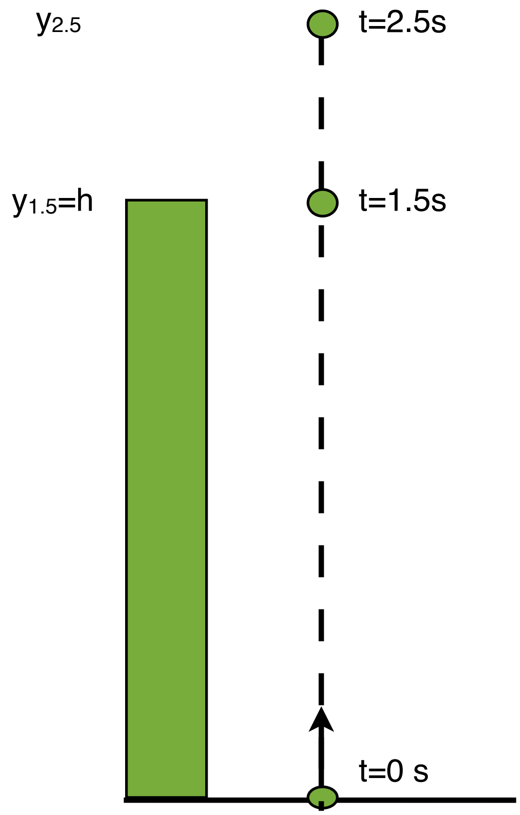
Chapter 2 Problems

2.58 A rock is thrown vertically uward from ground level at time t=0. At t=1.5s, it passes the to of a tall tower and 1.0 s later, it reaches its maximum height. What is the height of the tower



We begin by considering the t=0 to t=1.5s part of the trip.

$$y_i = 0m$$
$$y_{1.5} = h$$
$$v_i = ?$$
$$v_{1.5} = ?$$
$$a = -g$$
$$t = 1.5s$$

We can find the height if we have the initial velocity, using

$$y_{1.5} = y_i + v_i\, t + \frac{1}{2} a\, t^2$$

To find the initial velocity, we consider the entire 2.5 second trip.

$$y_i = 0m$$
$$y_{2.5} = ?$$
$$v_i = ? \qquad v_{2.5} = v_i - g \cdot 2.5s$$
$$v_{2.5} = 0 \qquad v_i = g \cdot 2.5s = 24.5m/s$$
$$a = -g$$
$$t = 2.5s$$

Now that we know the initial velocity, we can find the height of the tower.

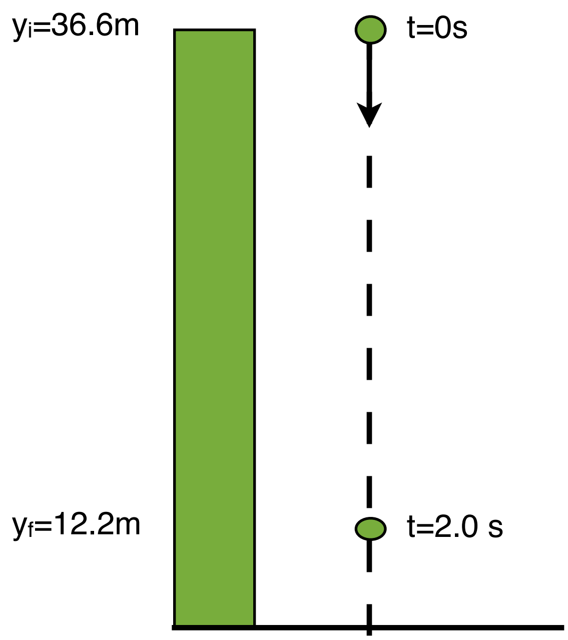
$$y_{1.5} = y_i + v_i t + \frac{1}{2} a t^2$$
$$y_{1.5} = y_i + 24.5m/s \cdot 1.5s - \frac{1}{2} \cdot 9.8m/s^2 \cdot (1.5s)^2 = 25.725m$$

2.67 How far does the runner whose velocity-time graph is shown in Fig 2036 travel in 16 s? The figure's vertical scaling is et by v_s=8.0m/s

The runner's displacement is the area under the curve. We can compute the area geometrically

$$A = \frac{1}{2} \cdot 2s \cdot 8m/s + 8s \cdot 8m/s + \frac{1}{2} \cdot 2s \cdot 4m/s + 2s \cdot 4m/s + 4s \cdot 4m/s = 100m$$

2.108 A ball is thrown vertically downward from the top of a 36.6m tall building. The ball passes the top of a window that is 12.2m above the ground 2.00s after being thrown. What is the speed of the ball as it passes the top of the window?



Once again, we write what we know

$$
\begin{aligned}
y_i &= 36.6m \\
y_f &= 12.2m \\
v_i &= ? \\
v_f &= ? \\
a &= -g \\
t &= 2.0s
\end{aligned}
$$

As in the previous problem, we need the initial velocity to find the result that we are looking for --in this case the final velocity. We can find the initial velocity

$$
y_f = y_i + v_i\,t + \frac{1}{2}a\,t^2 = y_i + v_i\,t - \frac{1}{2}g\,t^2
$$

$$
v_i = \frac{y_f - y_i + \frac{1}{2}g\,t^2}{t} = -2.4m/s
$$

We can now find the final velocity

$$
v_f = v_i + at = -2.4m/s - 9.8m/s^2 \cdot 2s = -22m/s
$$

Chapter 3 Problems

3.1 The x component of a certain vector is -25.0 units and the y component is +40 units. (a) What is the magnitude and direction of the vector? (b) What is the angle between n the direction of the vector and the positive direction of x.

(a). The magnitude is

$$
|\vec{v}| = \sqrt{(-25)^2 + (40)^2} = 47.17\ units
$$

(b) The direction with respect to +x is:

$$
\begin{aligned}
\tan\theta &= \frac{40}{-25} \\
\theta &= 122^\circ
\end{aligned}
$$

3.6 A displacement vector r in the xy plane is 15 m long and directed as shown in Fig 3-29. Determine the x-y coordinates.

$$
\begin{aligned}
r_x &= 15\cos 30 = 12.99m \\
r_y &= 15\sin 30 = 7.50m
\end{aligned}
$$

3.9 (a) In unit-vector notation , what is the sum of

$$\vec{a} = (4.0m)\,\hat{i} + (3.0m)\,\hat{j}$$
$$\vec{b} = (-13.0m)\,\hat{i} + (7.0m)\,\hat{j}$$

What are (b) the magnitude and (c) the direction of * relative to $\hat{i}$.

To compute the sum in vector notation, we just add the components.

$$\vec{a} + \vec{b} = (4.0 - 13.0)\hat{i} + (3.0 + 7.0)\,\hat{j}$$
$$= (-9.0m)\,\hat{i} + (10m)\,\hat{j}$$

We find the magnitude and direction...

$$\left|\vec{a} + \vec{b}\right| = \sqrt{9^2 + 10^2} = 13.45m$$
$$\tan\theta = \frac{10}{-9} \Rightarrow \theta = \tan^{-1}(\frac{10}{-9}) = -48.01°$$
$$\theta = -48.01° + 180° = 131.99°$$

Note that the calculator returns an angle of -48 degrees--but when we look at the components, we can see that the angle is actually in another quadrant that is 180 degrees away.

3.13 Two vectors are given by

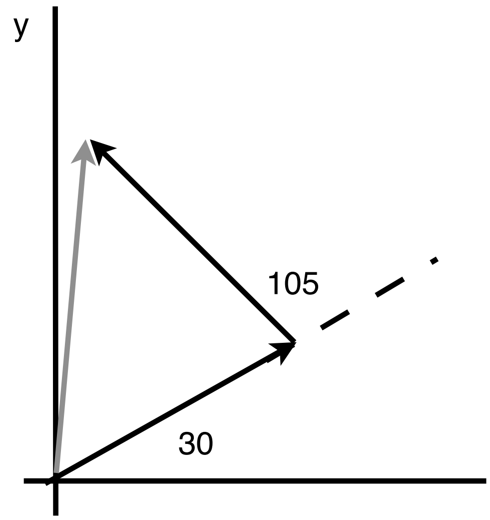
$$\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$$
$$\vec{b} = -\hat{i} + \hat{j} + 4\hat{k}$$

Find $\vec{a} + \vec{b}$, $\vec{a} - \vec{b}$ and a vector $\dot{c}$ such that $\vec{a} - \vec{b} + \vec{c} = 0$

$$\vec{a} + \vec{b} = 3\hat{i} - 2\hat{j} + 5\hat{k}$$
$$\vec{a} - \vec{b} = 5\hat{i} - 4\hat{j} - 3\hat{k}$$

$$0 = \vec{a} - \vec{b} + \vec{c}$$
$$\vec{c} = -(\vec{a} - \vec{b})$$
$$= -(5\hat{i} - 4\hat{j} - 3\hat{k})$$
$$= -5\hat{i} + 4\hat{j} + 3\hat{k}$$

3.17 The two vectors $\vec{a}$ and $\vec{b}$ in Fig 3-30 have equal magnitudes of 10.0 m and the angles are $\theta_1 = 30^\circ$ and $\theta_2 = 105^\circ$. Find the a) x and (b) y components of their vector sum $\vec{r}$ (c) the magnitude of $\vec{r}$, and (d) the angle $\vec{r}$ makes with the positive direction of the x axis.



We need to write out the two vectors in component notation. To do this, we need to remember that we need to write each angle with respect to the positive x axis

$$\vec{a} = 10\cos 30\,\hat{i} + 10\sin 30\,\hat{j} = 8.66\,\hat{i} + 5\,\hat{j}$$
$$\vec{b} = 10\cos 135\,\hat{i} + 10\sin 135\,\hat{j} = -7.07\,\hat{i} + 7.07\,\hat{j}$$
$$\vec{r} = \vec{a} + \vec{b} = (8.66 - 7.07)\,\hat{i} + (5 + 7.07)\,\hat{j} = 1.59\,\hat{i} + 12.07\,\hat{j}$$
$$|\vec{r}| = \sqrt{(1.59)^2 + (12.07)^2} = 12.17$$
$$\theta = \tan^{-1}\left(\frac{12.07}{1.59}\right) = 82.5^\circ$$

3.20 What is the sum of the following four vectors in unit-vector notation? For that sum, what are (b) the magnitude, (c) the angle in degrees, and (d) the angle in radians?

$$\vec{E} : 6.00\,m\ at\ +0.900\,rad \qquad \vec{F} : 5.00\,m\ at\ +-75.0^\circ$$
$$\vec{G} : 4.00\,m\ at\ +1.200\,rad \qquad \vec{H} : 6.00\,m\ at\ -210.0^\circ$$

We can write out each of these vectors in vector notation.

$$\vec{E} : 6.00\,m\ at\ +0.900\,rad \Rightarrow \vec{E} = 6\cos(0.9rad)\,\hat{i} + 6\sin(0.9rad)\,\hat{j} = 3.73\,\hat{i} + 4.70\,\hat{j}$$
$$\vec{G} : 4.00\,m\ at\ +1.200\,rad \Rightarrow \vec{G} = 4\cos(1.2rad)\,\hat{i} + 4\sin(1.2rad)\,\hat{j} = 1.45\,\hat{i} + 3.73\,\hat{j}$$
$$\vec{F} : 5.00\,m\ at\ +-75.0^\circ \Rightarrow \vec{F} = 5\cos(-75.0^\circ)\,\hat{i} + 5\sin(-75.0^\circ)\,\hat{j} = 1.29\,\hat{i} + -4.83\,\hat{j}$$
$$\vec{H} : 6.00\,m\ at\ -210.0^\circ \Rightarrow \vec{H} = 6\cos(-210.0^\circ)\,\hat{i} + 6\sin(-210.0^\circ)\,\hat{j} = -5.19\,\hat{i} + 3.00\,\hat{j}$$

$$\vec{R} = (3.73 + 1.45 + 1.29 - 5.19)\hat{i} + (4.70 + 3.73 - 4.83 + 3.00)\hat{j}$$
$$= 1.28\hat{i} + 6.60\hat{j}$$
$$\left|\vec{R}\right| = \sqrt{1.28^2 + 6.60^2} = 6.73m$$
$$\theta = \tan^{-1}(\frac{6.60}{1.28}) = 79.02°$$
$$= 1.379\,rad$$

3.25 If $\vec{B}$ is added to $\vec{C} = 3\,\hat{i} + 4\,\hat{j}$, the result is a vector in the positive direction of y axis with a magnitude equal to that of $\vec{C}$. What is the magnitude of $\vec{B}$.

We know that the magnitude of resultant vector is 5 because we are told that it has the magnitude of $\vec{C}$, which is 5. We are also told that the resultant is in the y direction. So

$$\vec{B} = B_x\,\hat{i} + B_y\,\hat{j}$$
$$\vec{C} = 3\,\hat{i} + 4\,\hat{j}$$
$$\vec{R} = 0\,\hat{i} + 5\,\hat{j}$$

We can see that to get this resultant, $\vec{B} = -\,3\hat{i} + 1\,\hat{j}$. The magnitude of B is 3.2

3.33 Two vectors, $\vec{r}$ and $\vec{s}$ lie in the xy plane. Their magnitudes are 4.50 and 7.30 units respectively and their directions are 320 degrees and 85 degrees respectively as measured counterclockwise from the positive x axis. What are the values of (a) $\vec{r}\cdot\vec{s}$ and (b) $\vec{r}\times\vec{s}$

If we plot these vectors, we can see that the angle between them is 85+40 =125 degrees. We can easily compute magnitudes

$$\vec{r}\cdot\vec{s} = |\,\vec{r}\,\|\,\vec{s}\,|\cos\theta = -\,18.84$$
$$|\,\vec{r}\times\vec{s}\,| == |\,\vec{r}\,\|\,\vec{s}\,|\sin\theta = 26.91$$

The cross-product is a vector, so we still need to specify its direction. The cross-product points out of the paper (+z) via the right hand rule.

3.40 For the following three vectors, what is $3\vec{C}\cdot(2\vec{A}\times\vec{B})$

$$\vec{A} = 2.00\hat{i} + 3.00\hat{j} - 4.00\hat{k}$$
$$\vec{B} = -3.00\hat{i} + 4.00\hat{j} + 2.00\hat{k}$$
$$\vec{C} = 7.00\hat{i} - 8.00\hat{j}$$

$$2\vec{A} = 4.00\hat{i} + 6.00\hat{j} - 8.00\hat{k}$$

$$3\vec{C} = 21.00\hat{i} - 24.00\hat{j}$$

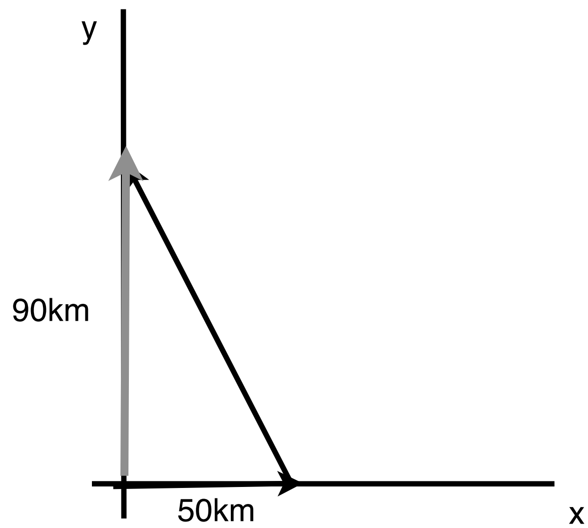
$$2\vec{A}\times\vec{B} = \det\begin{Bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & -8 \\ -3 & 4 & 2 \end{Bmatrix}$$

$$= \hat{i}\,(6\cdot 2 - (-8)\cdot 4) - \hat{j}\,(4\cdot 2 - (-8)\cdot(-3)) + \hat{k}\,(4\cdot 4 - 6\cdot(-3))$$

$$= 44\hat{i} + 16\hat{j} + 34\hat{k}$$

$$3\vec{C}\cdot(2\vec{A}\times\vec{B}) = (21)\cdot 44 + (-24)\cdot 16 = 54$$

3.51 A sailboat sets out from the U.S. Side of Lake Erie for a point on the Canadian side, 90 km due north. The sailor , however, ends up 50.0 km due east of the starting point (a) How far and (b) in what directions must the sailor now sail to reach the original destination?

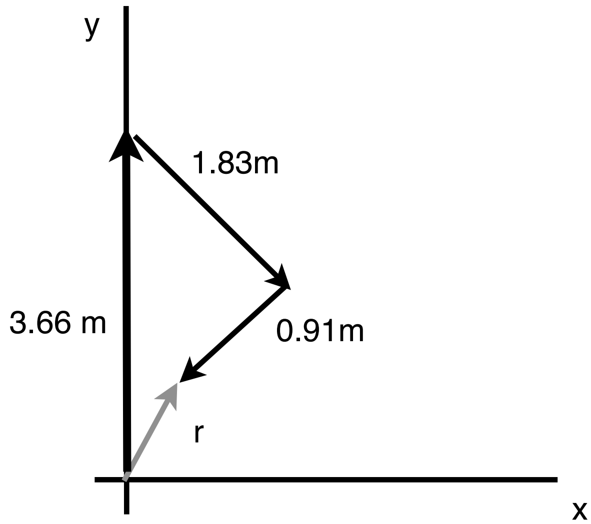


The key to this problem is getting the picture right. The sailor initially travels 50 km due easy, but he/she wants to reach the destination 90km due north of his/her original position. We can calculate the distance and angle that he/she needs to sail.

$$r = \sqrt{50^2 + 90^2} = 102.96km$$

$$\tan\theta = \frac{90}{50} \Rightarrow \theta = 60.96\ N\ of\ W$$

3.58 A golfer takes three putts to get the ball into the hole. The first putt displaces the ball 3.66m north, the second 1.83 m southeast, and the third 0.91 m southwest. What are (a) the magnitude and (b) direction of the displacement needed to get the ball into the hole on the first putt.



The putt we need to make the first time is the r vector shown in the picture. It is the vector sum of the three putts it took to get to the hole. We can do this most easily with components.

$$\vec{a} = 0\,\hat{i} + 3.66\,\hat{j}$$
$$\vec{b} = 1.83\sin 45\,\hat{i} - 1.83\cos 45\,\hat{j} = 1.29\,\hat{i} - 1.29\,\hat{j}$$
$$\vec{c} = -0.91\sin 45\,\hat{i} - 0.91\cos 45\,\hat{j} = -0.644\,\hat{i} - 0.644\,\hat{j}$$
$$\vec{r} = \vec{a} + \vec{b} + \vec{c} = 0.646\,\hat{i} + 1.726\,\hat{j}$$
$$|\vec{r}| = 1.843m$$
$$\tan\theta = \frac{1.726}{0.646} \Rightarrow \theta = 69.48^\circ$$