has due to its rotation about an axis that runs from the north to the south pole. Compute the total angular momentum and kinetic energy that the earth has in its circular orbit. (Note: You will need to look up the mass and radius of the earth).

$$\omega = \frac{2\pi rad}{24hrs \cdot 3600s/hr} = 7.27 \times 10^{-5} rad/s$$

$$I_{sphere} = \frac{2}{5}MR^2 = \frac{2}{5} \cdot 5.98 \times 10^{24} kg \cdot (6.37 \times 10^6 m)^2 = 9.71 \times 10^{37} kgm^2$$

$$L = I\omega = 9.71 \times 10^{37} kgm^2 \cdot 7.27 \times 10^{-5} rad/s = 7.06 \times 10^{33} kgm^2/s$$

$$E = \frac{1}{2}I\omega^2 = \frac{1}{2} 9.71 \times 10^{37} kgm^2 \cdot (7.27 \times 10^{-5} rad/s)^2 = 2.566 \times 10^{29} J$$

For orbital motion

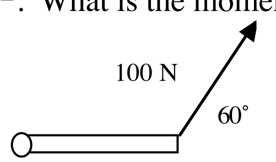
$$v = \frac{2\pi r}{t} = \frac{2\pi \cdot 1.5 \times 10^{11}}{365days \cdot 24hrs/day \cdot 3600s/hr} = 2.989 \times 10^4 m/s$$

$$p = mv = 5.98 \times 10^{24} kg \cdot 2.989 \times 10^4 kgm/s$$

$$L = rp\sin\theta = 1.5 \times 10^{11} m \cdot 5.98 \times 10^{24} kg \cdot 2.989 \times 10^4 m/s = 2.68 \times 10^{40} kgm^2/s$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2} 5.98 \times 10^{24} kg \cdot (2.989 \times 10^4 m/s)^2 = 2.671 \times 10^{33} J$$

2. A force of 100 N is applied to a bar that can rotate around one end. Compute the torque that is produced and be sure to indicate the direction of the torque. The bar responds by accelerating with an angular acceleration of 1 rad/s^2. What is the moment of the inertia of the bar?



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF\sin\theta = r \cdot 100N \cdot \sin 60$$

$$\tau = I\alpha$$

$$I = \frac{\tau}{\alpha} = \frac{r \cdot 100N \cdot \sin 60}{1 rad/s^2}$$

The torque points out of the paper.

3. A solid disk with a cord wrapped around it unwinds without slipping. Use energy methods to show that after it has fallen a distance h, its linear velocity is

$$v = \sqrt{\frac{4gh}{3}}$$

Starting with energy conservation

$$E_i = E_f$$
$$0 + mgh = (\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2) + 0$$
$$I = \frac{1}{2}mr^2$$
$$\omega = \frac{v}{r}$$
$$mgh = \frac{1}{2}(\frac{1}{2}mr^2)(\frac{v}{r})^2 + \frac{1}{2}mv^2$$
$$mgh = \frac{1}{4}mv^2 + \frac{1}{2}mv^2$$
$$mgh = \frac{3}{4}mv^2$$
$$v = \sqrt{\frac{4gh}{3}}$$

4. A wheel spins down from an initial angular velocity of $\omega_i = 200\,rad/s$ with an angular acceleration $\alpha = -20\,rad/s^2$. What is the *initial linear velocity v* of the edge of the wheel if the radius of the wheel is 1/2 m? What is the angular velocity after 10 seconds?

$$\omega_f = ?$$
$$\omega_i = 200\,rad/s$$
$$\alpha = -20\,rad/s^2$$
$$v_i = r\,\omega_i = 0.5m \cdot 200\,rad/s = 100m/s$$
$$\omega_f = \omega_i + \alpha\,t$$
$$= 200\,rad/s - 20\,rad/s^2 \cdot 10\,s$$
$$= 0\,rad/s$$

5. A sphere has a radius of 20,000 m and a mass of $1\times10^{30}kg$. The sphere makes one rotation (2π radians) every second. Compute the angular velocity of the sphere, the total angular momentum, the total kinetic energy of the sphere. (Note, these parameters are roughly for a neutron star). $I_{sphere} = \frac{2}{5}mr^2$

$$\omega = \frac{2\pi rad}{1s} = 6.28 rad/s$$

$$L = I\omega = \frac{2}{5}mr^2 \cdot \omega$$

$$= \frac{2}{5}\cdot 1\times10^{30}kg\cdot(20{,}000m)^2 \cdot 6.28 rad/s$$

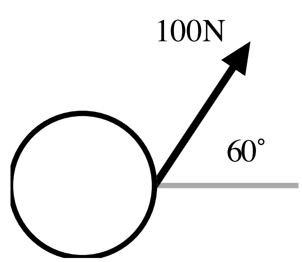
$$= 1.0\times10^{39} kg\cdot m^2/s$$

$$K = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\cdot(\frac{2}{5}\cdot 1\times10^{30}kg\cdot(20{,}000m)^2)\cdot(6.28 rad/s)^2$$

$$= 3.16\times10^{39}J$$

6. A force is applied to a disk as in the figure below. The mass of the disk is 80 kg and the radius is 1/2 m. Compute the torque and the angular acceleration that the disk experiences. $I_{disk} = \frac{1}{2}mr^2 = 10 kg\, m^2$ for this disk.



$$\tau = rF\sin\theta$$

$$= \frac{1}{2}m\cdot 100N\cdot\sin 60°$$

$$= 43.3Nm$$

$$I_{disk} = 10 kg\, m^2$$

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{43.3Nm}{10 kg m^2} = 4.33 rad/s^2$$

7. Compute the gravitational force between a 1 kg mass and a 10 kg mass that are separated by a distance of 10 meters. ($G = 6.67 \times 10^{-11} N \cdot m^2 / kg^2$)

$$F = G\frac{Mm}{r^2} = 6.67 \times 10^{-11} \frac{N\,m^2}{kg^2} \cdot \frac{1kg \cdot 10kg}{(10m)^2}$$
$$= 6.67 \times 10^{-12} N$$

8. Near the end of its life, the sun will swell in radius to engulf Mercury, Venus, and the Earth. What will happen to its angular velocity and why?

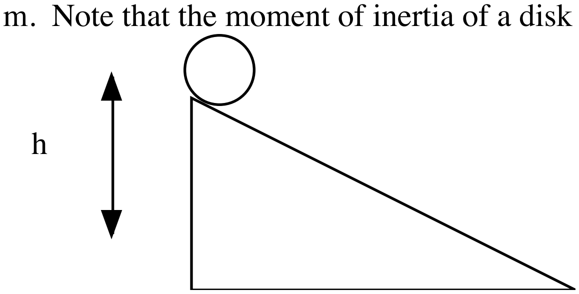
The angular velocity will fall. ***Angular momentum is conserved***, and as the sun expands, its moment of inertia will increase. The angular velocity will fall to conserve L.

9. Using the gravitational force between the earth and sun, find the period of the earth's orbit. Assume that the orbit is perfectly circular.

$$\frac{m_e v_e^2}{r_{es}} = G\frac{M_S m_e}{r_{es}^2}$$
$$v_e = \sqrt{G\frac{M_S}{r_{es}}}$$
$$T = \frac{2\pi r_{es}}{\sqrt{G\frac{M_S}{r_{es}}}} = \frac{2\pi r_{es}^{3/2}}{\sqrt{GM_S}}$$

Problems. Please work two (2) of the three problems and clearly indicate which problems you wish to have graded.

1. A solid disk with mass 1kg and radius 0.1 m rolls down an incline as shown below, falling a vertical distance h of 0.5 m. Note that the moment of inertia of a disk is $I_{disk} = \frac{1}{2}mr^2$



a. Using energy methods, find the final velocity of the disk at the bottom.

$$
\begin{aligned}
mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\
&= \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}mr^2)(\frac{v}{r})^2 \\
&= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 \\
&= \frac{3}{4}mv^2 \\
v &= \sqrt{\frac{4}{3}gh} \\
&= \sqrt{\frac{4}{3}\cdot 9.8m/s^2 \cdot \frac{1}{2}m} \\
&= 2.56m/s
\end{aligned}
$$

b. What angular velocity, angular momentum, and linear momentum does the disk have at the bottom?

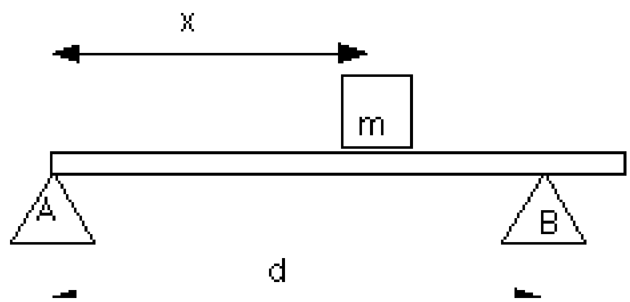
$$
\begin{aligned}
p &= mv = 1kg \cdot 2.56m/s = 2.56\,kg\,m/s \\
\omega &= \frac{v}{r} = \frac{2.56m/s}{0.1m} = 25.6rad/s \\
L &= I\omega = (\frac{1}{2}\cdot 1kg \cdot (0.1m)^2)\cdot 25.6rad/s \\
&= 0.128Nm
\end{aligned}
$$

c. A hoop with the same radius and the same mass rolls down the same incline. What is its velocity from at the bottom? Note that the moment of inertia of a hoop is $I_{hoop} = mr^2$. Does the hoop or the disk have a larger acceleration?

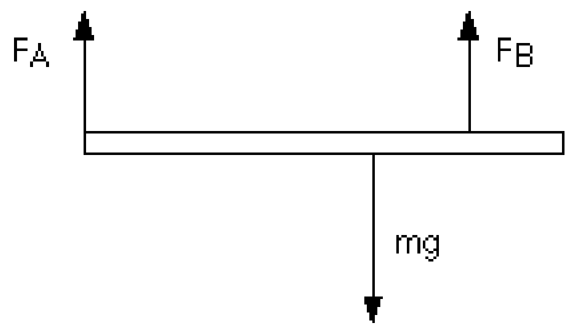
$$
\begin{aligned}
mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\
&= \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)(\frac{v}{r})^2 \\
&= mv^2 \\
v &= \sqrt{gh} \\
&= \sqrt{9.8m/s^2 \cdot \frac{1}{2}m} \\
&= 2.21m/s
\end{aligned}
$$

The disk has a larger acceleration (a larger final velocity after accelerating through the same distance).

2. A plank of negligible mass is supported by two supports as shown below.



a) Draw the forces that act on the plank.



b) Write the net torque acting on the plank assuming the axis at A. Use this expression to find the force produced by support B.

$$\tau_{net} = 0 = mgx - F_B d$$
$$F_B = \frac{x}{d} mg$$

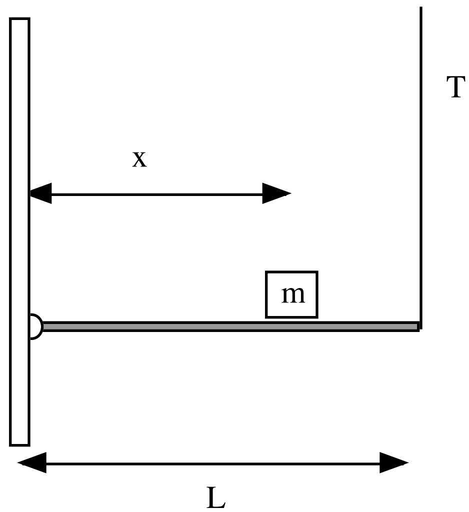
c) Write the net vertical force and use it to find the force produced by support A.

$$F_{net} = 0 = F_A + F_B - mg$$
$$F_A = mg - F_B$$
$$= mg - \frac{x}{d} mg$$
$$= (1 - \frac{x}{d}) mg$$

d) What would happen if x becomes greater d? Could the arrangement be stable?

The force at A would have to become negative. Given the picture, this can't happen. The arrangement would not be stable.

3. Consider the scaffolding below. It is suspended on the right by a cable with tension T. On the left, it is supported by a hinge that provides an upward force F. The scaffolding has negligible mass, and a mass m is located a distance x from the hinge.



a. Draw all of the forces on the scaffold.

Force up from hinge, force up due to tension, force down due to weight.

b. Consider the hinge as an axis of rotation. Write the torque about this axis due to the tension and the weight of the mass. Use this expression to find the tension.

$$\tau = 0 = mgx - LT$$
$$T = \frac{mgx}{L}$$

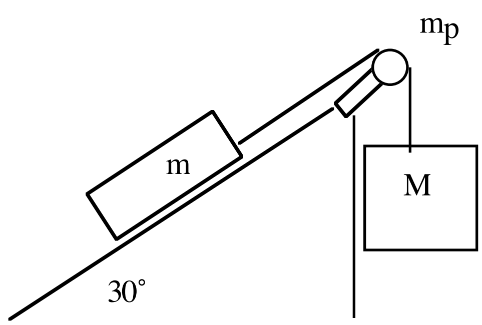
c. What upward force F does the hinge provide?

$$0 = T + F_u - mg$$
$$= \frac{mgx}{L} + F_u - mg$$
$$F_u = mg \cdot (1 - \frac{x}{L})$$

d. Suppose that the Tension is no longer vertical, but is angled outward with respect to the scaffold, pointing away from the wall. What additional force does the hinge need to provide and why? Note. DO NOT CALCULATE the force, simply describe what additional force is needed.

The hinge has to provide a force to the left to counter the outward force component of the tension.

4. Consider masses arranged as below. Assume that the pulley can be treated as a solid disk with radius r. You may also assume that M> m



a. Compute the acceleration, and the tensions T_1 and T_2 (the tensions on opposite sides of the pulley).

$$ma = T_2 - mg\sin\theta$$
$$Ma = Mg - T_1$$
$$I\alpha = r(T_1 - T_2)$$
$$T_1 = Mg - Ma$$
$$T_2 = ma + mg\sin\theta$$
$$I\alpha = r(Mg - Ma - ma - mg\sin\theta)$$
$$I\frac{a}{r} = r(Mg - Ma - ma - mg\sin\theta)$$
$$a(\frac{I}{r} + mr + Mr) = (Mgr - mgr\sin\theta)$$
$$a = \frac{(Mgr - mgr\sin\theta)}{(\frac{I}{r} + mr + Mr)}$$

b. Using energy methods, find the velocity of the system after M has fallen a distance h.

$$Mgh - mgh\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}Mv^2 + \frac{1}{2}I(\frac{v}{r})^2$$
$$v^2 = \frac{Mgh - mgh\sin\theta}{\frac{1}{2}m + \frac{1}{2}M + \frac{1}{2}\frac{I}{r^2}}$$

c. Verify that this result matches what you would compute using the acceleration that you found in part a.

$$v^2 = {v_0}^2 + 2ah$$

$$v^2 = 2\frac{(Mgr - mgr\sin\theta)}{(\frac{I}{r} + mr + Mr)}h$$