# Projectile Motion

## Introduction

The motion of an object in two dimensions under the influence of the downward acceleration due to gravity is referred to as *projectile motion*. Near the surface of the earth, this motion simplifies further since the acceleration due to gravity is nearly constant. If we ignore air friction, the description of the motion simplifies even further since $a_x = 0$ (acceleration in the x direction) and $a_y = -g$ (acceleration in the y direction).

$$v_{ix} = v\cos\theta$$

$$v_{iy} = v\sin\theta$$


**x motion**

$$x = x_i + v_{ix}t$$

$$v_{fx} = v_{ix}$$

**y motion**

$$y_f = y_i + v_{iy}t - \frac{1}{2}gt^2$$

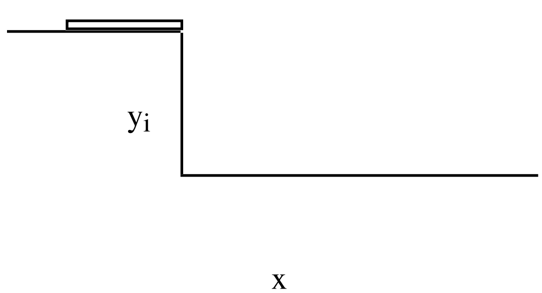
$$v_{fy} = v_{iy} - gt$$

$$v_{fy}^2 = v_{iy}^2 - 2g(y_f - y_i)$$

In this experiment, we will use projectile motion to determine initial velocity of a projectile in two different ways. We will refer to these as the Range-Fall method and the Range-Time of Flight Method. In each case, we take $x_0 = 0$

### Range-Fall Method

In the Range-Fall Method, we will fire the projectile in the perfectly horizontal direction from a known height $y_i$. We measure the distance to where it lands. In that case,




$$v_{ix} = v$$

$$v_{iy} = 0$$

$$x_i = 0$$

$$y_i = h$$

$$x_f = measured$$

$$y_f = 0$$

With these constraints, our equations of motion simplify even further.

**x motion**

$$x = v_{ix}t$$

**y motion**

$$0 = y_i - \frac{1}{2}gt^2$$

We solve for t in terms of $y_i$ and substitute in to find v.

$$t = \sqrt{\frac{2y_i}{g}}$$

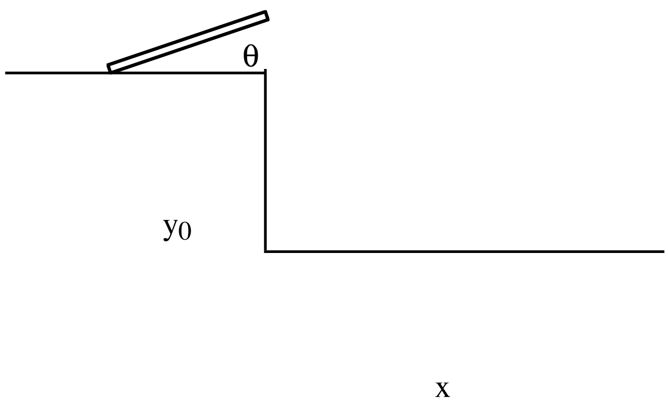
Substituting in and solving for the projectile velocity:

$$v = \frac{x_f}{t} = x_f \cdot \sqrt{\frac{g}{2y_i}} \qquad \textbf{(1)}$$

So with a measured range x for a projectile fired horizontally we can determine its initial velocity.

**Range-Time of Flight Method.**

In the Range-Time of Flight Method, we measure the time of flight as well as the range for a projectile fired at an angle θ. The calculation of the velocity is straightforward.




$$\begin{aligned} v_{ix} &= v\cos\theta \\ v_{iy} &= v\sin\theta \\ x_i &= 0 \\ y_i &= h \\ x_f &= measured \\ y_f &= 0 \end{aligned}$$

$$\begin{aligned} x_f &= v_{ix}t = v\cos\theta \cdot t \\ v &= \frac{x_f}{\cos\theta \cdot t} \end{aligned} \qquad \textbf{(2)}$$

We are interested in making the time large since small times will be difficult to measure, so we will choose to fire the projectile upward.

## Procedure

Set up a foam board as a landing pad. Attach a piece of paper to the foam board and carefully place the board so that the projectile will land on the paper. Mark the position of the board carefully on the floor using masking tape. This landing pad will allow us to measure the range x. NOTE: Use a second board to protect the wall or cabinets from the projectile.

## A. Range-Fall Method

**NOTE: Always make sure that the area in front of the launcher is clear before you fire. The person firing the projectile has the responsibility to make sure that no one will be hit by the projectile.**

1. Set up the projectile launcher so that it is level and fires perfectly horizontally.

2. Carefully measure the distance from the floor to the level that the projectile is launched from. This distance is $y_i$.

3. Fire the projectile and mark where it lands. Carefully realign the board and repeat to take a total of 10 shots.

4. Carefully determine the range of each shot by measuring the positions on the paper, remembering to compute the total horizontal distance covered.

5. Find the average range x and use equation (1) to find the magnitude of the initial velocity v.

## B. Range-Time of Flight Method

1. Using the wooden blocks provided, tilt the projectile launcher upward. Measure the angle made with the horizontal.

2. Measure and record the time and mark the landing position for 10 projectile shots. Carefully realign the board after each shot.

3. Carefully determine the range of each shot by measuring the positions on the paper, remembering to compute the total horizontal distance covered.

4. Compute a velocity $v_0$ for each shot using the measured time and range. Find the average velocity for the ten shots you have taken.

5. Compute an average time and an average range and use them in equation (2) to find the magnitude of the initial velocity v.

6. Calculate a percent difference between the two velocities found in the Range-Time of Flight method and the Range-Fall method that you determined with each method.

## C. Outdoor Projectile

We will attempt to fire a projectile using a catapult like device known as a "trebuchet". In this portion of the experiment, we will try to find the velocity of the projectile, the angle of launch, the maximum height, and the potential maximum range.

1. Measure the time of flight from when the projectile leaves the trebuchet until it hits the ground.

2. Measure the range of the projectile by pacing off the distance.

3. We can use the fact that the projectile was launched from zero height and ended at zero height to calculate the angle of launch. Show that the angle is given by

$$\tan\theta = \frac{g\,t^2}{2\,x_f}$$

Use this expression to find the angle.

4. Using the angle, compute the velocity using

$$v_0 = \frac{x_f}{\cos\theta\, t}$$

## Questions

1. Which of the two methods do you expect to be more accurate? Why?

2. Use the velocity that you measured in the Range-Fall Method to predict what time you should measure in the Range-Time of Flight Method. How did your measured times compare?

3 Use your maximum and minimum measured ranges to estimate the uncertainty in the velocity in the Range-Fall method? Why are the ranges changing at all? What does this imply about the launcher?

4. Use your maximum and minimum measured velocities to estimate the uncertainty in the velocity in the Range-Time of Flight method. What is the principle cause of this uncertainty.

5. Why did we tilt the launcher upward for the Range-Time of Flight method? Compute how much longer the time of flight was compared to the time of flight that you would have expected in the Range-Fall method.