Chapter 7.

7.2 If a Saturn V rocket with an Apollo spacecraft attached has a combined mass of \( m = 2.9 \times 10^5 \) kg and is to reach a speed of \( v = 11.2 \text{ km/s} = 11.2 \times 10^3 \text{ m/s} \), how much Kinetic Energy will it have

\[
K = \frac{1}{2}mv^2 = 1.82 \times 10^{13} \text{ J}
\]

7.5

7.8 A coin slides over a frictionless plane and across an xy coordinate system from the origin to a point with xy coordinates \((3.0 \text{ m}, 4.0 \text{ m})\) while a constant force acts on it. The force has magnitude 2.0N and is directed at a counterclockwise angle of 100 degrees from the positive direction of the x axis. How much work is done by the force on the coin during the displacement.

The easiest way to do this problem is to write out both the displacement vector and the force vector in component form and then use the definition of work. We begin with a picture of what is happening.

\[
\vec{d} = 3\hat{i} + 4\hat{j}
\]
\[
\vec{F} = 2\cos100^\circ\hat{i} + 2\sin100^\circ\hat{j}
\]
\[
W = \vec{F} \cdot \vec{d}
\]
\[
= 3\cdot2\cos100^\circ + 4\cdot2\sin100^\circ
\]
\[
= 6.84J
\]
\[
= 4.52J
\]

7.10

7.13 Figure 7-29 shows three forces applied to a greased trunk that moves leftward by 3m over a frictionless floor. The force magnitudes are \( F_1=5.00\text{N}, F_2=9.00\text{N}, \text{and } F_3=3.00\text{N} \). During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?
Since there is no motion in the vertical direction, there is no work done by forces and components of forces in that direction. We need only concern ourselves with the horizontal motion

\[ F_{net-x} = F_2 \cos 60^\circ - F_1 \]
\[ = 9.00 \text{N} \cdot \cos 60^\circ - 5.00\text{N} \]
\[ = -0.50\text{N} \]

The net force is to the left. This is in the same direction as the motion. The work done is therefore positive.

\[ W = F_{net-x}d \]
\[ = 0.50\text{N} \cdot 3\text{m} \]
\[ = 1.5\text{J} \]

The work done on the trunk is positive, so the kinetic energy increases.

7.17

7.20 In the figure, a horizontal force \( \vec{F}_a \) of magnitude 20.0 N is applied to a 3.00 kg psychology book as the book slides a distance \( d = 0.500\text{m} \) up a frictionless ramp at angle \( \theta = 30^\circ \). (a) During the displacement, what is the net work done on the book by \( \vec{F}_a \), the gravitation force on the book and the normal force on the book? (b) If the book has zero kinetic energy at the start of the displacement, what is its speed at the end of the displacement.

We begin with a drawing

We can calculate the work done by each force
To find the final speed, we can use the work to find the change in kinetic energy

\[ W_{\text{net}} = W_N + W_a + W_{mg} = 1.31J \]

\[ W_{\text{net}} = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \]

\[ = \frac{1}{2}mv_f^2 - 0 \]

\[ v_f = \sqrt{\frac{2W_{\text{net}}}{m}} = 0.9345 \text{ m/s} \]

7.23 In Fig. 7-33, a block of ice slides down a frictionless ramp at angle \( \theta = 50^\circ \) while an ice worker pulls on the block (via a rope with a force \( \vec{F}_r \) that has a magnitude of 50 N and is directed up the ramp. As the block slides through the distance \( d = 0.50m \) along the ramp, its kinetic energy increases by 80J. How much greater would its kinetic energy have been if the rope had not been used? We begin by computing the work done by the worker. The block slides down the incline, while the force exerted by the worker is up the incline. We begin by computing the work done by the force exerted by the worker

\[ W_w = F \cdot d \cos 180^\circ \]

\[ = 50N \cdot 0.5 \cdot (-1) \]

\[ = -25J \]

The total work done on the ice is composed of a piece due to gravity and the work that we have computed due to the worker’s force. We also know that the total work done is the change in the Kinetic energy.

\[ W_{\text{total}} = W_{\text{gravity}} + W_w \]

\[ 80J = W_{\text{gravity}} + (-25J) \]

\[ W_{\text{gravity}} = 105J \]

If there had been no rope force, the work done would be just the work of gravity. In that case, the work done would have been 105J, with a final K of 105 J. This is 25J greater than with the rope.

7.24 A cave rescue team lifts an injured spelunker directly upward and out of a sinkhole by means of a motor driven cable. The lift is performed in three stages, each requiring a vertical distance of 10.0 m: (a) initially stationary spelunker is accelerated to a speed of 5.00 m/s; (b) he is then lifted at the constant speed of 5.00 m/s; (c) finally he is decelerated to zero speed. How much work is done on the 80 kg rescuee by the force lifting him during each stage.
Two forces act on the rescuee: Tension in the cable and weight.

We are interested in finding the work done by the tension. We will do this by calculating the net work done first.

Stage I (final velocity = 5m/s, initial velocity = 0m/s)

\[ W_{net} = \Delta KE = \frac{1}{2}mv_f^2 - 0 = 1000J \]

\[ W_{mg} = -mgd = -80kg \cdot 9.8 \frac{m}{s^2} \cdot 10m = -7840J \]

\[ W_{net} = W_{mg} + W_{T} \]

\[ W_{T} = W_{net} - W_{mg} = 1000J - (-7840J) = 8840J \]

Stage II (final velocity = 5m/s, initial velocity = 5 m/s)

\[ W_{net} = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0J \]

\[ W_{mg} = -mgd = -80kg \cdot 9.8 \frac{m}{s^2} \cdot 10m = -7840J \]

\[ W_{net} = W_{mg} + W_{T} \]

\[ W_{T} = W_{net} - W_{mg} = 0J - (-7840J) = 7840J \]
\[
W_{\text{net}} = \Delta KE = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = 0 J
\]
\[
W_{mg} = -mgd = -80 kg \cdot 9.8 \frac{m}{s^2} \cdot 10 m = -7840 J
\]
\[
W_{net} = W_{mg} + W_T
\]
\[
W_T = W_{net} - W_{mg} = 0 J - (-7840 J) = 7840 J
\]

Stage III (final velocity = 0 m/s, initial velocity = 5 m/s)

\[
W_{net} = \Delta KE = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = 0 J - 1000 J
\]
\[
W_{mg} = -mgd = -80 kg \cdot 9.8 \frac{m}{s^2} \cdot 10 m = -7840 J
\]
\[
W_{net} = W_{mg} + W_T
\]
\[
W_T = W_{net} - W_{mg} = -1000 J - (-7840 J) = 6840 J
\]

7.26 During spring semester at MIT, residents of the parallel buildings of the East Campus dorms battle one another with large catapults that are made with surgical hose mounted on a window frame. A balloon filled with dyed water is placed in a pouch attached to the hose, which is stretched through the width of the room. Assume that the stretching of the hose obeys Hooke’s law with a spring constant of 100 N/m. If the hose is stretched by 5.00 m and then released, how much work does the force from the hose do on the balloon in the pouched by the time the hose reaches its relaxed length.

The magnitude of the work is

\[
W = \frac{1}{2} k x^2 = \frac{1}{2} \cdot 100 N / m \cdot (5.00 m)^2 = 1250 J
\]

The work done is positive because the force acts in the direction of the motion.

7.34 A 5.0 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in Fig 7.31. How much work is done by the force as the block moves from the origin to x=8m

The work done is the area under the curve.

\[
W = 10 N \cdot 2 m + \frac{1}{2} \cdot 2 m \cdot 10 N + \frac{1}{2} \cdot 2 m \cdot (-5 N) = 25 J
\]

7.43

7.46

7.54 A 250 g block is dropped onto a vertical spring with spring constant k=2.5N/cm. the block becomes attached to the spring and the spring compresses 12 cm before momentarily stopping.
While the spring is being compressed, what work is done on the block (a) by its weight, and (b) by the spring force? (c) What is the speed of the block just before it hits the spring? (d) If the speed is doubled, what is the maximum compression of the spring.

a. The work done by weight is positive. The force is downward and so is the motion.

\[ W_w = mgy = 0.25 \text{kg} \cdot 9.8 \text{m/s}^2 \cdot 0.12 \text{m} = 0.294 \text{J} \]

b) The work done by the spring is negative. The force is upward but the motion is downward.

\[ k = \frac{2.5 \text{N}}{cm} \cdot \frac{100 \text{cm}}{1 \text{m}} = 250 \text{N/m} \]
\[ W_s = -\frac{1}{2} k x^2 = -\frac{1}{2} \cdot 250 \text{N/m} \cdot (0.12 \text{m})^2 = -1.8 \text{J} \]

c) The net work done equals the change in Kinetic energy.

\[ W_{net} = W_{mg} + W_s = 0.294 \text{J} - 1.8 \text{J} = -1.506 \text{J} \]
\[ K_f - K_i = W_{net} \]
\[ 0 - \frac{1}{2} mv_i^2 = W_{net} \]
\[ v_i = \sqrt{\frac{-2W_{net}}{m}} = \sqrt{\frac{-2 \cdot (-1.506 \text{J})}{0.25}} = 3.47 \text{m/s} \]

(d) Use work to determine the final compression. The velocity is twice the value found in part c. We choose the negative solution, since we know that the mass drops below the zero starting value...
\[ v_i = 2 \cdot 3.47 \text{m/s} = 6.94 \text{m/s} \]

\[ W_{net} = \Delta K = 0 - \frac{1}{2} m v_i^2 \]

\[ W_{net} = m g y + \frac{1}{2} k y^2 \]

\[ m g y - \frac{1}{2} k y^2 = - \frac{1}{2} m v_i^2 \]

\[ \frac{1}{2} k y^2 - m g y - \frac{1}{2} m v_i^2 = 0 \]

\[ y = \frac{+m g \pm \sqrt{(m g)^2 - 4 \cdot \frac{1}{2} k \cdot (-\frac{1}{2} m v_i^2)}}{2 \cdot \frac{1}{2} k} \]

\[ = \frac{+m g \pm \sqrt{(m g)^2 + k m v_i^2}}{k} \]

\[ = \frac{+0.25 \text{kg} \cdot 9.8 \text{m/s}^2 \pm \sqrt{(0.25 \text{kg} \cdot 9.8 \text{m/s}^2)^2 + 250 \text{N/m} \cdot 0.25 \text{kg} \cdot (6.94 \text{m/s})^2}}{250 \text{N/m}} \]

\[ = -0.2099 \text{m} \]