## Chapter 8

8.2 In Fig. 8-27, a single frictionless roller-coaster car of mass $\mathrm{m}=825 \mathrm{~kg}$ tops the first hill with a speed of $v_{0}=17.0 \mathrm{~m} / \mathrm{s}$ at a height $h=42.0 \mathrm{~m}$. How much work does the gravitational force on the car from that point to (a) point A , (b) point B , and (c) point C ? If the mass m were doubled, would the change in gravitational potential energy
a) Between the initial point and point A , there is no change in height, so no work
b) The work done is given by

$$
\begin{aligned}
W & =-\Delta U=-m g\left(h_{B}-h_{i}\right) \\
& =-825 \mathrm{~kg} \cdot 9.8 \mathrm{~m} / \mathrm{s}^{2} \cdot(21.0 \mathrm{~m}-42 \mathrm{~m}) \\
& =169,785 \mathrm{~J}
\end{aligned}
$$

c) Point C is at zero height.

$$
\begin{aligned}
W & =-\Delta U=-m g\left(h_{B}-h_{i}\right) \\
& =-825 \mathrm{~kg} \cdot 9.8 \mathrm{~m} / \mathrm{s}^{2} \cdot(0.0 \mathrm{~m}-42 \mathrm{~m}) \\
& =339,570 \mathrm{~J}
\end{aligned}
$$

If the mass doubles, the work doubles.
8.4 Figure $8-31$ shows a ball with mass $m=0.341 \mathrm{~kg}$ attached to the end of a thin rod with length $L=0.452 \mathrm{~m}$ and negligible mass. The other end of the rod is pivoted so that the ball can move in a vertical circle The rod is held horizontally as sown and then given enough of a downward push to cause the ball to swing down and around and just reach the vertically up position with zero speed there. How much work is done on the ball by the gravitational force from the initial point (a) the lowest point, (b) the highest point, and (c) the point on the right level with the initial point? If the gravitation potential energy of the ball-earth system is taken to be zero at the initial point, what is it when the ball reaches (d) the lowest point, (e) the highest point and (f) the point on the right level with the initial point? (g) Suppose the rod were pushed harder sot that the ball passed through the highest point with nonzero speed. Would $\Delta U_{g}$ from the lowest point to the highest point then be greater than, less than, or the same as it was when the ball stopped at the highest point.

Since the gravitational force is conservative, the work done and change in potential energy only depend on the initial and final position.
(a) Work done in moving from initial position to lowest position

$$
W=m g L=1.51 \mathrm{~J}
$$

(b) Work done in moving from initial position to highest position

$$
W=-m g L=-1.51 \mathrm{~J}
$$

(c) Work done in moving from initial position to level position

$$
W=0
$$

Defining the potential to be zero at the initial point is actually defining the initial height as zero height. In that case
(d) Potential at lowest point

$$
U=m g y=m g(-L)=-1.51 \mathrm{~J}
$$

(e) Potential at highest point

$$
U=m g y=m g(L)=1.51 \mathrm{~J}
$$

(f) Potential at level point

$$
U=m g y=m g(0)=0 J
$$

(g) The work and potential are not dependent on velocity in this problem. The answers are unchanged.
8.4 In Fig 8-30, a 2.00 g ice flake is released from the edge of a hemispherical bowl whose radius is 22 cm . The flake=bowl contact is frictionless. (a) how much work is done on the flake by the gravitation force during the flake's descent to the bottom of the bowl? (b) What is the change in the potential energy of the flake Earth system during the descent? (c) If that potential energy is taken to be zero at the bottom of the bowl, what is its value when the flake is released? (d) If instead, the potential energy is taken to be zero at the release point, what is its value when the flake reaches the bottom of the bowl (e) If the mass of the flake were doubled, would the magnitudes of the answers to (a) through (d) increase, decrease, or remain the same?
a) The work done is

$$
\begin{aligned}
W & =-\Delta U \\
& =-\left(U_{f}-U_{i}\right) \\
& =-(0-m g y) \\
& =2 \times 10^{-3} \cdot 9.8 \mathrm{~m} / \mathrm{s}^{2} \cdot 0.22 \mathrm{~m} \\
& =4.312 \times 10^{-3} \mathrm{~J}
\end{aligned}
$$

For this part of the problem, assume the bottom of the bowl is zero height

$$
\begin{aligned}
U_{f} & =0 \\
U_{i} & =m g R \\
W & =-\left(U_{f}-U_{i}\right) \\
& =m g R
\end{aligned}
$$

b) See part (a). $\quad \Delta U=-4.312 \times 10^{-3} J$
c) See part (a) $U_{i}=4.312 \times 10^{-3} \mathrm{~J}$
d) Assume that the top of the bowl is 0 height. The bottom would be -R

$$
\begin{aligned}
& U_{i}=0 \\
& U_{f}=m g(-R)=-4.312 \times 10^{-3} \mathrm{~J}
\end{aligned}
$$

e) All of these answers are linear in $m$. If you double the mass, the results double.
8.6 In Fig. 8-31, a small block of mass $m=0.032 \mathrm{~kg}$ can slide along the frictionless loop-the-loop with a loop radius of $\mathrm{R}=12 \mathrm{~cm}$. The block is released from rest at a point P , at height 5.0 R above the bottom of the loop. A block slides on a track from a height 5R.
a) How much work does the weight do on the block as it travels from P to Q ? Assume that the potential energy at the bottom is 0 .

$$
\begin{aligned}
W & =-\Delta U \\
& =-\left(m g y_{f}-m g y_{i}\right) \\
& =-(m g R-m g \cdot 5 R) \\
& =4 m g R \\
& =4 \cdot 0.032 \mathrm{~kg} \cdot 9.8 \mathrm{~m} / \mathrm{s}^{2} \cdot 0.12 \mathrm{~m} \\
& =0.151 \mathrm{~J}
\end{aligned}
$$

b) How much work does the weight do on the block as it travels from P to the top of the loop? Assume that the potential energy at the bottom is 0 .

$$
\begin{aligned}
W & =-\Delta U \\
& =-\left(m g y_{f}-m g y_{i}\right) \\
& =-(m g \cdot 2 R-m g \cdot 5 R) \\
& =3 m g R \\
& =3 \cdot 0.032 \mathrm{~kg} \cdot 9.8 \mathrm{~m} / \mathrm{s}^{2} \cdot 0.12 \mathrm{~m} \\
& =0.113 \mathrm{~J}
\end{aligned}
$$

c-e) Calculate the potential energies. Assume that the potential energy at the bottom is 0 .

$$
\begin{array}{ll}
\text { At point } \mathrm{P} & U=m g(5 R)=0.188 \mathrm{~J} \\
\text { At point } \mathrm{Q} & U=m g(R)=0.0376 \mathrm{~J} \\
\text { At top of loop } & U=m g(2 R)=0.0753 \mathrm{~J}
\end{array}
$$

f) The potential energies and work done are unaffected by the initial velocity that the particle might have.
8.13 A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm if the marble is to just reach a target 20 m above the marble's position on the compressed spring. (a) What is the change $\Delta U_{g}$ in the gravitational potential energy of the marble-earth system during the 20 m ascent? (b) What is the change $\Delta U_{s}$ in elastic potential energy of the spring during its launch of the marble? (c) What is the spring constant of the spring?
The change in gravitation potential energy is calculated from the change in height.

$$
\Delta U_{g}=m g h-0=0.005 \cdot 9.8 \cdot 20=0.98 \mathrm{~J}
$$

All of the gain in gravitational potential energy came from loss in energy from the spring.

$$
\Delta U_{s}=-\Delta U_{g}=-0.98 \mathrm{~J}
$$

We can compute the spring constant since we know the energy and compression of the spring.

$$
\begin{aligned}
& \Delta U_{s}=-\frac{1}{2} k x^{2} \\
& k=\frac{-2 \Delta U_{s}}{x^{2}}=\frac{-2(-0.98 \mathrm{~J})}{(0.08 \mathrm{~m})^{2}}=306.25 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

8.19 Figure $8-34$ shows an 8.00 kg stone at rest on a spring. The spring is compressed 10.0 cm by the stone. (a) What is the spring constant? (b) The stone is pushed down an additional 30 cm and released. What is the elastic potential energy of the compressed spring just before that release? (c) What is the change in the gravitation potential energy of the stone-earth system when the stone moves from the release point to its maximum height? (d) What is that maximum height, measured from the release point?

There are a number of ways of approaching this problem.
(a). Since the stone is at rest, we know that the net force on it is zero.


If we examine the compression steps....


The total energy stored in the spring is

$$
\Delta U_{s}=\frac{1}{2} k x^{2}=\frac{1}{2} \cdot 784 \mathrm{~N} / \mathrm{m} \cdot(0.40 \mathrm{~m})^{2}=62.72 \mathrm{~J}
$$

The gravitational potential gets all of this energy when the stone reaches maximum height.

$$
\Delta U_{g}=\Delta U_{s}=62.72 \mathrm{~J}
$$

We can compute the maximum height from the compressed release point.

$$
\begin{aligned}
& \Delta U_{g}=m g y \\
& y=\frac{\Delta U_{g}}{m g}=\frac{62.72 \mathrm{~J}}{8 \mathrm{~kg} \cdot 9.8 \mathrm{~m} / \mathrm{s}^{2}}=0.8 \mathrm{~m}
\end{aligned}
$$

8.21 The string in Fig 8-36 is $\mathrm{L}=120 \mathrm{~cm}$ long, has a ball attached to one end, and is fixed at its other end. The distance d to the fixed peg at point P is 75.0 cm . When the initially stationary ball is released wit the string horizontal as shown, it will swing along the dashed arc. What is its speed when it reaches (a) its lowest point and (b) its highest point after the string catches on the peg.

We begin by computing the total energy.

$$
\begin{aligned}
E_{i} & =\frac{1}{2} m v_{i}^{2}+m g h_{i} \\
& =0+m g L \\
E_{i} & =m g L
\end{aligned}
$$

The total energy remains constant throughout this problem. We now consider the lowest point.

$$
\begin{aligned}
E_{\text {Low }} & =\frac{1}{2} m v_{\text {Low }}{ }^{2}+m g h_{\text {Low }} \\
h_{\text {Low }} & =0 \\
E_{\text {Low }} & =\frac{1}{2} m v_{\text {Low }}{ }^{2}+0 \\
E_{\text {Low }} & =E_{i} \\
\frac{1}{2} m v_{\text {Low }}{ }^{2} & =m g L \\
v_{\text {Low }} & =\sqrt{2 g L}=4.85 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

b) We now consider the high point after catching. Again, energy is conserved

$$
\begin{aligned}
E_{H i g h} & =\frac{1}{2} m v_{\text {High }}^{2}+m g h_{\text {High }} \\
E_{i} & =E_{\text {High }} \\
m g L & =\frac{1}{2} m v_{\text {High }}^{2}+m g(2 r) \\
\frac{1}{2} m v_{\text {High }}^{2} & =m g L-m g(2 r) \\
v_{\text {High }}^{2} & =2 g L-2 g(2 r) \\
v_{\text {High }} & =\sqrt{2 g L-2 g(2 r)} \\
& =2.42 m / s
\end{aligned}
$$

8.34 A boy is seated on top of a hemispherical mound of ice of radius $\mathrm{R}=13.8 \mathrm{~m}$. He begins to slide down the ice, with a negligible initial speed. Approximate the ice as being frictionless. At
what height does the boy lose contact with the ice. Show that he leaves the ice at point whose height is $2 \mathrm{R} / 3$ if the ice is frictionless.

We begin with a picture.


We are interested in finding when the Normal force disappears. We treat this as a circular motion problem

$$
\begin{aligned}
& \frac{m v^{2}}{R}=m g \cos \theta-N \\
& N=0 \\
& \frac{m v^{2}}{R}=m g \cos \theta \\
& v=\sqrt{R g \cos \theta}
\end{aligned}
$$

We now know the velocity when the boy comes off the ice. We now use conservation of energy to find out how high he is when this happens.

$$
\begin{aligned}
E_{i} & =\frac{1}{2} m v_{i}^{2}+m g h_{i}=0+m g R \\
E_{f} & =\frac{1}{2} m v_{f}^{2}+m g h_{f} \\
E_{f} & =E_{i} \\
\frac{1}{2} m v_{f}^{2}+m g h_{f} & =m g R \\
& \text { from above we substitute for } v_{f} \\
v_{f}^{2} & =R g \cos \theta \\
\cos \theta & =\frac{h_{f}}{R} \\
\frac{1}{2} m R g \cdot \frac{h_{f}}{R}+m g h_{f} & =m g R \\
h_{f} & =\frac{2}{3} R=9.2 m
\end{aligned}
$$

8.63 The cable of the 1800 kg elevator cab in Fig. 8-54 snaps when the cab is at rest at the first floor, where the cab bottom is a distance $d=3.7 \mathrm{~m}$ above a spring of spring constant $k=0.15 \mathrm{MN} / \mathrm{m}$. A safety device clamps the cab against guide rails so that a constant frictional force of 4.4 kN opposes the cab's motion. (a) Find the speed of the cab just before it its the spring. (b) Find the maximum distance $x$ that the spring is compressed (the frictional force still acts during this compression). (c) Find the distance that the cab will bounce back up the shaft. (d) using conservation of energy, find the approximate total distance that the cab will move before coming to rest. Assume that the frictional force on the cab is negligible when the cab is stationary.)

To find the speed, we find the work done by friction and gravity and then equate that work to the change in kinetic energy during the fall.

$$
\begin{aligned}
& W=m g y-F_{f} y=1800 \mathrm{~kg} \cdot 9.8 \mathrm{~m} / \mathrm{s}^{2} \cdot 3.7 \mathrm{~m}-4.4 \times 10^{3} \mathrm{~N} \cdot 3.7 \mathrm{~m}=48,988 \mathrm{~J} \\
& \Delta K E=W \\
& \frac{1}{2} m v_{f}^{2}-0=W \\
& v_{f}=\sqrt{\frac{2 W}{m}}=\sqrt{\frac{2 \cdot 48988 \mathrm{~J}}{1800 \mathrm{~kg}}}=7.38 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

During the stopping process, friction does work, the spring does work, and gravity does work.

$$
\begin{aligned}
& W=-F_{f} y-\frac{1}{2} k y^{2}+m g y \\
& K E_{f}-K E_{i}=-F_{f} y-\frac{1}{2} k y^{2}+m g y \\
& 0-K E_{i}=-F_{f} y-\frac{1}{2} k y^{2}+m g y \\
& -48988 \mathrm{~J}=-4400 y-\frac{1}{2} \cdot 0.15 \times 10^{6} \mathrm{~N} / \mathrm{m} \cdot \mathrm{y}^{2}+1800 \mathrm{~kg} \cdot 9.8 \mathrm{~m} / \mathrm{s}^{2} y \\
& y=0.901 \mathrm{~m}
\end{aligned}
$$

