Physics 130
Exam 4

Short Answer section. Please answer all of the short answer questions.

1. A metal hoop with a mass of 4 kg has a radius of 2m. If it spins with an angular velocity of $\omega = 30\,rad/s$, compute the rotational kinetic energy that the hoop has. Note: For a hoop, $I = m r^2$.

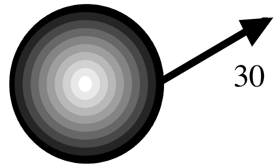
$$K_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\cdot 4kg\cdot(2m)^2\cdot(30\,rad/s)^2 = 7200J$$

2. A disk accelerates from rest to a final angular velocity $\omega_f = 20.94\,rad/s$. If this angular acceleration took place over 0.5 s, what was the angular acceleration α?

$$\omega_f = \omega_i + \alpha t$$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{20.94\ rad/s - 0\ rad/s}{0.5s} = 41.88\,rad/s^2$$

3. A force of 500 N is applied to a sphere of mass 25 kg and radius 2m as shown below. What torque and angular acceleration does this sphere experience? $I_{sphere} = \frac{2}{5}mr^2$ (Angle is in degrees)



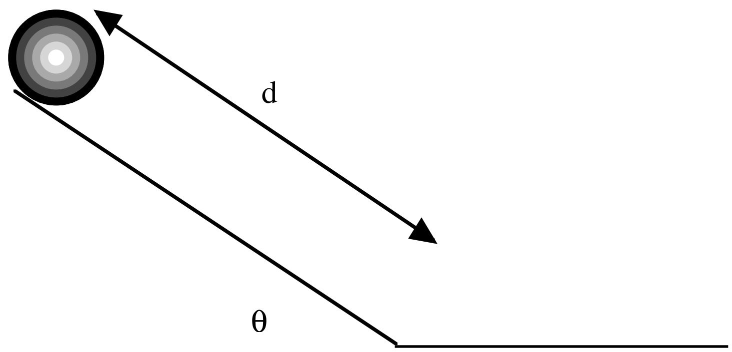
$$\tau = r\,F\sin\theta = 2m\cdot 500\cdot\sin 30 = 500Nm$$

$$\tau = I\,\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{500Nm}{\frac{2}{5}\cdot 25kg\cdot(2m)^2} = 12.5rad/s^2$$

4. A solid sphere (mass m and radius r, $I_{sphere} = \frac{2}{5}mr^2$) rolls down an incline as shown below. If the distance that it moves down the incline is d, show that its velocity at the bottom of the incline is

$$v = \sqrt{\frac{10gd\sin\theta}{7}}$$



$$E_f = E_i$$
$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgd\sin\theta$$
$$\frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mr^2\cdot\frac{v^2}{r^2} = mgd\sin\theta$$
$$\frac{1}{2}mv^2 + \frac{1}{5}mv^2 = mgd\sin\theta$$
$$\frac{7}{10}mv^2 = mgd\sin\theta$$
$$v = \sqrt{\frac{10gd\sin\theta}{7}}$$

5. A sphere that is spinning with initial angular velocity ω_i has a radius r_i. It expands to a final radius $r_f = 2r_i$. What will its final angular velocity be in terms of the initial angular velocity.

Note: $I_{sphere} = \frac{2}{5}mr^2$

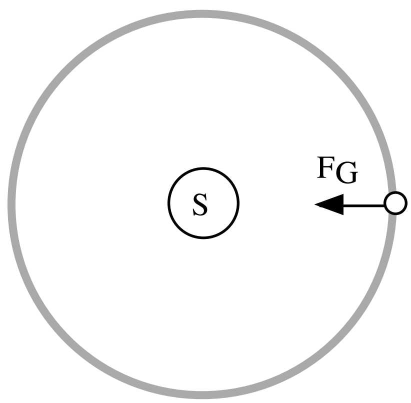
$$L_f = L_i$$
$$I_f\omega_f = I_i\omega_i$$
$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{\frac{2}{5}m\,r_i^2}{\frac{2}{5}m\,r_f^2}\omega_i = \frac{r_i^2}{r_f^2}\omega_i = \frac{r_i^2}{(2r_i)^2}\omega_i$$
$$\omega_f = \frac{1}{4}\omega_i$$

6. Compute the gravitational force between a proton and an electron in an atom. ($m_e = 9.11\times10^{-31} kg$, $m_p = 1.67\times10^{-27} kg$, $r = 1\times10^{-10} m$, $G = 6.67\times10^{-11} N\cdot m^2/kg^2$)

$$F_G = G\frac{Mm}{r^2} = 6.67\times10^{-11} N\cdot m^2/kg^2 \frac{1.67\times10^{-27} kg\cdot 9.11\times10^{-31} kg}{(1\times10^{-10} m)^2} = 1.01\times10^{-47} N$$

7. Using the gravitational force between the earth and the sun, find the mass of the sun. Assume that the orbit is perfectly circular. The velocity of the earth in its orbit and distance from the sun are:

$$v = 2.989\times10^4 m/s$$
$$r_{em} = 1.5\times10^{11} m$$

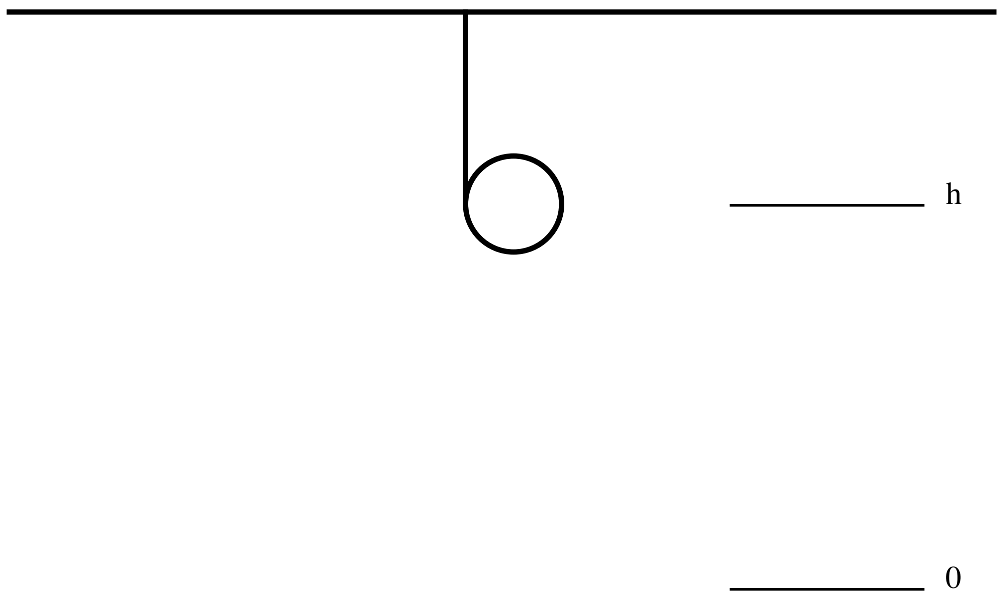


Note: This is a Newton's gravity/centripetal force problem.

$$\frac{m_E v^2}{r} = G\frac{M_S m_E}{r^2}$$
$$M_S = \frac{r v^2}{G} = \frac{1.5\times10^{11} m\cdot(2.989\times10^4 m/s)^2}{6.67\times10^{-11} N\cdot m^2/kg^2} = 2.01\times10^{30} kg$$

Problems. Please work two (2) of the three problems and clearly indicate which problems you wish to have graded.

1. A disk with mass m=5 kg and radius r=1m has a cord wrapped around it. It falls a distance h=10m and the cord unwraps as the mass falls. $I_{disk} = \frac{1}{2} m\, r^2$



a) Using energy methods find the velocity and angular velocity after the disk has fallen a distance h.

$$E_f = E_i$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{1}{2}mr^2\cdot\frac{v^2}{r^2} = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{4}mv^2 = mgh$$

$$\frac{3}{4}mv^2 = mgh$$

$$v = \sqrt{\frac{4gh}{3}}$$

$$= 11.43 m/s$$

$$\omega = \frac{v}{r} = \frac{11.43 m/s}{1m}$$

$$= 11.43 rad/s$$

b) In unrolling, find θ_f, the number of radians that the disk turned through (Remember that it traveled a distance x=h and you can relate x to θ_f. $\theta_i = 0\,rad$).

$$h = r\,\theta_f$$

$$\theta_f = \frac{h}{r} = 10 rad$$

c) Given that the initial angular velocity was zero and you have calculated the final angular velocity, and θ_f, what is the angular acceleration. Use this alpha to find the torque that the disk experienced?

$$\omega_f^{\,2} = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

$$\alpha = \frac{\omega_f^{\,2} - \omega_i^{\,2}}{2(\theta_f - \theta_i)} = \frac{(11.43 rad/s)^2 - 0^2}{2(10 rad - 0)} = 6.53 rad/s^2$$

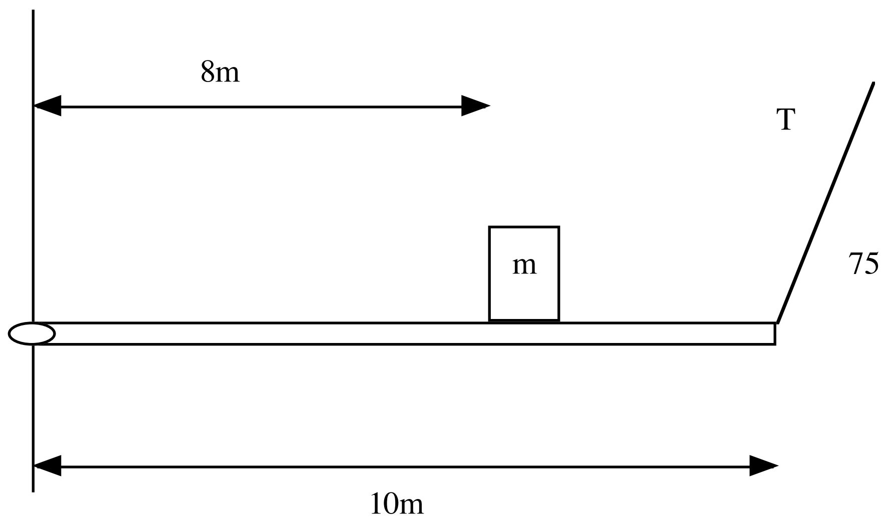
$$\tau = I\alpha = (\frac{1}{2}mr^2)\alpha = (\frac{1}{2}\cdot 5kg\cdot(1m)^2)\cdot 6.53 rad/s^2 = 16.33 Nm$$

d) Given the torque (and assuming that it was applied at 90 degrees) what force was present? The force was present at tension in the string.

$$\tau = r\,F\sin 90$$

$$F = \frac{\tau}{r\sin 90} = \frac{16.33 Nm}{1m} = 16.33 N$$

2. A scaffold is set up as shown below. The cable on the right can withstand a tension of up to **5000 N**. The scaffold plank itself is so light that we can neglect its mass.



a) Compute the torque about the hinge on the left, and use it to determine what mass can be placed as shown at 8m give the 5000 N tension T.

$$0 = 8 \cdot mg - 10T \sin 75$$
$$m = \frac{10T \sin 75}{8g} = \frac{10 \cdot 5000N \sin 75}{8 \cdot 9.8} = 616.0kg$$

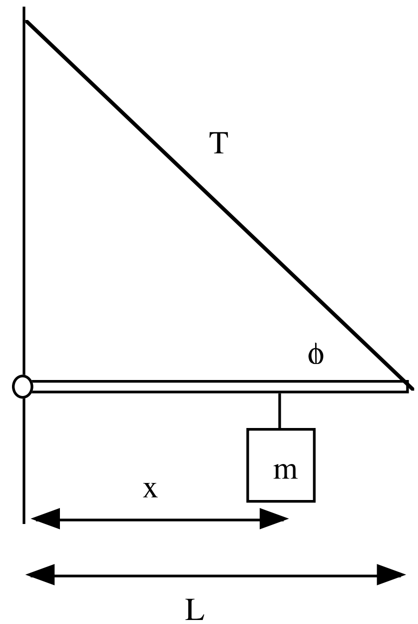
b) What horizontal force must the hinge provide in this case?

$$0 = T\cos 75 - F_x$$
$$F_x = T\cos 75 = 5000N\cos 75 = 1294.1N$$

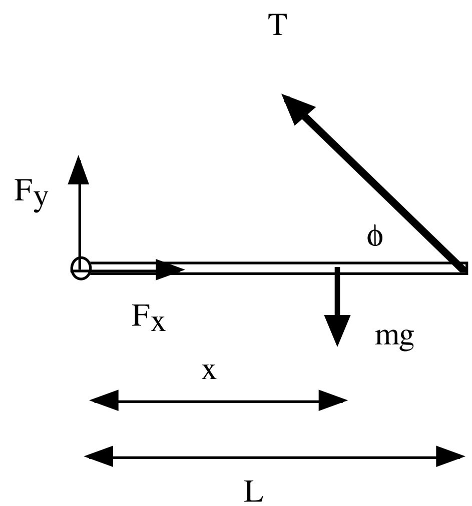
c) What vertical force must the hinge provide in this case?

$$0 = F_y + T\sin 75 - mg$$
$$F_y = mg - T\sin 75 = 616.0 \cdot 9.8 - 5000\sin 75 = 1207.4\,N$$

3. A rod of negligible mass is supported as shown below. A mass m hangs as shown.



a) Draw all of the forces acting on the bar. Be sure to remember the forces due to the hinge.



b) Write the net force in the x and y direction and net torque on the rod in terms of the tension and the forces due to the hinge.

$x - direction$	$y - direction$	$Torque\ about\ hinge$
$0 = F_x - T\cos\varphi$	$0 = F_y + T\sin\varphi - mg$	$0 = xmg - LT\sin(180 - \varphi)$

c) Find the tension using the net torque.

$$\text{Torque about hinge}$$
$$0 = x\,mg - LT\sin(180-\varphi)$$
$$T = \frac{x}{L\sin(180-\varphi)}mg$$

d) Find the horizontal and vertical forces due to the hinge.

$$x-\text{direction} \qquad\qquad y-\text{direction}$$
$$0 = F_x - T\cos\varphi \qquad\qquad 0 = F_y + T\sin\varphi - mg$$
$$F_x = T\cos\varphi \qquad\qquad F_y = mg - T\sin\varphi$$

Hints!

$$\tau = I\alpha$$
$$\tau = rF\sin\theta \qquad x = r\theta \qquad \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$
$$L = I\omega \qquad v = r\omega \qquad \omega_f = \omega_i + \alpha t$$
$$K_r = \frac{1}{2}I\omega^2 \qquad a = r\alpha \qquad \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

Conversions:

1 mile = 1.6 km

1 foot = 0.3048 m

1 inch = 2.54 cm

1 mile = 5280 ft.

Constants:

g = 9.8 m/s^2 = 32 ft/s^2

Misc. Equations:

Volume of a sphere = (4/3) π r^3

Volume of a cylinder = π r^2 L

Area of a circle = π r^2

Volume of a rect. obj. = L x W x D

Area of a triangle =(1/2)*base*height

sin θ = opp/hyp

cos θ = adj/hyp

tan θ = opp/adj

Derivative of a Polynomial.

$$\frac{d}{dx}(a\,x^n) = n\,a\,x^{n-1}$$

Example:

$$\frac{d}{dx}(a\,x^3) = 3\,a\,x^2$$

<u>Integral of a polynomial.</u>

$$\int a\,x^n dx = \frac{a\,x^{n+1}}{n+1}$$

Example:

$$\int b\,x^3 d\,x = \frac{b\,x^4}{4}$$