Chapter 9

9.2 Figure 9-37 shows a three particle system with masses $m_1 = 3.0\,kg$, $m_2 = 4.0\,kg$, and $m_3 = 8.0\,kg$. The scales are set by $x_s = 2.0\ m$ and $y_s = 2.0\ m$. What are (a) the x coordinate and (b) the y coordinate of the system's center of mass? (c) If m_3 is gradually increased, does the center of mass shift toward or away from that particle or does it remain stationary.

We begin by writing the coordinates of each mass and then we use those coordinates to find the cm position.

$m_1 : (0,0)$

$m_2 : (2,1)$

$m_3 : (1,2)$

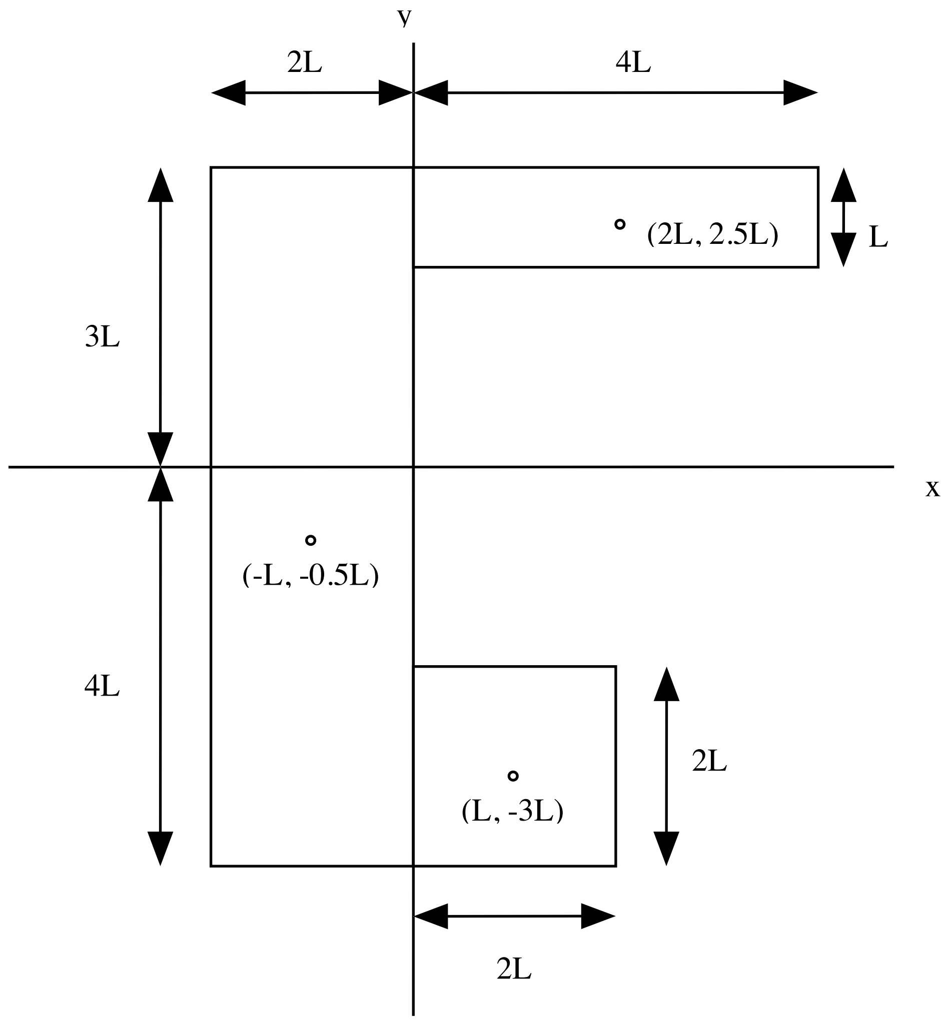
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = 1.067m$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = 1.333m$$

As m_3 is increased, the cm position will move toward the position of m_3 .

9.3 What are (a) the x coordinate and (b) the y coordinate of the center of mass for the uniform plate shown in Fig 9-38

For continuous objects, we can often use symmetry to find the cm. In the drawing below, we have labeled the location of the center of each piece of the plate. We can now consider each piece of the plate as a point mass at the center. The mass of each plate is proportional to the area.



$$
\begin{aligned}
x_{cm} &= \frac{1}{Total\ Area}\sum Area_i \cdot x_i \\
&= \frac{(2L\cdot 7L)\cdot(-L)+(4L\cdot L)\cdot 2L+(2L\cdot 2L)\cdot L}{(2L\cdot 7L)+(4L\cdot L)+(2L\cdot 2L)} \\
&= \frac{-2L^3}{22L^2} \\
&= -\frac{1}{11}L \\
&= -0.455cm
\end{aligned}
$$

$$
\begin{aligned}
y_{cm} &= \frac{1}{Total\ Area}\sum Area_i \cdot y_i \\
&= \frac{(2L\cdot 7L)\cdot(-0.5L)+(4L\cdot L)\cdot 2.5L+(2L\cdot 2L)\cdot(-3L)}{(2L\cdot 7L)+(4L\cdot L)+(2L\cdot 2L)} \\
&= \frac{-9L^3}{22L^2} \\
&= -\frac{9}{22}L \\
&= -2.045cm
\end{aligned}
$$

9.7 The figure shows a slab with dimensions $d_1 = 11.0\,cm$, $d_2 = 2.80\,cm$, and $d_3 = 11.0\,cm$. Half the slab consists of aluminum (density = $2.70\,g/cm^3$) and half consists of iron density = $7.85\,g/cm^3$) . What are the coordinates of the center of mass.

We begin finding the coordinates of the center of mass of each slab (Aluminum and Iron).

$$
\begin{aligned}
x_{Al} &= -\frac{d_3}{2} = -6.50cm & x_{Fe} &= -\frac{d_3}{2} = -6.50cm \\
y_{Al} &= d_1 + \frac{d_1}{2} = 16.5cm & y_{Fe} &= \frac{d_1}{2} = 5.50cm \\
z_{Al} &= \frac{d_2}{2} = 1.40cm & z_{Fe} &= \frac{d_2}{2} = 1.40cm
\end{aligned}
$$

The volume of each slab is the same. We use the volume to find the mass of each slab.

$$
\begin{aligned}
V &= d_1\, d_2\, d_3 \\
m_{Al} &= \rho_{Al} V \\
m_{Fe} &= \rho_{Fe} V
\end{aligned}
$$

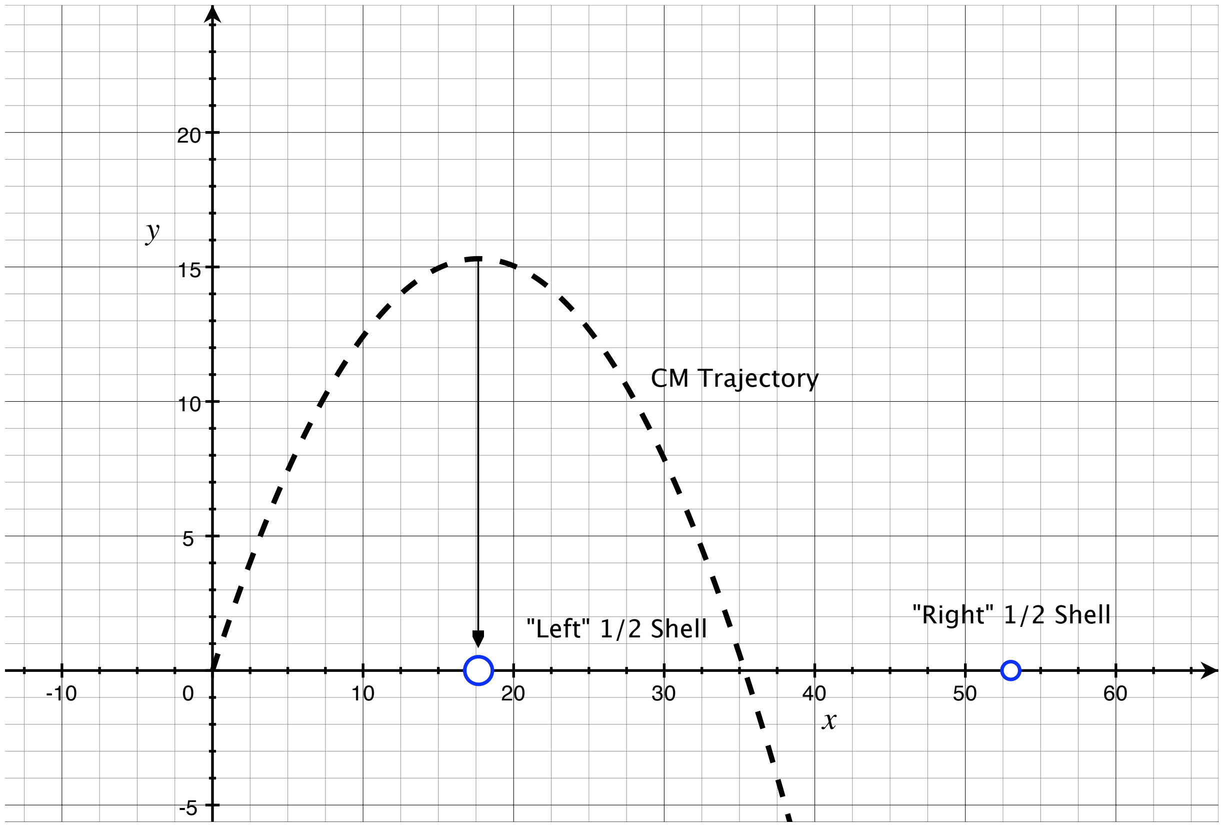
$$
\begin{aligned}
x_{cm} &= \frac{m_{Al}x_{Al} + m_{Fe}x_{Fe}}{m_{Al} + m_{Fe}} \\
&= \frac{\rho_{Al} V\ x_{Al} + \rho_{Fe} V\ x_{Fe}}{\rho_{Al} V + \rho_{Fe} V} \\
&= \frac{\rho_{Al}\ x_{Al} + \rho_{Fe}\ x_{Fe}}{\rho_{Al} + \rho_{Fe}} = \frac{2.70 \cdot -6.50 + 7.85\ \cdot -6.50}{2.70 + 7.85} \\
&= -6.50
\end{aligned}
$$

$$
\begin{aligned}
y_{cm} &= \frac{m_{Al}y_{Al} + m_{Fe}y_{Fe}}{m_{Al} + m_{Fe}} \\
&= \frac{\rho_{Al}\ y_{Al} + \rho_{Fe}\ y_{Fe}}{\rho_{Al} + \rho_{Fe}} = \frac{2.70 \cdot 16.5 + 7.85\ \cdot 5.50}{2.70 + 7.85} \\
&= 8.32 cm
\end{aligned}
$$

$$
\begin{aligned}
z_{cm} &= \frac{m_{Al}z_{Al} + m_{Fe}a_{Fe}}{m_{Al} + m_{Fe}} \\
&= \frac{\rho_{Al}\ z_{Al} + \rho_{Fe}\ z_{Fe}}{\rho_{Al} + \rho_{Fe}} = \frac{2.70 \cdot 1.40 + 7.85\ \cdot 1.40}{2.70 + 7.85} \\
&= 1.40 cm
\end{aligned}
$$

9.15 A shell is fired from a gun with a muzzle velocity of 20m/s at an angle of 60˚. At the top of the trajectory, the shell explodes into two equal mass fragments. One fragment, whose speed immediately after the explosion is zero falls vertically. How far from the gun does the other fragment land, assuming that the terrain is level and that the air drag is negligible.

Let’s sketch what happens first



Examine how long it took to reach the explosion point.

$$v_{fy} = 0$$
$$v_{iy} = 20m/s \cdot \sin 60 = 17.32m/s$$
$$v_{ix} = 20m/s \cdot \cos 60 = 10m/s$$
$$a = -g$$
$$v_{fy} = v_{iy} - gt$$
$$t = \frac{v_{iy}}{g} = 1.767s$$

Next we find the distance traveled before the explosion.

$$x_L = v_{ix}t = 17.67m$$

The "left" half-shell lands at $x_L = 17.67m$. If the shell had not exploded, we know that it would land at $x_{cm} = 2 \cdot 17.67m = 35.34m$ Since the forces involved in the explosion were entirely internal, the center of mass of the two shells still lands at exactly this point. Knowing where the cm is allows us to find where the second half shell will land.

$$
\begin{aligned}
x_{cm} &= \frac{\frac{1}{2}m \cdot x_L + \frac{1}{2}m \cdot x_R}{m} \\
x_{cm} &= \frac{1}{2}x_L + \frac{1}{2}x_R \\
x_R &= 2x_{cm} - x_L \\
&= 2 \cdot 35.34 - 17.67 \\
&= 53.01m
\end{aligned}
$$

9.23 A force in the negative direction of an x axis is applied for 27 ms to a 0.40 kg ball initially moving at 14 m/s in the positive direction of the axis. The force varies in magnitude and the impulsed has magnitude 32.4 Ns. What are the ball's (a)speed and (b) direction of travel just after the force is applied? What are the (c) average magnitude of the force and (d) the direction of the impulse on the ball

The impulse is the change in momentum. Since the force is in the negative direction, the impulse is also negative.

$$
\begin{aligned}
\Delta p &= p_f - p_i \\
p_f &= p_i + \Delta p \\
mv_f &= mv_i + \Delta p \\
v_f &= v_i + \frac{\Delta p}{m} = 14m/s + \frac{-32.4Ns}{0.4} = -67m/s
\end{aligned}
$$

$$
\left|\vec{F}\right| = \left|\frac{\Delta \vec{p}}{\Delta t}\right| = 1200N
$$

Force is negative.

9.27 A 1.2 kg ball drops vertically onto a floor, hitting with a speed of 25 m/s. It rebounds with an initial speed of 10 m/s. It rebounds with an initial speed of 10 m/s. (a) What impulse acts on the ball during the contact? (b) If the ball is in contact with the floor for 0.020 s, what is the magnitude of the average force on the floor from the ball

$$
\begin{aligned}
\Delta \vec{p} &= \vec{p}_f - \vec{p}_i \\
&= 1.2kg \cdot (-25m/s) - 1.2kg \cdot (10m/s) \\
&= 42\,kg\,m/s \\
F &= \frac{\Delta p}{\Delta t} = \frac{42\,kg\,m/s}{0.020s} \\
&= 2100N
\end{aligned}
$$

9.39 A 91 kg man lying on a surface of negligible friction shoves a 68 g stone away from himself, giving it a speed of 4 m/s. What speed does he acquire as a result?

This is a momentum conservation problem. The total momentum is zero (man and stone at rest at the beginning). The total momentum must remain zero. In the final state

$$\begin{aligned}
\vec{p}_f &= 0 = \vec{p}_m + \vec{p}_s \\
\vec{p}_m &= -\vec{p}_s \\
m_m \vec{v}_m &= -m_s \vec{v}_s \\
\vec{v}_m &= -\frac{m_s}{m_m} \vec{v}_s \\
&= -\frac{0.068kg}{91.0kg} \cdot 4m/s \\
&= 0.002989m/s
\end{aligned}$$

9.40. A space vehicle is traveling at 4300km/h relative to the earth when he exhausted rocket motor is disengage and sent backward with a velocity of 82km/h relative to the command module
M=4m

This problem is similar to the balloon. First define the velocity of the booster relative to earth.v' is the velocity of the module relative to earth after the separation.

$$V' = v' - 82km/h$$

Again we write conservation of momentum

$$\begin{aligned}
&v = 4300km/h \\
&M = 4m \\
&(m+M)v = mv' + MV' \\
&(m+M)v = mv' + M(v' - 82km/h) \\
&v' = \frac{(m+M)v + M \cdot 82km/h}{m+M} = \frac{5m \cdot 4300 + 4m \cdot 82km/h}{5m} = 4365.56m/s
\end{aligned}$$

9.49 A bullet of mass 10 g strikes a ballistic pendulum of mass 2.0 kg. The center of mass of the pendulum rises a vertical distance of 12 cm. Assuming that the bullet remains embedded in the pendulum, calculate the bullet's initial speed.

This is both a momentum conservation problem and an energy conservation problem. The energy conservation part is the motion of the pendulum. The total energy of the pendulum at the top of the swing is equal to the total energy of the pendulum after it has been struck by a bullet.

$$E_f = (M+m)g\,y$$

$$E_i = \frac{1}{2}(M+m)v^2$$

$$E_i = E_f$$

$$\frac{1}{2}(M+m)v^2 = (M+m)g\,y$$

$$v = \sqrt{2g\,y}$$

$$= \sqrt{2 \cdot 9.8 \cdot 0.12}$$

$$= 1.534\ m/s$$

Now that we know the velocity of the bullet-pendulum immediately after the bullet strikes the pendulum, we can use momentum conservation to find the bullet's velocity

$$\vec{p}_i = \vec{p}_f$$

$$mv_b = (M+m)v$$

$$v_b = \frac{(M+m)}{m}v$$

$$= \frac{(2.0kg + 0.01kg)}{0.01kg} \cdot 1.534m/s$$

$$= 308.3m/s$$

9.55 In Fig. 9-63, a ball of mass $m = 60g = 0.060kg$ is shot with a speed $v_i = 22\,m/s$ into a the barrel of a spring gun of mass $M = 240g = 0.240kg$ initially at rest on a frictionless surface. The ball sticks in the barrel at the point of maximum compression of the spring. Assume that the increase n thermal energy due to friction between the ball and the gun after the ball stops in the barrel is negligible. (a) What is the speed of the spring gun after the ball stops in the barrel? (b) What fraction of the initial kinetic energy of the ball is stored in the spring?

Since the momentum is conserved, we can use conservation of momentum to find the final velocity of the gun-ball system. We can consider this to be a completely inelastic collision.

$$p_i = mv_i$$

$$p_f = (m+M)v_f$$

$$p_f = p_i$$

$$(m+M)v_f = mv_i$$

$$v_f = \frac{m}{m+M}v_i = \frac{0.060kg}{0.060kg + 0.240kg} \cdot (-22m/s) = 4.4m/s$$

Now that we know the final velocity, we can compute the final kinetic energy. The "missing" kinetic energy is stored in the spring.

$$KE_i = \frac{1}{2}mv_i^2 = 14..52J$$

$$KE_f = \frac{1}{2}(m+M)v_f^2 = 2.904J$$

$$\Delta KE = KE_f - KE_i = -11.616J$$

$$fraction\, in\, spring = \left|\frac{\Delta KE}{KE_i}\right| = 0.8$$

9.66 A steel ball of mass 0.5 kg is fastened to a cord that is 70 cm long an fixed at the far end. The ball is then released when the cord is horizontal (Fig. 9-65). At the bottom of its path, the ball strikes a 2.5 kg steel block initially at rest on a frictionless surface. The collision is elastic. Find (a) the speed of the ball and (b) the speed of the block, both just after the collision.

We begin by finding the initial velocity of the ball using energy conservation.

$$\frac{1}{2}mv^2 = mgh$$

$$v^2 = 2gh$$

$$v_b = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 0.7m}$$

$$= 3.7m/s$$

Now consider the collision. Both momentum and energy are conserved in this elastic collision.

$$mv_b = mv_b' + Mv_B'$$

$$\frac{1}{2}mv_b{}^2 = \frac{1}{2}mv_b'^2 + \frac{1}{2}Mv_B'^2$$

We have two equations and two unknowns. We solve the momentum equation for one of the unknowns and then plug into the energy equation.

$$mv_b' = mv_b - Mv_B'$$

$$v_b' = v_b - \frac{M}{m}v_B'$$

$$\frac{1}{2}mv_b{}^2 = \frac{1}{2}mv_b'^2 + \frac{1}{2}Mv_B'^2$$

$$\frac{1}{2}mv_b{}^2 = \frac{1}{2}m(v_b - \frac{M}{m}v_B')^2 + \frac{1}{2}Mv_B'^2$$

$$\frac{1}{2}mv_b{}^2 = \frac{1}{2}mv_b{}^2 + \frac{1}{2}\frac{M^2}{m}v_B'^2 - Mv_bv_B' + \frac{1}{2}Mv_B'^2$$

$$0 = \frac{M^2}{m}v_B'^2 - 2Mv_bv_B' + Mv_B'^2$$

$$0 = \frac{M}{m}v_B'^2 - 2v_bv_B' + v_B'^2$$

$$= v_B'\left(\left(\frac{M}{m}+1\right)v_B' - 2v_b\right)$$

$$0 = \left(\left(\frac{M}{m}+1\right)v_B' - 2v_b\right)$$

$$v_B' = \frac{2v_b}{\left(\frac{M}{m}+1\right)} = \frac{2\cdot 3.7m/s}{\left(\frac{2.5kg}{0.5kg}+1\right)}$$

$$= 1.23m/s$$

Notice that if the mass of the ball and the mass of the block are the same, the ball will stop and the block will take all of the velocity of the ball. This is exactly what we saw in the elastic equal mass collision in lab. We finally find the ball's velocity after the collision, by substituting back in.

$$v_b' = v_b - \frac{M}{m}v_B'$$

$$= 3.7m/s - \frac{2.5}{0.5}\cdot 1.23m/s$$

$$= -2.45m/s$$

9.76