Measurement of g Using Free-Fall

Introduction

The acceleration of all masses near the surface of the earth is nearly a constant value of 9.8 m/s^2 (980 cm/s^2). The measurement of this quantity has a long history that dates back to Galileo. In recent times, very precise measurements of the acceleration due to gravity have been used for a variety of applications. The measurement of g has been to investigate claims that new forces exist, to find mineral and oil deposits, to investigate geological formations, and to enhance guidance and targeting systems in the military.

In this experiment, we hope to accomplish two tasks. First, we will investigate the constant acceleration equations of motion:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + at$$

Is the acceleration due to gravity constant as we have been told? Do position and velocity for a free falling object obey these equations?

If we are successful in demonstrating that acceleration is a constant, we wish to determine the value of g. We can do this in two ways. We can fit the position vs. time curve directly using the polynomial:

$$y = \alpha t^2 + \beta t + \gamma$$

The coefficients α, β and γ can be associated with the parameters of the motion.

$$\begin{aligned} \alpha &\to \frac{1}{2} a \\ \beta &\to v_0 \\ \gamma &\to y_0 \end{aligned}$$

We can also do a linear fit of the velocity vs. time using

$$v = mt + b$$

and associating the slope and intercept with the motion:

$$\begin{aligned} m &\to a \\ b &\to v_0 \end{aligned}$$

After we have extracted g in these two ways, we can compare the two values with each other and with the given value of g.

Procedure

To the acceleration g, we need to record position at regular time intervals. To do this in a small lab setting, we need to record positions over very small time intervals since we expect that g is of order $10\frac{m}{s^2}$. We will use a free falling mass that falls along two charged wires. The mass is held in place by an electromagnet until it is released. After the magnet is powered off, the wires will be charged every 1/60 of a second and a spark will arc between the wires and the falling mass. The spark will leave a small burn spot on a piece of spark tape. The burn mark records the position of the mass every 1/60 of a second.

Data Acquisition

1. Check to see that the free fall apparatus is leveled properly. It is important that the mass fall perfectly vertically between the charged wires.

2. Check to see that a clean piece of spark tape is in place for your data run.

3. Turn on the power to the electromagnet. You should only use enough voltage to hold the mass in place.

4. When the sparker is engaged, the wires on the apparatus are are charged with high voltage. **If you touch the wires while the sparker is engaged, you will be shocked. Please be careful.** While the current present is not enough to seriously harm you, you will certainly be startled.

Make sure that everyone is clear of the apparatus before you engage the spark source. After you are sure that you and your partners are all clear of the apparatus, you may engage the sparker and turn off the power to the electromagnet. The mass should fall and leave a trail of sparks.

5. Carefully remove the spark tape and set the new tape to be ready for the next group.

Data Analysis

1. Tape the spark record down on the lab bench so that it is flat and straight. Examine the tape and check that you can see at least 20 spark marks. They should get farther apart as you move down the tape. Pick a spark near the top of the tape (but not one of the first few) and circle and number each spark mark.

2. Align a two-meter stick with the line of sparks and record the position in cm of each spark. Spark one will be at 0 s, spark 2 at (1/60)s, and so on. Be sure to set the two-meter stick on end to do this.

3. Estimate the uncertainty associated with your position measurement and record it with the measurement.

4. Carefully plot the position vs. time. Be sure to draw an error bar on each point that is the size of the uncertainty of your measurement. Does this curve look parabolic?

5. Compute and record the average velocity for each interval of motion. This is the average velocity over the time interval.

$$v = \frac{x_{i+1} - x_i}{\Delta t}$$

where x are the positions as marked by successive sparks. Δ t = 1/60 s. You should see the velocities getting uniformly larger. You should record the velocities as being at $t = \frac{1}{120}s, \frac{3}{120}s, \frac{5}{120}s, ...$

6. Carefully plot the velocity vs. time in your notebook. You will later indicate the uncertainty in each velocity with an error bar. Does the plot look linear?

7. Using a ruler, draw the best fit line. (Physicists refer to this as an “eyeball fit”-- which turns out to be amazingly good!). The best fit line should go through as many of the data as possible. Of the points that do not lie on the line, half should be below the line and half should be above.

8. Find the slope of your line. This is your experimental value for g in cm/s^2.

9.* Compute the uncertainty in the velocities, σv. We will assume that the uncertainty in the time is negligible You will need to estimate the uncertainty in the position measurements. The uncertainty will be

$$\begin{aligned} \sigma_v &= v\sqrt{(\frac{\sigma_x}{x_{i+1}})^2 + (\frac{\sigma_x}{x_i})^2 + (\frac{\sigma_t}{t})^2} \\ &= v\sqrt{(\frac{\sigma_x}{x_{i+1}})^2 + (\frac{\sigma_x}{x_i})^2} \end{aligned}$$

You and your lab partner should divide your data set in half so that each of you does half.

10.* Return to your velocity vs. time graph. Add the errors bars that you have calculated and draw the line that has the maximum slope that could still be construed as going through the data and their errors. Draw a line that has the minimum slope that could still be construed as going through the data and their error bars. Compute the slopes of these two lines and compare them to your result from above to determine the error in your slope, and thus the error in your experimental value of g. Here you will have effectively found the uncertainty in g using graphical means.

11.* Be sure to interpret what your data mean and what all of the constants mean. Can you confirm that the acceleration due to gravity is constant? What specific features in your data allows you to make this conclusion.

12.* Compute a percent error between each measured value and the given value for g. Is g within one error bar?

13. One of your lab group should keep the spark tape in his or her lab notebook.

Computing. This can be done at any time during the upcoming week.

In this section, you will learn how to use some new data analysis tools. These are tools that you might find in a research level physics environment. You will 1) Create files that contain your time/position data and time/velocity data, 2) graph and fit those data, and finally 3) record the fit results and print.

1. Login to your account on the computers in the lab.

2. Find the application X11.app in Applications > Utilities and start it. A window should appear that is labeled "xterm"

3. We will create the file using and editor. Editors are related to word processors, but they are not word processors. Editors do not include font information--they create flat text files. The editor that we will use is called "xedit" Type in the window that appears:

```
xedit position.dat &
```

You should see a new window appear. You will type your data in this window.

4. Enter data. Make two columns using spaces or tabs. The left column should be the time in decimal form. The right column is your position in cm.

5. Periodically hit the save button. When you are all done, you can hit save and then quit.

6. Now we will begin graphing and fitting. Return to the xterm window and type

```
xmgrace &
```

The graphing window should appear.

7. You can import your data by going to Data > Import > ascii

8. You can control the look of the graph with menus under Plot. In particular, consider Plot > Set Appearance

9. You can fit your data with Data > Transformations > Regression You will need to select the type of fit. For position data, select "Quadratic" and for velocity data select "Linear" Be sure to record the results.

10. Be sure to save your plot using File > Save. Your plot name should have a suffix of .agr

11. Repeat for your velocity data. Begin by opening a new file by typing in the xterm window

```
xedit velocity.dat
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12. Email Dr. Fasano when you are satisfied with your graph. He will gather all of the plots electronically and send printable copies to include with your lab write up.

Questions

1. Use a computer program of *your choosing* (like Excel, Minitab or Mathematica) to fit the velocity vs. time graph using linear regression. What value for g does this procedure determine? What value for the initial velocity does the computer fit yield? How do these computer generated values compare to your "eyeball fit" and the given value of g? If you need help with this, see either your lab instructor or your lab teaching assistant.

2. Plot velocity vs. time for a mass falling from rest for 30 seconds. Do you think that a mass continues to accelerate without limit. Sketch what happens and explain why you think this.

3. What assumptions have we made in doing this experiment? How do these assumptions affect your results.

4. Explain in some detail why we need such a short time interval to make our measurement of g?

5. Why is your v_0 not equal to 0? How did you determine it from your data? Didn't we drop the mass from rest?

6. How could this experiment be improved?

BONUS:

Use a computer program of *your choosing* to do a nonlinear fit of the position vs. time for your data You should use the equation

$$y = \alpha t^2 + \beta t + \gamma$$

Use the computer determined values for b, c and d to determine the acceleration due to gravity, the initial velocity and the initial position? How do these values compare to the values you have determined in other parts of the experiment?