Physics 130 Exam 1

Short Answer. Please answer all of the short answer questions.

1. For reasons that remain unclear, some glaciers can "surge" at speeds much higher than normal. If a glacier surges at a speed of 300 m/day, what is its speed in m/s and mi/hr?

$$s = \frac{300m}{d} \cdot \frac{1d}{24hr} \cdot \frac{1hr}{3600s} = 3.47 \times 10^{-3} \, m \, / \, s$$
$$= \frac{300m}{d} \cdot \frac{1d}{24hr} \cdot \frac{1km}{1000m} \cdot \frac{1mi}{1.6km} = 7.81 \times 10^{-3} \, mi \, / \, hr$$

2. Estimate how much air you breathe in one year. What is the mass of the air that you have breathed in one year? Air has a density of 1.2 g/cm^3

You can make this estimate by thinking about the size of your chest cavity. You could imagine your lungs are each the same as about a 1 liter bottle. They don't each fill completely (they're not empty bottles, after all). We could estimate each lung takes 1/2 or 1/3 of a liter on average. You probably breath 10-20 times per minute on average

$$V = \frac{1/3 L}{breath} \cdot \frac{15breath}{\min} \cdot (365 days \cdot \frac{24hr}{1day} \cdot \frac{60\min}{1hr}) = 2.6 \times 10^6 L$$

$$1L = 1000 cm^3$$

$$V = 2.6 \times 10^6 L \cdot \frac{1000 cm^3}{L} = 2.6 \times 10^9 cm^3$$

$$m = \rho V = \frac{1.2g}{cm^3} \cdot 2.6 \times 10^9 cm^3 = 3.12 \times 10^9 g$$

There is a lot of variability in both chest size and breathing rate.

3. One of the ways that we hope to travel to distant places in the galaxy is to go very fast. Suppose that somehow you are able to travel at 0.5 the speed of light. If your deceleration (to slow down) has a magnitude of 2.0 g, how far do you travel (from when you apply the brakes) to when you stop? The speed of light is $3.0 \times 10^8 m / s$

$$y_{i} = 0$$

$$y_{f} = ?$$

$$v_{i} = \frac{1}{2}c$$

$$v_{f} = 0$$

$$a = -2g$$

$$v_{f}^{2} = v_{i}^{2} + 2a(y_{f} - y_{i})$$

$$0 = v_{i}^{2} + 2(-2g)(y_{f} - 0)$$

$$y_{f} = \frac{v_{i}^{2}}{2 \cdot 2g} = 5.74 \times 10^{14} m$$

4. Using the information in problem 3 how long will it take you to stop to visit a planet?

$$y_{i} = 0$$

$$y_{f} = ?$$

$$v_{i} = \frac{1}{2}c$$

$$v_{f} = v_{i} + at$$

$$0 = v_{i} - 2gt$$

$$v_{f} = 0$$

$$t = \frac{v_{i}}{2g} = 7.65 \times 10^{6}s$$

$$a = -2g$$

This is about 3 months!

5. A golfer hits a ball and it rises directly upward. If it reaches a maximum height of 30m, what speed did it have when it left the ground and how long until it reaches its maximum height?

$$y_{i} = 0 \qquad v_{f}^{2} = v_{i}^{2} + 2a(y_{f} - y_{i}) y_{f} = 30m \qquad 0 = v_{i}^{2} + 2(-g)(y_{f} - 0) \qquad v_{f} = v_{i} + at 0 = v_{i} - gt v_{f} = 0 \qquad v_{i}^{2} = 2gy_{f} v_{i} = \sqrt{2gy_{f}} = 24.2m/s \qquad t = \frac{v_{i}}{g} = 2.47s$$

6. Consider vectors $\vec{a} = -4\hat{i} + -3\hat{j}$ and $\vec{b} = -5\hat{i} + 12\hat{j}$ Compute the magnitude and direction of \vec{a} and just the magnitude of \vec{b} . Compute the result of $\vec{a} - \vec{b}$, $\vec{a} \cdot \vec{b}$, and $\vec{a} \times \vec{b}$. What is the angle between the two vectors?

$$|\vec{a}| = \sqrt{(-4)^2 + (-3)^2} = 5$$

$$\tan \theta_a = \frac{-3}{-4} \Rightarrow \theta_a = \tan^{-1} \left(\frac{-3}{-4}\right) = 36.87^\circ + 180^\circ$$

$$|\vec{b}| = \sqrt{(-5)^2 + (12)^2} = 13$$

$$\vec{a} - \vec{b} = (-4 - (-5))\hat{i} + (-3 - 12)\hat{j} = 1\hat{i} - 15\hat{j}$$

$$\vec{a} \cdot \vec{b} = (-4 \cdot -5) + (-3 \cdot 12) = -16$$

$$\vec{a} \times \vec{b} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & -3 & 0 \\ -5 & 12 & 0 \end{bmatrix}$$

$$= \hat{i}(-3 \cdot 0 - 0 \cdot 12) - \hat{j}(-4 \cdot 0 - 0 \cdot -5) + \hat{k}(-4 \cdot 12 - (-3 \cdot -5))$$

$$= 0\hat{i} + 0\hat{j} + -63\hat{k}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{-16}{5 \cdot 13} \Rightarrow \theta = \cos^{-1}\left(\frac{-16}{5 \cdot 13}\right) = 104.25^{\circ}$$

7. The position of an object is given by

$$x(t) = 2t - 5t^2 + 15t^4$$

Find the average velocity over the interval 0 s to 4 s, the instantaneous velocity at 2s, and the instantaneous acceleration at 2s.

$$\begin{aligned} x(t) &= 2t - 5t^{2} + 15t^{4} & x(0) = 0m \\ v(t) &= 2 - 10t + 60t^{3} & x(4) = 3768m & v(2) = 462m / s \\ a(t) &= -10 + 180t^{2} & v_{avg} = \frac{x(4) - x(0)}{4 - 0} = 942m / s & a(2) = 710m / s^{2} \end{aligned}$$

8. The Edmund Fitzgerald sank in Lake Superior in 1975 in a storm. Before it sank, the ship sailed from port approximately 20 degrees north of east for 400 km and then 200 km at 45 degrees south of east. It sank at this point Sketch the trip and using components, find the location of the shipwreck. (Note: This shipwreck, one of the worst on Lake Superior, is immortalized in a song by Gordon Lightfoot).



$$\begin{aligned} \vec{d}_1 &= 400\cos 20^\circ \hat{i} + 400\sin 20^\circ \hat{j} \\ \vec{d}_2 &= 200\cos(-45^\circ)\hat{i} + 200\sin(-45^\circ)\hat{j} \\ \vec{r} &= (400\cos 20^\circ + 200\cos(-45^\circ))\hat{i} + (400\sin 20^\circ + 200\sin(-45^\circ))\hat{j} \\ \vec{r} &= 517.3km\hat{i} + -4.613\hat{j} \end{aligned}$$

9. Sketch the position vs. time, velocity vs. time and acceleration vs. time for a free falling object with no friction?

From lab....

Problems. Please work 2 of the three problems and clearly indicate which problems you wish to have graded.

1. Giraffes are deceptively quick. They can reach a top speed of 37 mi/hr.

a) If a giraffe can accelerate from rest to 37 mi/hr (16.44 m/s) in just 5 seconds, what acceleration can a giraffe achieve in m/s^2 ? What is this acceleration in g's?

$$x_{i} = 0$$

$$x_{f} = ?$$

$$v_{i} = 0$$

$$v_{f} = 16.44 m / s$$

$$a = \frac{v_{f} - v_{i}}{t} = 3.288 m / s^{2}$$

$$a = ?$$

$$t = 5s$$

$$\# g = \frac{3.288 m / s^{2}}{9.8 m / s^{2}} = 0.336$$

b) How far does the giraffe go in this time?

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$
$$v_f^2 = 0 + 2a(x_f - 0)$$
$$x_f = \frac{v_f^2}{2a} = 41.1m$$

A cheetah is much faster than a giraffe--it can run at 70 mi/hr (31.11 m/s), but only for a short distance before it must stop.

c) If a cheetah is chasing a giraffe, and they are both running at their top speed, how far ahead must the giraffe be to not get caught? Assume that the cheetah can only run at top speed for 60 s.

(Hint: Take the initial position of the cheetah to be zero and the initial position of the giraffe to be x_{ig} . Both animals run for 60 seconds, and just getting away means that the giraffe and

cheetah have the same position at the end of their 60 second run. Remember that they are both running at their constant top speeds).

$$\begin{array}{ll} x_{ig} = ? & x_{ic} = ? & x_{fg} = x_{fc} \\ x_{fg} = ? & x_{fc} = ? & x_{ig} = x_{fc} \\ v_{ig} = 16.44 \, m \, / \, s & v_{ic} = 31.11 \, m \, / \, s & x_{ig} + v_{ig}t + \frac{1}{2}a_gt^2 = x_{ic} + v_{ic}t + \frac{1}{2}a_ct^2 \\ v_{fg} = 16.44 \, m \, / \, s & v_{fg} = 31.11 \, m \, / \, s & x_{ig} + v_{ig}t = 0 + v_{ic}t \\ a = 0 & a = 0 & x_{ig} + = 0 + v_{ic}t - v_{ig}t \\ t = 60 \, s & t = 60 \, s & = 880.2 \, m \end{array}$$

2. A sand bag drops from a hot air balloon that is sinking straight down at 15 m/s. The balloon is at 200m when the sandbag is dropped. Ignore frictional effects.

a) What are the initial position and velocity of the sandbag when it is dropped?

$$y_i = 200m$$
$$v_i = -15m / s$$

b) How long does it take for the sandbag to reach the ground?

$$y_{f} = y_{i} + v_{i}t + \frac{1}{2}at^{2}$$

$$y_{i} = 200m$$

$$y_{f} = 0m$$

$$v_{i} = -15m / s$$

$$a = -g$$

$$t = \frac{-(-15) \pm \sqrt{(-15)^{2} - 4(-\frac{9.8}{2})200}}{2(-\frac{9.8}{2})}$$

= 5.04s

c) What is the speed of the sandbag when it hits the ground?

$$v_{f}^{2} = v_{i}^{2} + 2a(y_{f} - y_{i})$$

$$v_{f}^{2} = v_{i}^{2} + 2(-g)(0 - y_{i}) \text{ or } v_{f} = -64.4m / s$$

$$v_{f} = -64.4m / s$$

Note. You could have solved c) first and then found the time

Immediately after the balloon drops the sandbag, it instantly slows to *constant* a downward speed of 7.5 m/s. It is at 200 m when this happens.

d) How much after the sandbag lands does the balloon reach the ground?

We first calculate how long it takes for the balloon to reach the ground. We can then compute h

$$y_{f} = y_{i} + v_{i}t + \frac{1}{2}at^{2}$$

$$0 = y_{i} + v_{i}t$$

$$t = -\frac{y_{i}}{v_{i}} = 26.67s$$

$$\Delta t = 26.67s - 5.04s = 21.63s$$

3. One way of losing a chasing aircraft if you are flying is to fly a difficult path at high speed. If you have practiced the path many times and the person chasing you has not, they will be forced to abandon the chase. Swiss fighter pilots spend many hours flying complicated routes in the Alps for precisely this reason.

The chase consists of 4 legs.

Leg 1: Vertical take-off from base from 0m height to 3000 m in 60s.

Leg 2: High speed flight at 3000 m due north at 250 m/s for 600s

Leg 3: High speed flight at 3000 m due west at 300 m/s for 900 s

Leg 4: Vertical descent from 3000 m to 0 m at a new base in 60s..

a) Write each leg in unit vector (i,j,k) notation.

$$\vec{d}_1 = 0\,\hat{i} + 0\,\hat{j} + 3000\,\hat{k}$$
$$\vec{d}_2 = 0\,\hat{i} + 150,000\,\hat{j} + 0\,\hat{k}$$
$$\vec{d}_3 = -270,000\,\hat{i} + 0\,\hat{j} + 0\,\hat{k}$$
$$\vec{d}_4 = 0\,\hat{i} + 0\,\hat{j} + -3000\,\hat{k}$$

b) Draw Legs 2 and 3 of the trip. (This is the part that occurs in a plane at constant altitude.



c) What is the displacement for the entire trip in unit vector notation and magnitude and direction? Note that the beginning and ending points are at zero height, but they are not the same place.

$$\begin{aligned} \vec{d}_1 &= 0\,\hat{i} + 0\,\hat{j} + 3000\,\hat{k} \\ \vec{d}_2 &= 0\,\hat{i} + 150,000\,\hat{j} + 0\,\hat{k} \\ \vec{d}_3 &= -270,000\,\hat{i} + 0\,\hat{j} + 0\,\hat{k} \\ \vec{d}_4 &= 0\,\hat{i} + 0\,\hat{j} + -3000\,\hat{k} \\ \Delta \vec{r} &= -270,000\,\hat{i} + 150,000\,\hat{j} + 0\,\hat{k} \end{aligned}$$

d) What is the average velocity vector for the entire trip?

$$\Delta \vec{r} = 3000\,\hat{i} + 3000\,\hat{j} + 0\,\hat{k}$$
$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{-270,000\,\hat{i} + 150,000\,\hat{j} + 0\,\hat{k}}{60 + 600 + 900 + 60} = \frac{-270,000\,\hat{i} + 150,000\,\hat{j} + 0\,\hat{k}}{1620\,s}$$

Conversions:

1 mile = 1.6 km 1 inch = 2.54 cm 1 mile = 5280 ft. $pico=10^{-12}$

Constants:

 $\overline{g} = 9.8 \text{ m/s}^2 = 32 \text{ ft/s}^2$

Misc. Equations:

Volume of a sphere = $\frac{4}{3}\pi r^3$ Volume of a cylinder = $\pi r^2 L$ Area of a circle = πr^2 Volume of a rect. obj. = L x W x D

 $\sin \theta = \text{opp/hyp}$ $\cos \theta = \text{adj/hyp}$ $\tan \theta = \text{opp/adj}$

Derivative of a Polynomial.

$$\frac{d}{dt}(\alpha t^n) = n \,\alpha t^{n-1}$$