## Chapter 4

**4.1** A positron undergoes a displacement of $\Delta\vec{r} = 2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k}$, ending with the the position vector $\vec{r}_f = 3.0\hat{i} - 4.0\hat{j}$.in meters. What was the positron's initial position vector?

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
$$\vec{r}_i = \vec{r}_f - \Delta\vec{r}$$
$$\vec{r}_i = (3.0\hat{i} - 4.0\hat{j}) - (2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k})$$
$$\vec{r}_i = 1.0\hat{i} - 1..0\hat{j} - 6.0\hat{k}$$

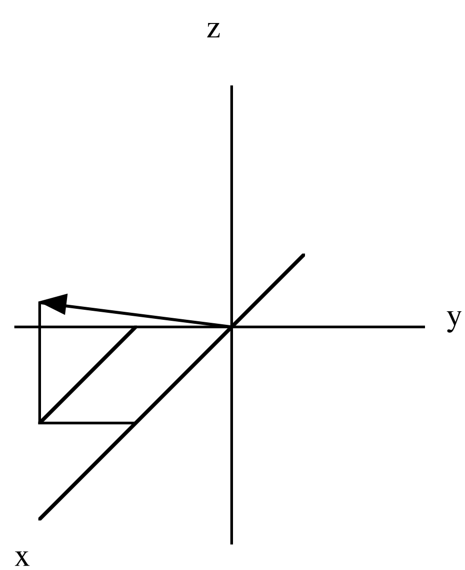
**4.3** The position vector of an electron is

$$\vec{r}_i = 5.0\hat{i} - 3.0\hat{j} + 2.0\hat{k}$$

(a) Find the magnitude of $\vec{r}$ (b) Sketch the vector on a right handed coordinate system.

$$|\vec{r}| = \sqrt{5^2 + (-3)^2 + 2^2} = 6.16\,m$$

(b) Sketch...




**4.5** An ion's initial and final position vectors are:

$$\vec{r}_i = 5.0\,\hat{i} - 6.0\,\hat{j} + 2.0\,\hat{k}$$
$$\vec{r}_f = -2.0\hat{i} + 8.0\,\hat{j} - 2.0\,\hat{k}$$

The change in position occurs over 10 s. What is the average velocity?

$$\vec{v}_{avg} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t} = -\frac{7}{10}\hat{i} + \frac{14}{10}\hat{j} - \frac{4}{10}\hat{k}\ m/s$$

**4.6** The position of an electron is given by

$$\vec{r} = 3.00\,t\,\hat{i} - 4.00t^2\,\hat{j} + 2.00\hat{k}$$

(a) What is the electron's velocity v(t)? At t = 2.00s what is v in (b) in unit-vector notation and as (c) a magnitude and (d) an angle relative to the positive direction of the x axis.

(a) We find the velocity.

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = 3.00\ \hat{i} - 8.00t\,\hat{j}$$

(b) At t = 2s.

$$\vec{v}(2) = 3.00\ \hat{i} - 16.00\,\hat{j}\ \ m/s$$

$$|\vec{v}(2)| = \sqrt{3^2 + 16^2} = 16.28\,m/s$$

$$\tan\theta = \frac{-16}{3} \Rightarrow \theta = \tan^{-1}(-\frac{16}{3}) = -79.4°$$

**4.11** A particle moves so that its position (in meters) as a function of time in seconds is

$$\vec{r}(t) = 1\ \hat{i} + 4\,t^2\ \hat{j} + t\,\hat{k}$$

Write expressions for (a) its velocity and (b) its accelerations as a function of time.

(a) Velocity is the derivative of the position

$$\vec{v}\,(t) = \frac{d\vec{r}}{dt} = 8t\,\hat{j} + 1\,\hat{k}$$

(b) The acceleration is the derivative of the velocity vector.

$$\vec{a}(t) = \frac{d\vec{v}}{d\,t} = 8\,\hat{j}$$

**4.16** A moderate wind accelerates a pebble over a horizontal plane with a constant acceleration $\vec{a} = (5.00m/s^2)\hat{i} + (7.00m/s^2)\ \hat{j}$. At time t=0, the velocity is $\vec{v} = 4.00m/s\hat{i}$. What are the magnitude and (b) angle of its velocity when it has been displaced by 12.00 m parallel to the x axis.

To proceed, we need to find out how long it takes for the x component of the motion to reach 12 m.

$$x_i = 0 \qquad y_i = 0$$
$$x_f = 12m \qquad y_f = 0$$
$$v_{ix} = 4m/s \qquad v_{iy} = 0m/s$$
$$v_{fx} = ? \qquad v_{fy} = ?$$
$$a_x = 5m/s^2 \qquad a_y = 7m/s^2$$

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$
$$x_f = 0 + v_{ix}t + \frac{1}{2}a_x t^2$$
$$0 = \frac{1}{2}a_x t^2 + v_{ix}t - x_f$$
$$0 = \frac{1}{2}\cdot 5\cdot t^2 + 4t - 12.0$$
$$t = \frac{-4 \pm \sqrt{4^2 - 4\cdot\frac{5}{2}\cdot -12}}{2\cdot\frac{5}{2}}$$
$$= 3.13s$$

4.21

**4.24** A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally away from the edge of the table. (a) How long is the ball in the air? (b)What is the speed at the instant it leaves the table.

a) We begin by writing the conditions that are given.

$$x_i = 0 \qquad y_i = 1.20$$
$$x_f = 1.52m \qquad y_f = 0$$
$$v_{ix} = ? \qquad v_{iy} = 0m/s$$
$$v_{fx} = v_{ix} \qquad v_{fy} = ?$$
$$a_x = 0 \qquad a_y = -g$$
$$t = ?$$

We now compute how long it was in the air

$$y_f = y_i + v_{iy}t + \frac{1}{2}at^2$$
$$0 = y_i + 0 - \frac{1}{2}g\,t^2$$
$$t = \sqrt{\frac{2y_i}{g}} = 0.495\,\text{sec}$$

b) Now that we know the time, we can find the horizontal velocity

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$
$$x_f = 0 + v_{ix}t + 0$$
$$v_{ix} = \frac{x_f}{t} = \frac{1.52m}{0.495s} = 3.1m/s$$

**4.27** A certain airplane has a speed of 290km/h and is diving at an angle of 30degrees below the horizontal when the pilot releases a radar decoy. The horizontal distance between the release point and the point where the decoy strikes the ground is 700m. (a) How long is the decoy in the air? (b) How high was the released point?

$$v = 290\frac{km}{h}\cdot\frac{1000m}{1km}\cdot\frac{1h}{3600s} = 80.56m/s$$
$$x_i = 0$$
$$x_f = 700m$$
$$v_{ix} = 80.56\cos 30$$
$$v_{fx} = v_{ix}$$
$$a_x = 0$$

$$y_i = ?$$
$$y_f = 0m$$
$$v_{iy} = -80.56\sin 30$$
$$v_{fy} = v_{iy}$$
$$a_x = -g$$

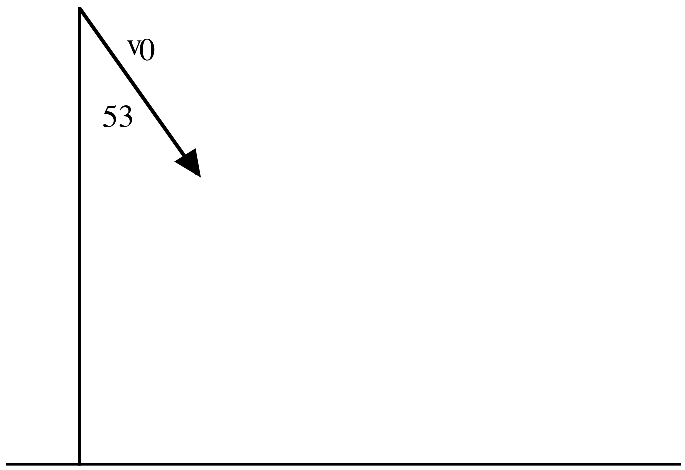
We solve for the time of flight.

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$
$$x_f = 0 + v_{ix}t + 0$$
$$t = \frac{x_f}{v_{ix}} = \frac{700m}{80.56\cos 30} = 10.03s$$

Now we can use the time of flight to find the release point

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$
$$0 = y_i + v_{iy}t - \frac{1}{2}gt^2$$
$$y_i = \frac{1}{2}gt^2 - v_{iy}t$$
$$= \frac{1}{2}\cdot 9.8\cdot(10.03)^2 - (-80.56\sin 30)\cdot(10.03)$$
$$= 897m$$

**4.31** An airplane, diving at an angle of 53 degrees with the vertical, releases a projectile at an altitude of 730m. The projectile hits the ground 5.00s after being released. (a) What is the speed of the aircraft? (b) How far did the projectile travels horizontally during its flight? What were the (c)horizontal and (d) vertical components of its velocity just before striking the ground.




We begin by realizing that the speed of the plane is the initial speed of the particle. We write the initial conditions

$$x_i = 0 \qquad y_i = 730$$
$$x_f = ? \qquad y_f = 0m$$
$$v_{ix} = v_0 \sin 53° \qquad v_{iy} = -v_0 \cos 53$$
$$v_{fx} = v_{ix} \qquad v_{fy} = ?$$
$$a_x = 0 \qquad a_y = -g$$
$$t = 5s$$

We can now solve for the speed of the aircraft.

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$
$$0 = y_i - v_0 \cos 53 t - \frac{1}{2}gt^2$$
$$v_0 = \frac{y_i - \frac{1}{2}gt^2}{(\cos 53)t} = \frac{730m - \frac{1}{2}\cdot 9.8m/s^2 \cdot (5s)^2}{(\cos 53)\cdot 5s}$$
$$= 201.9\, m/s$$

Now that we know the speed, we can find the range

$$
\begin{aligned}
x_f &= x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\
&= 0 + (v_0 \sin 53)t + 0 \\
&= 201.9m/s \cdot \sin 53 \cdot 5s \\
&= 806.2m
\end{aligned}
$$

We also find the final velocity just before impact...

$$
\begin{aligned}
v_{fx} &= v_{ix} = 201.9 \sin 53 \\
&= 161.2m/s \\
v_{fy} &= v_{iy} + at \\
&= -v_0 \cos 53 - g\,t \\
&= -201.9 \cos 53 m/s - 9.8m/s^2 \cdot 5s \\
&= -170.5m/s
\end{aligned}
$$

**4.41**

**4.57**

**4.59** When a large star becomes a supernova, its core may be compressed so tightly that it becomes a neutron star, with a radius of about 20km (about the size of the San Francisco area). If a neutron star rotates once every second, (a) what is the speed of a particle on the star's equator and (b) what is the magnitude of the particle's centripetal acceleration? (c) If the neutron star rotates faster, does the answers to (a) and (b) increase, decrease, or stay the same.

$$
v = \frac{2\pi r}{t} = \frac{2\pi \cdot 20{,}000m}{1s} = 1.27 \times 10^5 m/s
$$

$$
a_c = \frac{v^2}{r} = \frac{(1.27 \times 10^5 m/s)^2}{20{,}000m} = 7.89 \times 10^5 m/s^2
$$

Both velocity and centripetal acceleration increase if the star rotates faster