## Physics 130

Exam 4

## Short Answer section. Please answer all of the short answer questions.

1. A metal hoop with a mass of 5 kg has a radius of 1 m . If it spins with an angular velocity of $\omega=10 \mathrm{rad} / \mathrm{s}$, compute the rotational kinetic energy and angular momentum that the hoop has.

Note: For a hoop, $I=m r^{2}$.

$$
\begin{aligned}
& K E_{f}=\frac{1}{2} I \omega^{2}=\frac{1}{2}\left(m r^{2}\right) \omega^{2}=\frac{1}{2}\left(5 \mathrm{~kg} \cdot(1 \mathrm{~m})^{2}\right)(10 \mathrm{rad} / \mathrm{s})^{2}=250 \mathrm{~J} \\
& L=I \omega=\left(5 \mathrm{~kg} \cdot(1 \mathrm{~m})^{2}\right)(10 \mathrm{rad} / \mathrm{s})=50 \mathrm{kgm}^{2} \mathrm{~s}
\end{aligned}
$$

2. What angular momentum does the moon have in its orbit? Take the distance from the earth to the moon, the mass of the moon, and the velocity of the moon in its orbit to be

$$
\begin{aligned}
& r_{e m}=3.844 \times 10^{8} \mathrm{~m} \\
& m_{m}=7.348 \times 10^{22} \mathrm{~kg} \\
& v_{m}=1,023 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
L=m r v=7.348 \times 10^{22} \mathrm{~kg} \cdot 3.844 \times 10^{8} \mathrm{~m} \cdot 1,023 \mathrm{~m} / \mathrm{s}=2.89 \times 10^{34} \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}
$$

3. A disk accelerates from rest $\omega_{i}=0 \mathrm{rad} / \mathrm{s}$ to a final angular velocity $\omega_{f}=20.0 \mathrm{rad} / \mathrm{s}$. If this angular acceleration took place over 0.5 s , what was the angular acceleration $\alpha$ ?

$$
\begin{aligned}
& \omega_{f}=\omega_{i}+\alpha t \\
& \alpha=\frac{\omega_{f}-\omega_{i}}{t}=\frac{20 \mathrm{rad} / \mathrm{s}-0}{0.5 \mathrm{~s}}=40 \mathrm{rad} / \mathrm{s}^{2}
\end{aligned}
$$

4. A force is applied to the edge of a sphere of mass 10 kg with radius 0.5 m (as shown below) to make it spin about an axis that runs through its center. The sphere experiences an angular acceleration $\alpha=0.1 \mathrm{rad} / \mathrm{s}^{2}$. What torque does the sphere experiences? What force F is producing this torque? What direction is the torque pointing? $I_{\text {sphere }}=\frac{2}{5} m r^{2}$

$\tau=I \alpha=\frac{2}{5} m r^{2} \cdot \alpha=\frac{2}{5} \cdot 10 \mathrm{~kg} \cdot(0.5 \mathrm{~m})^{2} \cdot 0.1 \mathrm{rad} / \mathrm{s}^{2}=0.1 \mathrm{Nm}$
$\tau=r F \sin \theta$
$F=\frac{\tau}{r \sin \theta}=\frac{0.1 \mathrm{Nm}}{0.5 \cdot \sin 30}=0.4 \mathrm{~N}$

Using the right hand rule, the torque points out of the paper.
5. A disk of mass $m$ and radius $r$ has an initial linear velocity of $v_{0}$ at the bottom of an incline. It rolls up the incline and reaches a high point. Using energy methods, what vertical change in height h can it achieve? $I_{\text {disk }}=\frac{1}{2} m r^{2}$. Hint: Remember that the disk rolls without slipping.


$$
\begin{aligned}
& K E_{i}=\frac{1}{2} I \omega^{2}+\frac{1}{2} m v^{2}=\frac{1}{2} \cdot \frac{1}{2} m r^{2} \cdot\left(\frac{v_{0}}{r}\right)^{2}+\frac{1}{2} m v_{0}^{2}=\frac{3}{4} m v_{0}^{2} \\
& U_{i}=0 \\
& E_{i}=K E_{i}+U_{i}=\frac{3}{4} m v_{0}^{2} \\
& K E_{f}=0 \\
& U_{f}=m g h \\
& E_{f}=K E_{f}+U_{f}=m g h \\
& E_{f}=E_{i} \\
& m g h=\frac{3}{4} m v_{0}^{2} \\
& h=\frac{3}{4} \frac{v_{0}^{2}}{g}
\end{aligned}
$$

6. Explain why an ice skater speeds up while spinning when she pulls her arms in close to her body.

Conservation of angular momentum. When she pulls her arms in, her moment of inertia falls, so to conserve angular momentum, her angular velocity must rise.
7. A sphere that is spinning with initial angular velocity $\omega_{i}$ has a radius $r_{i}$. It contracts to a final radius $r_{f}=\frac{1}{2} r_{i}$. What will its final angular velocity be in terms of the initial angular velocity. $I_{\text {sphere }}=\frac{2}{5} m r^{2}$

$$
\begin{aligned}
& L_{f}=L_{i} \\
& I_{f} \omega_{f}=I_{i} \omega_{i} \\
& \frac{2}{5} m r_{f}^{2} \omega_{f}=\frac{2}{5} m r_{i}^{2} \omega_{i} \\
& \omega_{f}=\frac{r_{i}^{2}}{r_{f}^{2}} \omega_{i}=\frac{r_{i}^{2}}{\left(\frac{1}{2} r_{i}\right)^{2}} \omega_{i}=4 \omega_{i}
\end{aligned}
$$

8. Compute the gravitational force between a proton and an electron in an atom. $\left(m_{e}=9.11 \times 10^{-31} \mathrm{~kg}, m_{p}=1.67 \times 10^{-27} \mathrm{~kg}, r=1 \times 10^{-10} \mathrm{~m}, G=6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)$

$$
F_{G}=\frac{G M m}{r^{2}}=1.01 \times 10^{-47} \mathrm{~N}
$$

Problems. Please work two (2) of the three problems and clearly indicate which problems you wish to have graded.

1. A spherical shell with mass $\mathrm{m}=5 \mathrm{~kg}$ and radius $\mathrm{r}=1 \mathrm{~m}$ has a cord wrapped around its equator. It falls a distance $\mathrm{h}=10 \mathrm{~m}$ and the cord unwraps without slipping as the shell falls. $I_{\text {shell }}=\frac{3}{5} m r^{2}$

a) Using energy methods find the velocity $v_{f}$ and angular velocity $\omega_{f}$ after the shell has fallen a distance $\mathrm{h}=10 \mathrm{~m}$.

$$
\begin{aligned}
& K E_{i}=0 \\
& U_{i}=m g h \\
& E_{i}=K E_{i}+U_{i}=m g h \\
& K E_{f}=\frac{1}{2} I \omega^{2}+\frac{1}{2} m v^{2}=\frac{1}{2} \cdot \frac{3}{5} m r^{2} \cdot\left(\frac{v_{f}}{r}\right)^{2}+\frac{1}{2} m v_{f}^{2}=\frac{4}{5} m v_{f}^{2} \\
& U_{f}=0 \\
& E_{f}=K E_{f}+U_{f}=\frac{4}{5} m v_{f}^{2}+0 \\
& E_{f}=E_{i} \\
& \frac{4}{5} m v_{f}^{2}=m g h \\
& v_{f}=\sqrt{\frac{5 g h}{4}}=11.07 \mathrm{~m} / \mathrm{s} \\
& \omega_{f}=\frac{v_{f}}{r}=11.07 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

b) In unrolling, how many radians did it turn through (Remember that it traveled a distance $x=h$ and you can relate x to $\left.\theta_{f} . \quad \theta_{i}=0 \mathrm{rad}\right)$.

$$
\begin{aligned}
& x=r \theta \\
& \theta=\frac{x}{r}=\frac{h}{r}=10 \mathrm{rad}
\end{aligned}
$$

c) Given that the initial angular velocity was zero and the you have calculated the final angular velocity, what angular acceleration $\alpha$ and torque $\tau$ did the disk experience?

$$
\begin{aligned}
& \omega_{f}^{2}=\omega_{i}^{2}+2 \alpha\left(\theta_{f}-\theta_{i}\right) \\
& \omega_{f}^{2}=0^{2}+2 \alpha\left(\theta_{f}-0\right) \\
& \alpha=\frac{\omega_{f}^{2}}{2 \theta_{f}}=6.13 \mathrm{rad} / \mathrm{s}^{2} \\
& \tau=I \alpha=\frac{3}{5} m r^{2} \cdot \alpha=18.4 \mathrm{Nm}
\end{aligned}
$$

d) Given the torque (and assuming that the force that caused it was applied at 90 degrees) what force produce this torque? Note: This force is the tension in the string.

$$
\begin{aligned}
& \tau=r F \sin \theta \\
& F=\frac{\tau}{r \sin \theta}=18.4 \mathrm{~N}
\end{aligned}
$$

2. A scaffold is set up as shown below. The cable on the right can withstand a tension of up to $\mathbf{5 0 0 0} \mathbf{N}$. The scaffold plank itself is so light that we can neglect its mass.

a. Draw all the forces acting on the plank and define clockwise motion of rotation about the hinge as positive torque.

b. Compute the torque about the hinge on the left, and use it to determine what mass can be placed as shown at 8 m . (Hint: Assume the maximum tension in the cable. $\mathrm{T}=5000 \mathrm{~N}$ )

$$
\begin{aligned}
& \tau=0=8 \cdot m g-10 \cdot T \sin 75 \\
& m=\frac{10 \cdot T \sin 75}{8 g}=616.0 \mathrm{~kg}
\end{aligned}
$$

c. What horizontal force must the hinge provide in this case?

$$
\begin{aligned}
& 0=T \cos 75-F_{h} \\
& F_{h}=T \cos 75=1294.1 \mathrm{~N}
\end{aligned}
$$

d. What vertical force must the hinge provide in this case?

$$
\begin{aligned}
& 0=F_{v}+T \sin 75-m g \\
& F_{v}=m g-T \sin 75=1207.2 N
\end{aligned}
$$

Hint: Recall that the net force in both the x and y direction should be zero.
3. Consider the teeter-totter below.

a. For the teeter-totter to remain balanced as shown, at what x must the 150 kg mass be placed? (Hint: Compute the torque about the pivot at the center and set it to zero)

b. What upward force must the pivot provide?

$$
\begin{aligned}
& 0=F_{\text {pivot }}-150 g-50 g \\
& F_{\text {pivot }}=150 g+50 g=1960 \mathrm{~N}
\end{aligned}
$$

Now the large mass is moved all the way to the (corrected: should be left) as shown below.

c. What torque about the pivot would the plank experience in the new configuration?

$$
\begin{aligned}
\tau & =5 \cdot 50 \mathrm{~g}-5 \cdot 150 \mathrm{~g} \\
& =-4900 \mathrm{Nm}
\end{aligned}
$$

Note: the sign implies that the torque is counter-clockwise (out of paper).
d. Given the torque that you computed in (c), if the moment of inertial of the plank is $I=50 \mathrm{~kg} \mathrm{~m}^{2}$, what angular acceleration would it experience?

$$
\begin{aligned}
& \tau=I \alpha \\
& \alpha=\frac{\tau}{I}=\frac{-4900 \mathrm{Nm}}{50 \mathrm{kgm}^{2}}-98 \mathrm{rad} / \mathrm{s}^{2}
\end{aligned}
$$

## Serious Hints!

$$
\begin{aligned}
& \tau=I \alpha \\
& \vec{\tau}=\vec{r} \times \vec{F} \\
& \tau=r F \sin \theta \\
& L=I \omega \\
& \vec{L}=\vec{r} \times \vec{p} \\
& L=r p \sin \theta \\
& K E_{\text {rotational }}=\frac{1}{2} I \omega^{2}
\end{aligned}
$$

$$
\begin{aligned}
& x=r \theta \\
& v=r \omega \\
& a=r \alpha
\end{aligned}
$$

$$
\theta_{f}=\theta_{i}+\omega_{i} t+\frac{1}{2} \alpha t^{2}
$$

$$
\omega_{f}=\omega_{i}+\alpha t
$$

$$
\omega_{f}^{2}=\omega_{i}^{2}+2 \alpha\left(\theta_{f}-\theta_{i}\right)
$$

## Conversions:

| 1 mile $=1.6 \mathrm{~km}$ | 1 foot $=0.3048 \mathrm{~m}$ |
| :--- | :--- |
| 1 inch $=2.54 \mathrm{~cm}$ |  |
| 1 mile $=5280 \mathrm{ft}$. |  |

## Constants:

$\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}=32 \mathrm{ft} / \mathrm{s}^{2}$

## Misc. Equations:

Volume of a sphere $=(4 / 3) \mathrm{pr}^{3} \quad$ Volume of a cylinder $=\mathrm{pr}^{2} \mathrm{~L}$

Area of a circle $=\mathrm{pr}^{2} \quad$ Volume of a rect. obj. $=\mathrm{L} \times \mathrm{W} \times \mathrm{D}$

Area of a triangle $=(1 / 2)^{*}$ base*height
$\sin \theta=\frac{o p p}{\text { hyp }}$.
$\cos \theta=\frac{a d j}{\text { hyp }}$.
$\tan \theta=\frac{o p p .}{\text { adj. }}$

Derivative of a Polynomial.

$$
\frac{d}{d x}\left(a x^{n}\right)=n a x^{n-1}
$$

Example:

$$
\frac{d}{d x}\left(a x^{3}\right)=3 a x^{2}
$$

Integral of a polynomial.

$$
\int a x^{n} d x=\frac{a x^{n+1}}{n+1}
$$

Example:

$$
\int b x^{3} d x=\frac{b x^{4}}{4}
$$

