Physics 130
Exam 1

Short Answer. Please answer all of the short answer questions.

1. An F5 tornado can have winds that reach 317 mi/hr. What is the speed are these winds in m/s?

$$s = \frac{317mi}{hr} \cdot \frac{1600m}{1mi} \cdot \frac{1hr}{3600s} = 140.9m/s$$

2. Estimate how many Cheerios are in your bowl of breakfast cereal.

$$V_{Cheerio} \approx \pi r^2 h = \pi (0.7cm)^2 \cdot 0.3cm = 0.46cm^3$$
$$V_{bowl} = \frac{1}{2} \cdot \frac{4}{3} \pi r^3 = \frac{1}{2} \cdot \frac{4}{3} \pi (8cm)^3 = 1072.33cm^3$$
$$V_{cereal} = \frac{1}{3} \cdot 1072.33cm^3 = 357.443$$
$$\# Cheerios = \frac{V_{cereal}}{V_{Cheerio}} = \frac{\sim 350}{\sim 0.5} = \sim 700$$

This number is really too big. The Cheerios don't pack that well and there are lots of spaces between them....

3. An aircraft taking off of an aircraft carrier deck must reach its takeoff speed of 266 km/h from rest in only two seconds. What acceleration does this aircraft experience? How many g's is this?

$$v_i = 0.0m/s$$
$$v_f = \frac{266km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = 73.89m/s$$
$$t = 2s$$
$$a = \frac{v_f - v_i}{t} = \frac{73.89m/s - 0m/s}{2s} = 36.94m/s^2$$
$$\# g's = \frac{36.94m/s^2}{9.8m/s^2} = 3.77g's$$

4. What length of runway on a carrier is necessary for the aircraft in short answer 3?

$$v_i = 0.0m/s$$
$$v_f = \frac{266km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = 73.89m/s$$
$$t = 2s$$
$$a = 36.94m/s^2$$

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$
$$x_f = 0 + 0 + \frac{1}{2} a t^2$$
$$x_f = \frac{1}{2} \cdot 36.94m/s^2 \cdot (2s)^2$$
$$= 73.88m$$

5. The position of a particle is given by

$$x(t) = -5t^2 + 20t^3$$

Find the average velocity over the interval 0 s to 1 s, the instantaneous velocity at 1s, and the instantaneous acceleration at 1s.

$$x(t) = -5t^2 + 20t^3$$
$$v(t) = -10t + 60t^2$$
$$a(t) = -10 + 120t$$

$$x(0) = 0m$$
$$x(1) = 15m$$
$$v_{avg} = \frac{x(1) - x(0)}{1s} = 15m/s$$
$$v(1) = -10 \cdot 1s + 60 \cdot (1s)^2 = 50m/s$$
$$a(1) = -10 + 120 \cdot 1s = 110m/s^2$$

6. Consider vectors $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 9\hat{i} + 12\hat{j}$ Compute the magnitude and direction of $\vec{a}$ and the magnitude of $\vec{b}$**.** Compute the result of $\vec{a} - \vec{b}$ **,** $\vec{a} \cdot \vec{b}$ and $\vec{a} \times \vec{b}$ **.** What is the angle between the two vectors?

$$\vec{a} = 4\hat{i} + 3\hat{j}$$
$$a = \sqrt{4^2 + 3^2} = 5$$
$$\tan\theta = \frac{3}{4} \Rightarrow \theta = 36.87°$$

$$\vec{b} = 9\hat{i} + 12\hat{j}$$
$$b = \sqrt{9^2 + 12^2} = 15$$

$$\vec{a} - \vec{b} = -5\hat{i} + -9\hat{j}$$

$$\vec{a} \cdot \vec{b} = (4 \cdot 9) + (3 \cdot 12) = 72$$

$$\vec{a} \times \vec{b} = \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 0 \\ 9 & 12 & 0 \end{bmatrix} = (4 \cdot 12 - 3 \cdot 9)\hat{k} = 21\hat{k}$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta$$
$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{ab} = \frac{72}{5 \cdot 15} = \frac{72}{75}$$

7. A person walks 4 miles East and then 3 miles South What is her position in unit-vector notation? What is her position in terms of angle and direction?

$$\vec{r}_1 = 4\hat{i} + 0\hat{j}$$
$$\vec{r}_2 = 0\hat{i} - 3\hat{j}$$
$$\vec{r} = \vec{r}_1 + \vec{r}_2 = 4\hat{i} - 3\hat{j}$$

$$\vec{r} = 4\hat{i} - 3\hat{j}$$
$$r = \sqrt{4^2 + 3^2} = 5$$
$$\tan\theta = \frac{-3}{4} \Rightarrow \theta = -36.87° \quad (36.87° \textit{ South of East})$$

8. You and your friends drive East to go to the Steak n Shake in Galesburg at 2:45 AM. The 12 mile drive takes you approximately 20 minutes. . You spend 90 minutes at the restaurant having a steakburger, fries and a shake. Your drive back takes 25 minutes because you drive a little slower because you're watching for deer. What is the average speed for your trip to Galesburg, from Galesburg, and for the entire trip? (Hint: Don't forget to include your eating time in the total time for the entire trip calculation). What is the average velocity for this round trip to Galesburg?

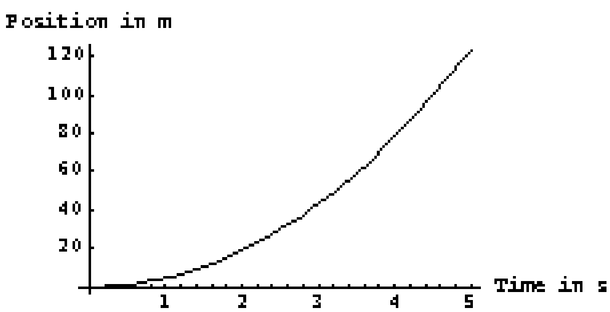
$$s_{to} = \frac{12 mi}{1/3 hr} = \frac{36 mi}{hr}$$

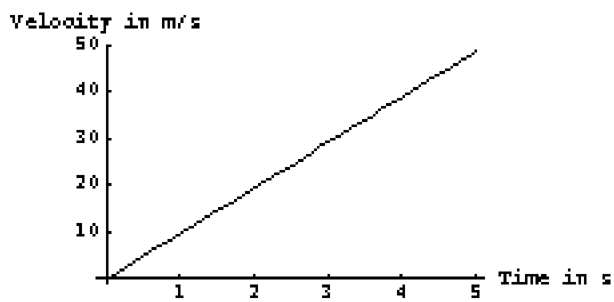
$$s_{from} = \frac{12 mi}{25/60 hr} = \frac{28.8 mi}{hr}$$

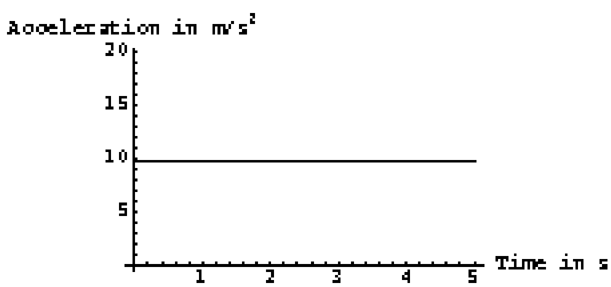
$$s_{total} = \frac{24 mi}{(25+20+90)/60 hr} = \frac{10.67 mi}{hr}$$

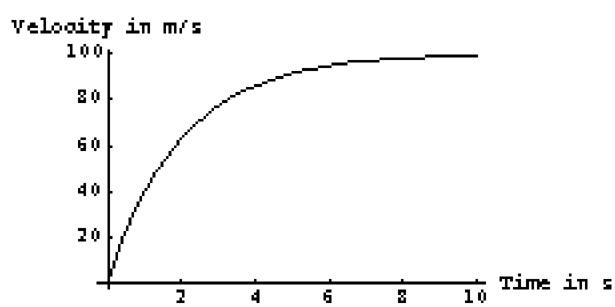
$$\vec{v}_{avg} = 0$$

9. Sketch the position vs. time, velocity vs. time, and acceleration vs. time graphs for free-fall with no friction. What does the velocity vs. time graph look like if you include friction? Hint: You should think about the free-fall lab.









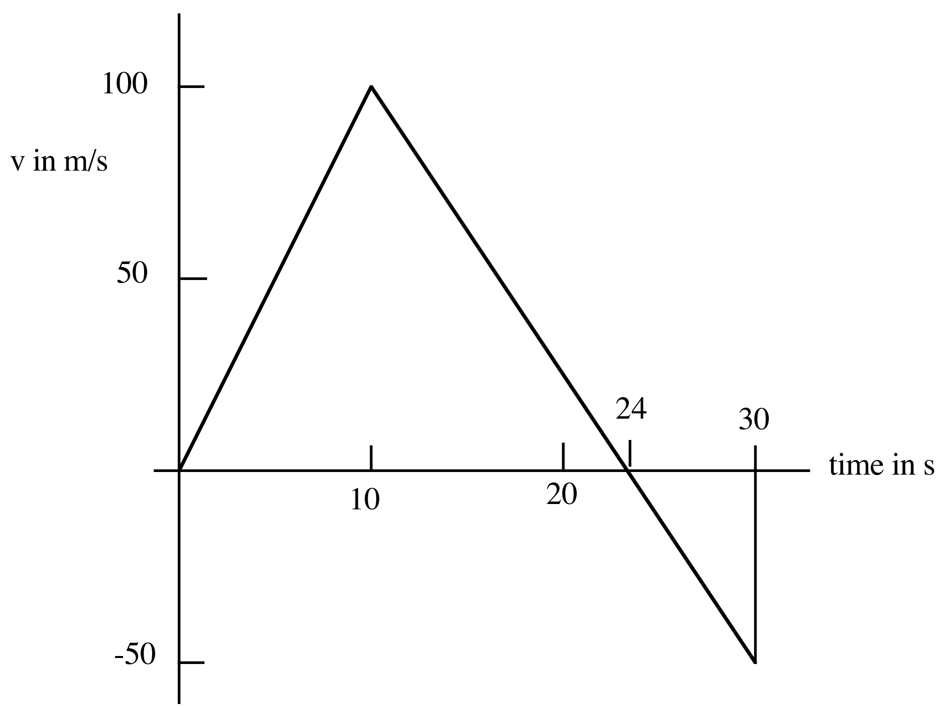
10. A bolt falls off of a hovering helicopter. The helicopter is hovering at an altitude of 500m. How long does it take for the bolt to reach the ground? How fast is it going just before it hits the ground?

$$y_i = 500m$$
$$y_f = 0\,m$$
$$v_i = 0\,m/s$$
$$v_f = ?$$
$$a = -g$$
$$t = ?$$

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$
$$0 = y_i - \frac{1}{2}gt^2$$
$$t = \sqrt{\frac{2y}{g}} = 10.1s$$
$$v_f = v_i + at$$
$$= 0 - gt$$
$$= 98.98\,m/s$$

bonus) Consider the velocity of an object that is shown below. How far does the object move in the time interval 0s to 24s? How far does the object move in the interval 0 s to 30s?

$$x_{0s-24s} = \frac{1}{2} \cdot 24s \cdot 100\,m/s = 1200m$$
$$x_{0s-30s} = \frac{1}{2} \cdot 24s \cdot 100\,m/s + \frac{1}{2} \cdot 6s \cdot -50\,m/s = 1050m$$



Problems. Please work two of the three problems and clearly indicate which problems you wish to have graded.

1. A person standing on a cliff that is 300m high throws a ball upward with an initial velocity of 20m/s. The ball rises to a maximum height and then falls to the ground below the cliff. The ball's trajectory is perfectly vertical--it goes up and falls down.

a) How long does it take for the ball to reach its maximum altitude?

$$y_i = 300m$$
$$y_f = ?$$
$$v_i = 20m/s$$
$$v_f = 0m/s$$
$$a = -g$$
$$t = ?$$

$$v_f = v_i + at$$
$$0 = v_i - gt$$
$$t = \frac{v_i}{g} = \frac{20m/s}{9.8m/s^2} = 2.041s$$

b) What maximum altitude does the ball reach?

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$
$$y_f = y_i + v_i t - \frac{1}{2}gt^2$$
$$= 320.41m$$

c) When does the ball reach the ground at the base of the cliff?

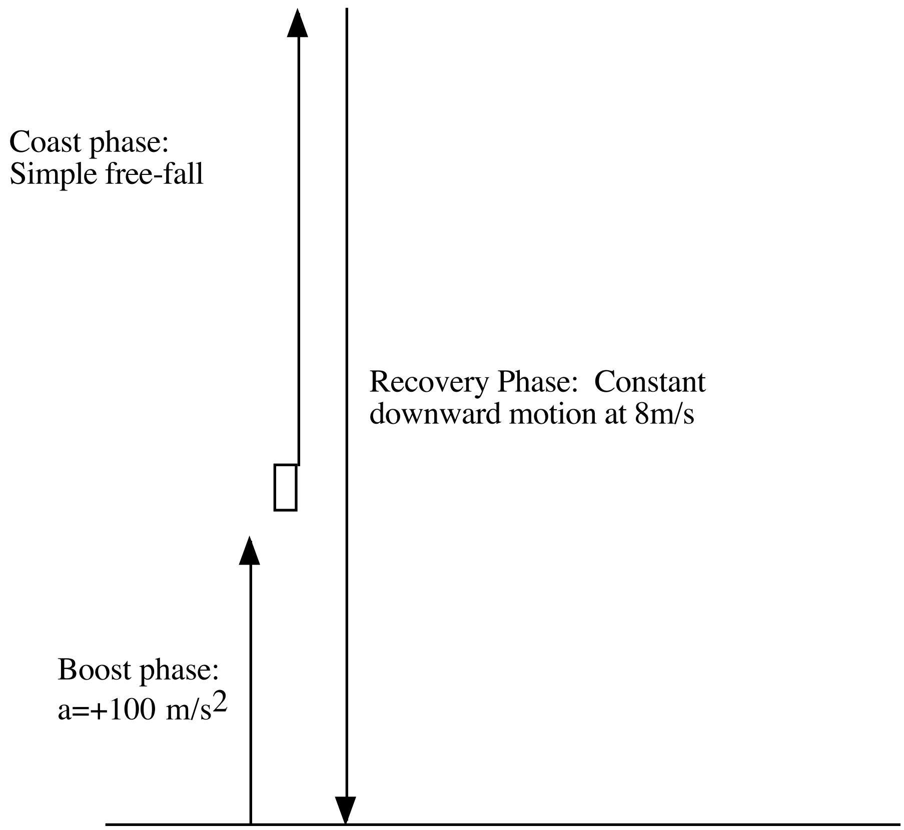
$$y_i = 320.41m$$
$$y_f = 0m$$
$$v_i = 0m/s$$
$$v_f = ?m/s$$
$$a = -g$$
$$t = ?$$

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$
$$0 = y_i + 0 - \frac{1}{2}gt^2$$
$$t = \sqrt{\frac{2y_i}{g}} = 8.086s$$
$$t_{total} = 2.041 + 8.086$$
$$= 10.127s$$

d) What is the speed of the ball when it hits the ground?

$$v_f = v_i + at$$
$$= 0 - gt$$
$$= -g \cdot 8.086s$$
$$= -79.24m/s$$

2. A model rocket flight consists of three separate sections: a) Boost phase: while the engine is running, the rocket accelerates upward, with $a = +100 m/s^2$ for a time of 0.5 s b) Coast phase: After the engine stops, the rocket is in free-fall and finally c) Recovery: The parachute pops and the rocket drifts downward at constant speed of 8 m/s



a) What height does the rocket have after its acceleration just ends (t=0.5s)

$$y_i = 0m$$
$$y_f = ?$$
$$v_i = 0 m/s$$
$$v_f = ?$$
$$a = 100 m/s^2$$
$$t = 0.5s$$

$$y_f = y_i + v_i t + \frac{1}{2} a t^2$$
$$= 0 + 0 + \frac{1}{2} \cdot 100 m/s^2 \cdot (0.5s)^2$$
$$= 12.5m$$

b) What velocity does the rocket have after its acceleration ends?

$$v_f = v_i + a t$$
$$= 0 + at$$
$$= 100 m/s^2 \cdot 0.5$$
$$= 50 m/s$$

c) How long does the rocket take to reach its maximum height above the ground

$$\begin{aligned} y_i &= 12.5m \\ y_f &= ? \\ v_i &= 50m/s \\ v_f &= 0 \\ a &= -g \\ t &= ? \end{aligned}$$

$$\begin{aligned} v_f &= v_i + at \\ 0 &= v_i - gt \\ t &= \frac{v_i}{g} \\ &= 5.1s \end{aligned}$$

d) How high does the rocket go?

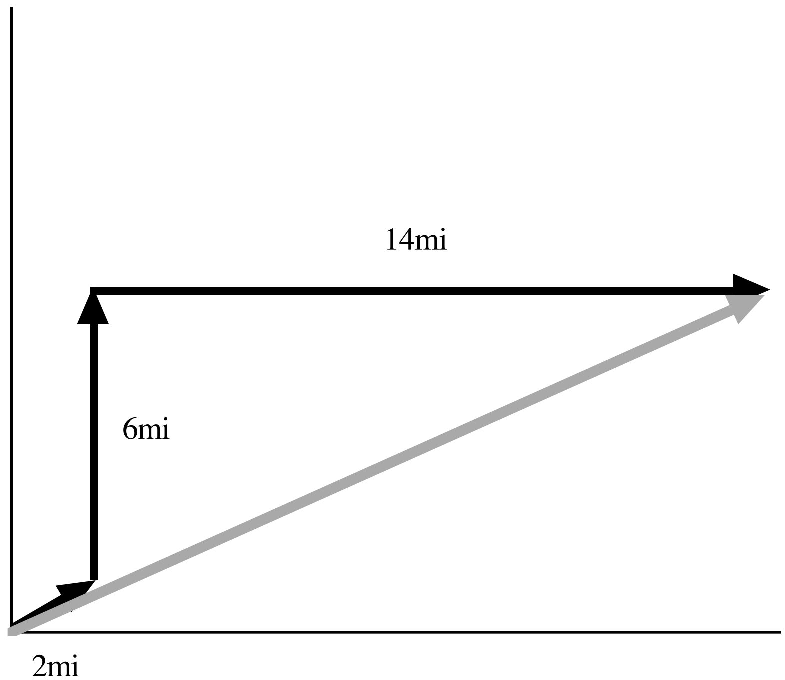
$$\begin{aligned} y_f &= y_i + v_i t + \frac{1}{2}at^2 \\ &= 12.5m + 50m/s \cdot 5.1s - \frac{1}{2} \cdot 9.8m/s^2 \cdot (5.1s)^2 \\ &= 140.05m \end{aligned}$$

bonus. How long is the entire flight--from liftoff at t=0 until the rocket lands safely?

$$\begin{aligned} y_i &= 140.05m \\ t_{float} &= \frac{140.05m}{8m/s} = 17.51s \\ t_{total} &= 0.5 + 5.1s + 17.51 \\ &= 23.5s \end{aligned}$$

3. Sailing on Lake Michigan or anywhere that land is not visible is a challenging act of seamanship. Consider the following trip. A sailor starts from the origin at a speed of 4 mi/hr for 30 minutes at a heading of 30 degrees North of East. He then changes heading as the wind shifts to travel due North for 1 hour at 6 mi/hr. Finally, he heads due East at 7 mi/hr for two hours before finally dropping anchor.

a) Draw the trip.



b) What was the boat's average speed for the entire trip?

$$s = \frac{2mi + 6mi + 14mi}{0.5 + 1.0 + 2.0} = 6.29 mi/h$$

c) What is the position of the boat after the trip?

$$\vec{r}_1 = 2\cos 30\ \hat{i} + 2\sin 30\ \hat{j} = 1.732\ \hat{i} + 1\ \hat{j}$$
$$\vec{r}_2 = 0\ \hat{i} + 6\ \hat{j}$$
$$\vec{r}_3 = 14\ \hat{i} + 0\ \hat{j}$$
$$\vec{r} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3$$
$$= 15.732\ \hat{i} + 7\ \hat{j}$$

d) What velocity vector would the boat need to sail directly back to the origin in 3 hours.

$$\vec{v} = \frac{\vec{r}_f - \vec{r}_i}{3h} = \frac{(0\ \hat{i} + 0\ \hat{j}) - (15.732\ \hat{i} + 7\ \hat{j})}{3h}$$
$$= -5.244\ \hat{i} + -2.333\ \hat{j}$$

e) What magnitude and direction would the boat need to sail directly back to the origin in 3 hours.

$$v = -5.244\ \hat{i} + -2.333\ \hat{j}$$
$$v = \sqrt{(-5.244)^2 + (-2.333)^2} = 5.74 mi/h$$
$$\tan\theta = \frac{2.333}{5.244} \Rightarrow \theta = 23.984^\circ\ S\ of\ W$$

Conversions:

1 mile = 1.6 km
1 inch = 2.54 cm
1 mile = 5280 ft.
1 hour=3600s
1 foot = 0.3048 m

Constants:
g = 9.8 m/s^2 = 32 ft/s^2

Misc. Equations:

Volume of a sphere = (4/3) π r^3
Area of a circle = π r^2
Volume of a cylinder = π r^2 L
Volume of a rect. obj. = L x W x D

sin θ = opp/hyp
cos θ = adj/hyp
tan θ = opp/adj

Derivative of a Polynomial.

$$\frac{d}{dt}(t^n) = nt^{n-1}$$