## Chapter 9

9.2 Figure 9-37 shows a three particle system with masses $m_{1}=3.0 \mathrm{~kg}, m_{2}=4.0 \mathrm{~kg}$, and $m_{3}=8.0 \mathrm{~kg}$. The scales are set by $x_{s}=2.0 \mathrm{~m}$ and $y_{s}=2.0 \mathrm{~m}$. What are (a) the x coordinate and (b) the y coordinate of the system's center of mass? (c) If $m_{3}$ is gradually increased, does the center of mass shift toward or away from that particle or does it remain stationary.

We begin by writing the coordinates of each mass and then we use those coordinates to find the cm position.

$$
\begin{array}{ll}
m_{1}:(0,0) & x_{c m}=\frac{m_{1} x_{1}+m_{2} x_{2}+m_{3} x_{3}}{m_{1}+m_{2}+m_{3}}=1.067 \mathrm{~m} \\
m_{2}:(2,1) & y_{c m}=\frac{m_{1} y_{1}+m_{2} y_{2}+m_{3} y_{3}}{m_{1}+m_{2}+m_{3}}=1.333 m \\
m_{3}:(1,2) &
\end{array}
$$

9.5 What are (a) the $x$ coordinate and (b) the $y$ coordinate of the center of mass for the uniform plate shown in Fig 9-38 if $\mathrm{L}=5.0 \mathrm{~cm}$

For continuous objects, we can often use symmetry to find the cm . In the drawing below, we have labeled the location of the center of each piece of the plate. We can now consider each piece of the plate as a point mass at the center. The mass of each plate is proportional to the area.
9.12 Two skaters, one with mass 65 kg and the other with mass 40 kg stand on an ice rink hold a ole with a length of 10 m and negligible mass. Starting from the ends of the pole, the skaters pull themselves along the pole until they meet. How far will the 40 kg skater move?

Assume that the 65 kg is at 10 and the 40 kg is at 0 m . The cm does not move, so both skaters end up at the cm . The 40 kg skater moves from 0 to the cm .

$$
x_{c m}=\frac{40 \cdot 0+65 \cdot 10}{105}=6.19 \mathrm{~m}
$$

The 40 kg skater moves 6.19 m .


$$
\begin{aligned}
x_{c m} & =\frac{1}{\text { Total Area }} \sum \text { Area }_{i} \cdot x_{i} \\
& =\frac{(2 L \cdot 7 L) \cdot(-L)+(4 L \cdot L) \cdot 2 L+(2 L \cdot 2 L) \cdot L}{(2 L \cdot 7 L)+(4 L \cdot L)+(2 L \cdot 2 L)} \\
& =\frac{-2 L^{3}}{22 L^{2}} \\
& =-\frac{1}{11} L \\
& =-0.455 \mathrm{~cm}
\end{aligned}
$$

$$
\begin{aligned}
y_{c m} & =\frac{1}{\text { Total Area }} \sum \text { Area }_{i} \cdot y_{i} \\
& =\frac{(2 L \cdot 7 L) \cdot(-0.5 L)+(4 L \cdot L) \cdot 2.5 L+(2 L \cdot 2 L) \cdot(-3 L)}{(2 L \cdot 7 L)+(4 L \cdot L)+(2 L \cdot 2 L)} \\
& =\frac{-9 L^{3}}{22 L^{2}} \\
& =-\frac{9}{22} L \\
& =-2.045 \mathrm{~cm}
\end{aligned}
$$

9.16 Ricardo, of mass 80 kg , and Carmelita, who is lighter, are enjoying Lake Merced at dusk in a 30.0 kg canoe. When the canoe is at rest in the placid water, they exchange seats, which are 3.0 me apart and symmetrically located with respect to the canoe's center. If the canoe moves 40 cm horizontally relative to a pier post, what is Carmelita's mass.

The picture is absolutely essential for this picture. The correct picture is shown below. The top picture is before Roberto and Carmelita switch places. The bottom picture is after the switch, showing that the boat has moved.

d

$\mathrm{d}-0.4 \mathrm{~m}$

The physics of this problems is that the center of mass cannot move. All of the forces are "internal forces" so the CM of the system must remain in the same place. We can now write the position of the cm before the switch and after the switch, and then set them equal. There are three masses that contribute: Ricardo, Carmelita, and the boat. The center of mass of the boat is
at the center of the boat. We measure the distance from the pole to the center of the boat, and this moves by 0.4 m .

$$
\begin{aligned}
& x_{c m}=\frac{m_{R}(d-1.5)+m_{b} d+m_{C}(d+1.5)}{m_{R}+m_{b}+m_{R}} \quad \text { (before) } \\
& x_{c m}=\frac{m_{R}(d-0.4+1.5)+m_{b}(d-0.4)+m_{C}(d-0.4-1.5)}{m_{R}+m_{b}+m_{R}} \quad \text { (after) } \\
& \frac{m_{R}(d-1.5)+m_{b} d+m_{C}(d+1.5)}{m_{R}+m_{b}+m_{R}}=\frac{m_{R}(d-0.4+1.5)+m_{b}(d-0.4)+m_{C}(d-0.4-1.5)}{m_{R}+m_{b}+m_{R}} \\
& m_{R}(d-1.5)+m_{b} d+m_{R}(d+1.5)=m_{R}(d-0.4+1.5)+m_{b}(d-0.4)+m_{C}(d-0.4-1.5) \\
& m_{R}(-1.5)+m_{C}(+1.5)=m_{R}(+1.1)+m_{b}(-0.4)+m_{C}(-1.9) \\
& m_{C}(+1.5+1.9)=m_{R}(+1.1+1.5)+m_{b}(-0.4) \\
& m_{C}=\frac{m_{R}(+1.1+1.5)+m_{b}(-0.4)}{(+1.5+1.9)}=\frac{80 \mathrm{~kg}(2.6)+30 \mathrm{~kg}(-0.4)}{3.4} \\
& m_{C}=57.6 \mathrm{~kg}
\end{aligned}
$$

(provided that I haven't made a typo!)
9.18 A 0.70 kg ball is moving horizontally with a speed of $5.0 \mathrm{~m} / \mathrm{s}$ when it strikes a vertical wall. The ball rebounds with a speed of $2.0 \mathrm{~m} / \mathrm{s}$. What is the magnitude of the change in the linear momentum of the ball

$$
\begin{aligned}
\Delta \vec{p} & =\vec{p}_{f}-\vec{p}_{i} \\
& =0.7 \mathrm{~kg} \cdot 5 \mathrm{~m} / \mathrm{s}-0.7 \mathrm{~kg} \cdot(-2 \mathrm{~m} / \mathrm{s}) \\
& =4.9 \mathrm{kgm} / \mathrm{s}
\end{aligned}
$$

9.24 In February of 1955, a paratrooper fell 370 m from an airplane without being able to open his chute but happened to land in snow, suffering only minor injuries. Assume that his speed at impact was $56 \mathrm{~m} / \mathrm{s}$ (terminal speed), that his mass (including gear) was 85 kg , and that the magnitude of the force on him from the snow was at the survivable limit of $1.2 \times 10^{5} \mathrm{~N}$. What are (a) the minimum depth of snow that would have stopped him safely and (b) the magnitude of the impulse.

The snow does work in stopping this very lucky paratrooper. The work that the snow does is equal to the change in kinetic energy

$$
\begin{aligned}
\Delta K E & =K E_{f}-K E_{i} \\
& =0-\frac{1}{2} m v_{i}^{2} \\
& =-1.333 \times 10^{5}
\end{aligned}
$$

Since we know the force, we can write the work done by the snow in terms of the depth.

$$
\begin{aligned}
\Delta K E & =W \\
\Delta K E & =F d \cos 180^{\circ}=-F d \\
d & =-\frac{\Delta K E}{F}=-\frac{-1.333 \times 10^{5} \mathrm{~J}}{1.2 \times 10^{5} \mathrm{~N}} \\
& =1.111 \mathrm{~m}
\end{aligned}
$$

The impulse is the change in momentum.

$$
\begin{aligned}
\Delta \vec{p} & =\vec{p}_{f}-\vec{p}_{i} \\
& =0-85 \mathrm{~kg} \cdot-56 \mathrm{~m} / \mathrm{s} \\
& =4760 \mathrm{kgm} / \mathrm{s}
\end{aligned}
$$

9.39 A 91 kg man lying on a surface of negligible friction shoves a 68 g stone away from himself, giving it a speed of $4 \mathrm{~m} / \mathrm{s}$. What speed does he acquire as a result?

This is a momentum conservation problem. The total momentum is zero (man and stone at rest at the beginning). The total momentum must remain zero. In the final state

$$
\begin{aligned}
\vec{p}_{f} & =0=\vec{p}_{m}+\vec{p}_{s} \\
\vec{p}_{m} & =-\vec{p}_{s} \\
m_{m} \vec{v}_{m} & =-m_{s} \vec{v}_{s} \\
\vec{v}_{m} & =-\frac{m_{s}}{m_{m}} \vec{v}_{s} \\
& =-\frac{0.068 \mathrm{~kg}}{91.0 \mathrm{~kg}} \cdot 4 \mathrm{~m} / \mathrm{s} \\
& =0.002989 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

9.40. A space vehicle is traveling at $4300 \mathrm{~km} / \mathrm{h}$ relative to the earth when he exhausted rocket motor is disengage and sent backward with a velocity of $82 \mathrm{~km} / \mathrm{h}$ relative to the command module ( $\mathrm{M}=4 \mathrm{~m}$ )

First define the velocity of the booster relative to earth. v' is the velocity of the module relative to earth after the separation.

$$
V^{\prime}=v^{\prime}-82 \mathrm{~km} / \mathrm{h}
$$

Again we write conservation of momentum

$$
\begin{aligned}
& v=4300 \mathrm{~km} / \mathrm{h} \\
& M=4 m \\
& (m+M) v=m v^{\prime}+M V^{\prime} \\
& (m+M) v=m v^{\prime}+M\left(v^{\prime}-82 \mathrm{~km} / \mathrm{h}\right) \\
& v^{\prime}=\frac{(m+M) v+M \cdot 82 \mathrm{~km} / \mathrm{h}}{m+M}=\frac{5 m \cdot 4300+4 \mathrm{~m} \cdot 82 \mathrm{~km} / \mathrm{h}}{5 m}=4365.56 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

9.49 A bullet of mass 10 g strikes a ballistic pendulum of mass 2.0 kg . The center of mass of the pendulum rises a vertical distance of 12 cm . Assuming that the bullet remains embedded in the pendulum, calculate the bullet's initial speed.

This is both a momentum conservation problem and an energy conservation problem. The energy conservation part is the motion of the pendulum. The total energy of the pendulum at the top of the swing is equal to the total energy of the pendulum after it has been struck by a bullet.

$$
\begin{aligned}
E_{f} & =(M+m) g y \\
E_{i} & =\frac{1}{2}(M+m) v^{2} \\
E_{i} & =E_{f} \\
\frac{1}{2}(M+m) v^{2} & =(M+m) g y \\
v & =\sqrt{2 g y} \\
& =\sqrt{2 \cdot 9.8 \cdot 0.12} \\
& =1.534 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Now that we know the velocity of the bullet-pendulum immediately after the bullet strikes the pendulum, we can use momentum conservation to find the bullet's velocity

$$
\begin{aligned}
\vec{p}_{i} & =\vec{p}_{f} \\
m v_{b} & =(M+m) v \\
v_{b} & =\frac{(M+m)}{m} v \\
& =\frac{(2.0 \mathrm{~kg}+0.01 \mathrm{~kg})}{0.01 \mathrm{~kg}} \cdot 1.534 \mathrm{~m} / \mathrm{s} \\
& =308.3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

9.57 In Fig. 9-61, a ball of mass $m=60 g=0.060 \mathrm{~kg}$ is shot with a speed $v_{i}=22 \mathrm{~m} / \mathrm{s}$ into a the barrel of a spring gun of mass $M=240 \mathrm{~g}=0.240 \mathrm{~kg}$ initially at rest on a frictionless surface.
The ball sticks in the barrel at the point of maximum compression of the spring. Assume that the increase n thermal energy due to friction between the ball and the gun after the ball stops in the barrel is negligible. (a) What is the speed of the spring gun after the ball stops in the barrel? (b) What fraction of the initial kinetic energy of the ball is stored in the spring?

Since the momentum is conserved, we can use conservation of momentum to find the final velocity of the gun-ball system. We can consider this to be a completely inelastic collision.

$$
\begin{aligned}
& p_{i}=m v_{i} \\
& p_{f}=(m+M) v_{f} \\
& p_{f}=p_{i} \\
& (m+M) v_{f}=m v_{i} \\
& v_{f}=\frac{m}{m+M} v_{i}=\frac{0.060 \mathrm{~kg}}{0.060 \mathrm{~kg}+0.240 \mathrm{~kg}} \cdot(-22 \mathrm{~m} / \mathrm{s})=4.4 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Now that we know the final velocity, we can compute the final kinetic energy. The "missing" kinetic energy is stored in the spring.

$$
\begin{aligned}
& K E_{i}=\frac{1}{2} m v_{i}^{2}=14 . .52 \mathrm{~J} \\
& K E_{f}=\frac{1}{2}(m+M) v_{f}^{2}=2.904 \mathrm{~J} \\
& \Delta K E=K E_{f}-K E_{i}=-11.616 \mathrm{~J} \\
& \text { fractionin spring }=\left|\frac{\Delta K E}{K E_{i}}\right|=0.8
\end{aligned}
$$

9.102 A man of mass 80 kg is on a ladder hanging from a balloon of mass 320 kg . The balloon is initially stationary with respect to the ground. If the man begins to climb the ladder with at
speed of $2.5 \mathrm{~m} / \mathrm{s}$ relative to the ladder, in what direction and with what velocity will the balloon move? (b) What is the state of motion after the man stops climbing?
(a) If we take the velocity $v$ to be upward relative to the ladder and we assume that the ladder is moving with velocity V downward, the velocity of the balloon, the velocity of the man with respect to the earth is

$$
v^{\prime}=v-V
$$

Momentum is conserved and the CM does not move, so when we observe from the earth

$$
\begin{aligned}
0 & =m v^{\prime}-M V \\
0 & =m(v-V)-M V \\
V & =\frac{m v}{M+m}=\frac{80 \mathrm{~kg} \cdot 2.5 \mathrm{~m} / \mathrm{s}}{320 \mathrm{~kg}+80 \mathrm{~kg}} \\
& =0.5 \mathrm{~m} / \mathrm{s} \text { downward }
\end{aligned}
$$

(b) After the man stops climbing, the balloon stops moving.

