Sample Exam 1 - Physics 130

Short Answer Section

1. Estimate the volume of gold in a solid gold wedding ring. The density of gold is $19.3\times 10^{3}\, kg / m^{3}$.

Estimate the dimensions of the ring (inner radius, outer radius, height).

$$
\begin{aligned}
r_{inner} &= 1cm = 1\times 10^{-2} m \\
r_{outer} &= 1.2cm = 1.2\times 10^{-2} m \\
h &= 0.4\, cm = 4.0\times 10^{-3} m \\
volume &= \pi \cdot (r_{outer}{}^{2} - r_{inner}{}^{2}) \cdot h \\
&= 0.55 cm^{3}
\end{aligned}
$$

2. Your fingernails grow approximately 6 inches per year. What is the speed in m/s?

$$
\begin{aligned}
s &= \frac{8\ in}{yr}\cdot\frac{2.54\, cm}{1 in}\cdot\frac{1m}{100cm}\cdot\frac{1yr}{365 days}\cdot\frac{1 day}{24 hrs}\cdot\frac{1hr}{3600\, s} \\
&= 6.44\times 10^{-9} m / s
\end{aligned}
$$

3. You attend a rock concert on the grounds of a football field that is packed with people. If the field is 150m long and 100m wide, estimate the size of the crowd at the concert. Take the space that each person has to be a circle of radius 1m. Is your estimate too low or too high?

$$
\begin{aligned}
A_{total} &= \#\ people \cdot A_{person} \\
\#\ people &= \frac{A_{total}}{A_{person}} = \frac{150m\cdot 100m}{\pi\cdot (1m)^{2}} = 4774
\end{aligned}
$$

This estimate is probably too low if the field is really packed. People will be closer than 2m.

4. Boulders on movie sets are often made of light materials like Styrofoam. If we approximate the shape of a boulder as a cylinder of radius 1.0 m, and length 3m, what is the mass of this false boulder. The density of Styrofoam is 0.92 kg/m^3

$$
m = \rho\ V = 0.92\frac{kg}{m^{3}}\cdot \pi\cdot (1m)^{2}\cdot 3m = 8.67kg
$$

5. When the space shuttle re-enters the earth's atmosphere, it is traveling at 25 times the speed of sound. If the speed of sound is 340 m/s, calculate the shuttle's speed in m/s, and mi/hr.

$$
\begin{aligned}
s &= 25 \cdot 340\frac{m}{s} = 8.50 \times 10^3 \frac{m}{s} \\
&= 8.50 \times 10^3 \frac{m}{s} \cdot \frac{1mi}{1600m} \cdot \frac{3600s}{1hr} \\
&= 19{,}125\frac{mi}{hr}
\end{aligned}
$$

6. A hailstone forms in the clouds of a thunderstorm approximately 1000ft above the earth's surface and falls to the ground. What velocity does it have when it hits the earth. If this is so, why aren't hailstones lethal?

$$
\begin{aligned}
&a = -g = -32\,ft/s^2 \\
&y_f = -1000\,ft \\
&v_i = 0\,ft/s \\
&v_f^{\,2} = v_i^2 + 2a(y_f - y_i) \\
&v_f^{\,2} = 2 \cdot 32\frac{ft}{s^2} \cdot 1000\,ft \\
&v_f = 253\frac{ft}{s}
\end{aligned}
$$

60mi/hr=88ft/s. Hailstones don’t go this fast because of friction.

7. How long did it take the hailstone to reach the earth in the previous question?

$$
\begin{aligned}
&y_f = y_i + v_i t + \frac{1}{2}at^2 \\
&a = -g = -32\,ft/s^2 \\
&y_f = -1000\,ft \\
&v_i = 0\,ft/s \\
&t = \sqrt{\frac{2(y_f - y_i)}{g}} = 7.9s
\end{aligned}
$$

8. Consider two vectors: **a** = 1 **i** + 2 **j** and **b** = 4 **i** + 5 **j** Compute the magnitude and direction of **a**, the magnitude of **b**, **a** + **b**, the scalar product of **a** and **b**, **a** x **b**, and the angle between **a** and **b**.

$$\vec{a} = 1\hat{i} + 2\hat{j}$$
$$\vec{b} = 4\hat{i} + 5\hat{j}$$
$$|\vec{a}| = \sqrt{1^2 + 2^2} = \sqrt{5} = 2.236$$
$$\tan\theta_a = \frac{2}{1} \Rightarrow \theta_a = 63.43°$$
$$|\vec{b}| = \sqrt{4^2 + 5^2} = 6.4$$
$$\vec{a} + \vec{b} = 5\hat{i} + 7\hat{j}$$
$$\vec{a} \bullet \vec{b} = 1 \cdot 4 + 2 \cdot 5 = 14$$
$$\vec{a} \times \vec{b} = \det\begin{Bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 4 & 5 & 0 \end{Bmatrix}$$
$$= \hat{i}(2 \cdot 0 - 0 \cdot 5) - \hat{j}(1 \cdot 0 - 0 \cdot 4) + \hat{k}(1 \cdot 5 - 2 \cdot 4) = -3\hat{k}$$
$$\cos\theta = \frac{\vec{a} \bullet \vec{b}}{|\vec{a}||\vec{b}|} = \frac{14}{2.236 \cdot 6.4}$$

9. The position of the a particle is given by $x(t) = 10 + 5t^2 + 1t^4$. Is the acceleration constant in this motion? Why or why not? Compute the average velocity and the average acceleration over the interval 0s to 2s. Compute the instantaneous velocity and the instantaneous acceleration at 2s.

$$x(t) = 10 + 5t^2 + t^4$$
$$v(t) = 10t + 4t^3$$
$$a(t) = 10 + 12t^2$$
$$x(0) = 10m$$
$$x(2) = 46m$$
$$v_{avg} = \frac{x(2) - x(0)}{2s} = 18m/s$$
$$v(0) = 0m/s$$
$$v(2) = 52m/s$$
$$a_{avg} = \frac{v(2) - v(0)}{2s} = 26m/s^2$$
$$a(2) = 58m/s^2$$

10. You drive approximately due east to Chicago from Moline. The 165 mi trip takes you approximately 3.5 hrs. You return to Moline from Chicago and the trip takes you a little longer because of traffic -- 4.5 hrs. What was your average speed on each leg of the trip and for the entire trip. What was your average velocity for the entire trip?

$$\text{Moline to Chicago: } s = \frac{165mi}{3.5hrs} = 47.1\frac{mi}{hr}$$

$$\text{Chicago to Moline: } s = \frac{165mi}{4.5hrs} = 36.7\frac{mi}{hr}$$

$$\text{Moline to Moline: } s = \frac{330mi}{8hrs} = 41.25\frac{mi}{hr}$$

The average velocity is zero. There is no displacement.

11. Explain the relationship between the position vs. time, velocity vs. time, and acceleration vs time graphs. You may want to do this with a sketch.

Velocity is the slope of the position vs. time graph.
Acceleration is the slope of the velocity vs. time graph.

12. An acceleration has length 10 m/s^2 and angle 60 degrees. Compute the x and y components of the acceleration?

$$a_x = 10\cos 30 = 8.66 m/s^2$$

$$a_y = 10\sin 30 = 5.00 m/s^2$$

Problems: Practice doing all of them. In the real exam, you would choose 2 of 3.

1. A playful cat named Ernie leaps perfectly vertically to a height of 5 ft from a standing start when trying to chase things at the window

a) What velocity does he have when he leaves the ground?

First we write what we know...

$$y_i = 0ft$$
$$y_f = 5ft$$
$$v_i = ?$$
$$v_f = 0f/s$$
$$a = -g = -32ft/s^2$$

Now we solve for the initial velocity.

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$
$$0 = v_i^2 - 2g(y_f - 0)$$
$$v_i = \sqrt{2gy_f} = 17.8ft/s$$

b) How long does it take him to reach the maximum height?

$$v_f = v_i + at$$
$$0 = v_i - gt$$
$$t = \frac{v_i}{g} = 0.56s$$

c) What velocity does he have when he lands?

The path is perfectly symmetric. $v_{landing} = -v_i = -17.8 ft/s$

He always lands on his feet, as all cats do, and she comes to a complete stop in about 0.1 seconds.

d. What acceleration does he feel in the landing after he has touched the ground?

$$a = \frac{\Delta v}{\Delta t} = \frac{17.8 ft/s}{0.1s} = 178 ft/s^2$$

This is a large acceleration. Our estimate of the stopping time was likely too short.

2. A cliff diver enters the water perfectly vertically with a downward velocity of 10 m/s. While in the water, he experiences a constant acceleration UPWARD of 10 m/s^2 (since he floats, after all). a) What maximum depth does the diver reach? From what maximum height did he fall ?

$$y_i = 0m$$
$$y_f = ?$$
$$v_i = -10m/s$$
$$v_f = 0m/s$$
$$a = 10m/s^2$$

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$
$$0 = v_i^2 + 2a(y_f - 0)$$
$$y_f = -\frac{v_i^2}{2a} = -\frac{(-10m/s)^2}{2 \cdot 10m/s^2} = -5m$$

The final y is negative since went below the surface of the water, (which was zero height in our case).

The second part of this question is asking from what height would you need to fall to be going 10m/s?

$$y_i = ?$$
$$y_f = 0$$
$$v_i = 0m/s$$
$$v_f = -10m/s$$
$$a = -g$$
$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$
$$v_f^2 = 0 - 2g(0 - y_i)$$
$$y_i = \frac{v_f^2}{2g} = \frac{(-10m/s)^2}{2 \cdot 9.8m/s^2} = 5.1m$$

Notice that this height almost the same as the depth that he achieved. This is because the gravitational acceleration at $9.8m/s^2$ and the acceleration due to the water, $10m/s^2$ have almost the same magnitude.

b) How long must the diver hold his breath (i.e. go down and come up)?

$$y_i = 0m$$
$$y_f = 0m$$
$$v_i = -10m/s$$
$$v_f = ?$$
$$a = 10m/s^2$$
$$t = ?$$
$$y_f = y_i + v_i t + \frac{1}{2}at^2$$
$$0 = 0 + v_i t + \frac{1}{2}at^2$$
$$t = -\frac{2v_i}{a} = 2s$$

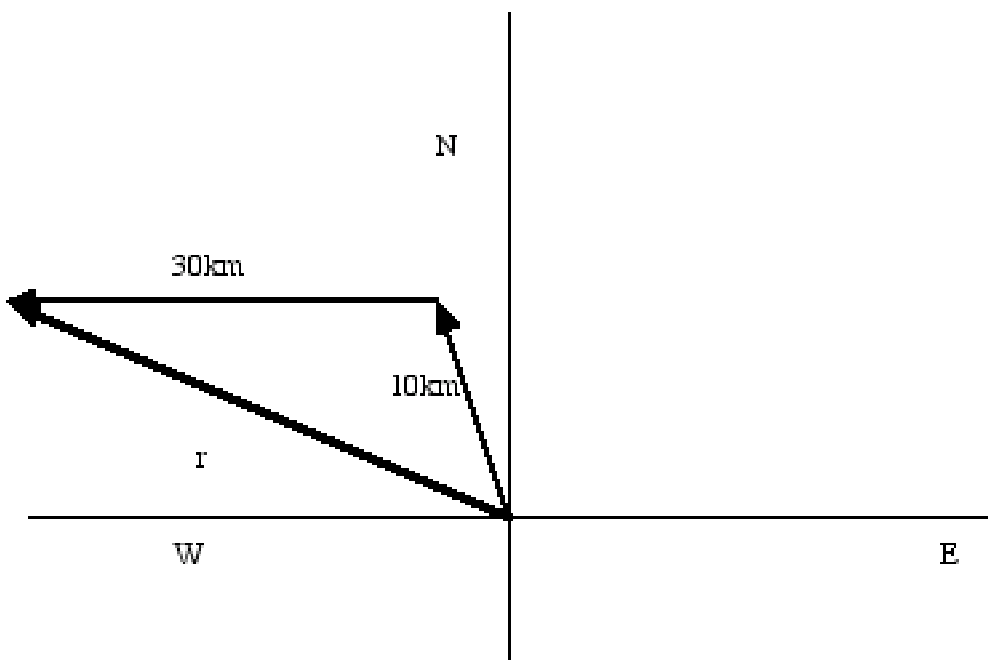
c) What acceleration would the diver need to experience to only reach a depth of 2m. How does this compare to gravitational acceleration?

$$y_i = 0m$$
$$y_f = -2m$$
$$v_i = -10m/s$$
$$v_f = 0m/s$$
$$a = ?$$
$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$
$$0 = v_i^2 + 2a(y_f - 0)$$
$$a = -\frac{v_i^2}{2y_f} = -\frac{(-10m/s)^2}{2 \cdot (-2m)} = 25m/s^2$$

...over 2g's

3. A woman hikes 10 km 30 degrees west of north in approximately 2 hours. She stops for lunch, and then hikes due west for 30 km in 4 hours. She stops and sets up camp for the night, spending 8 hours at this location. She hikes directly back to her starting point in 5 hours the next day.

a. Draw the hike.



b. Where does she spend the night.

We can find her location via vector addition.

$$\vec{r}_1 = -10\sin 30\, km\,\hat{i} + 10\cos 30\, km\,\hat{j}$$
$$\vec{r}_2 = -30km\,\hat{i}$$
$$\vec{r} = \vec{r}_1 + \vec{r}_2$$
$$= -35km\,\hat{i} + 8.67\,km\,\hat{j}$$

To compute the average velocity

$$\vec{v} = \frac{-35km}{6hr}\hat{i} + \frac{8.67\,km}{6hr}\hat{j}$$

c. What is her average speed on each leg of the trip to her night camp spot? What is her average speed for the trip to night camp? What is her average velocity for the entire trip to night camp?

Assuming that she spent almost no time on lunch...

Avg speed for leg 1=10km/2hrs=5km/hr
Avg. speed for leg 2 = 30km/4hrs = 7.5km/hr
Avg speed for both legs = 40km/6hrs = 6.67 km/hr.

d. In what direction must she hike in the morning to get directly back to her starting point? What is her average speed for this return trip? What is her average velocity for this return trip?

To proceed, we find her position in magnitude ...

$$|\vec{r}\,| = \sqrt{(35km)^2 + (8.67\,km)^2} = 36.06km$$

We now know how far she must hike (in a straight line) and in a direction opposite her position, We can calculate the change in position, starting with the overnight camp as the initial position, and 0 as her final position.

$$\vec{r}_i = -35km\,\hat{i} + 8.67\,km\,\hat{j}$$
$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = (0km\,\hat{i} + 0\,km\,\hat{j}) - (-35km\,\hat{i} + 8.67\,km\,\hat{j})$$
$$= 35\,km\,\hat{i} - 8.67\,km\,\hat{j}$$

Avg speed for return = 36.06 km/5hrs = 7.2km/hr

Avg velocity: She hikes in the -r direction in 5 hours.

$$\Delta\vec{r} = 35km\,\hat{i} - 8.67\,km\,\hat{j}$$

$$v_{avg} = \frac{35\,km}{5hrs}\hat{i} - \frac{8.67\,km}{5hrs}\hat{j}$$

$$\tan\theta = \frac{-\dfrac{8.67\,km}{5\,hrs}}{\dfrac{35\,km}{5\,hrs}} = \frac{-8.67\,km}{35km} = -\frac{8.67}{35}$$

$$\theta = -13.91$$

e. What is her average velocity for the entire hike?

She begins and ends at the same spot so her average velocity is zero.