Chapter 2 and Chapter 3 Problems

2.49 A hot-air balloon is ascending at the rate of 12m/s and is 80m above the ground when a package is dropped over the side. (a) How long does the package take to reach the ground? With what speed does it hit the ground?

$$y_i = 80m$$
$$y_f = 0$$
$$v_i = +12m/s$$
$$v_f = ?$$
$$a = -g$$
$$t = ?$$

(a) We determine the time

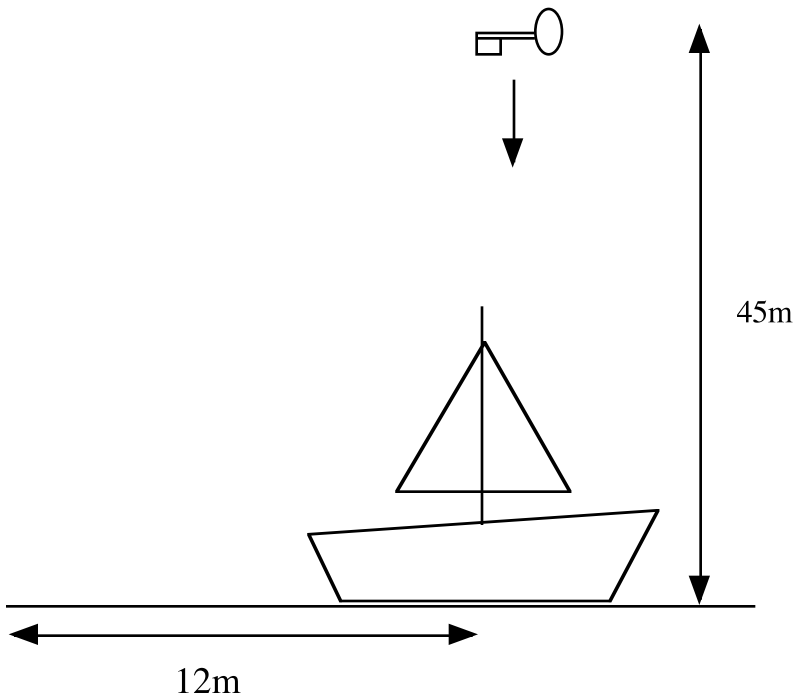
$$y_f = y_i + v_i t + \frac{1}{2} a t^2$$
$$0 = y_i + v_i t - \frac{1}{2} g t^2$$
$$0 = 80 + 12t - \frac{1}{2} \cdot 9.8 t^2 = 80 + 12t - 4.9t^2$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{12^2 - 4 \cdot (-4.9) \cdot 80}}{2 \cdot (-4.9)}$$
$$= 5.44s$$

(choose the positive time).

Now that we know the time, find the speed

$$v_f = v_i + at$$
$$= v_i - gt$$
$$= 12m/s - 9.8m/s^2 \cdot 5.44s$$
$$= -41.3m/s$$

2.51 A key falls from a bridge that is 45 m above the water. It falls directly into a model boat moving with a constant velocity, that is 12m from the point of impact when the key is released. What is the speed of the boat?



Since we know the distance that the boat travels, we could find the speed if we knew how long it took to travel the 12 m. We can find this time by finding the time for the key to drop, since we know that the boat and key were released at the same time.

$$v_i = 0$$
$$v_f = ?$$
$$y_i = 45m$$
$$y_f = 0m$$
$$a = -g = -9.8\ m/s^2$$

$$y_f = y_i + v_i t + \frac{1}{2} a_y t^2$$
$$0 = y_i + 0 - \frac{1}{2} g t^2$$
$$t = \sqrt{\frac{2y_i}{g}} = 3.03s$$
$$s_{boat} = \frac{d}{t} = \frac{12m}{3.03s} = 3.96m/s$$

2.58

2.67

3.1 The x component of a certain vector is -25.0 units and the y component is +40 units. (a) What is the magnitude and direction of the vector? (b) What is the angle between n the direction of the vector and the positive direction of x.

(a). The magnitude is

$$|\vec{v}| = \sqrt{(-25)^2 + (40)^2} = 47.17$$

(b) The direction with respect to +x is:

$$\tan\theta = \frac{40}{-25}$$
$$\theta = 122°$$

You need to pay attention to which quadrant the result is in. You may need to add 180 to the result given by your calculator.

3.6 A displacement vector r in the xy plane is 15 m long and directed as shown in Fig 3-26. Determine the xy coordinates.

$$r_x = 15\cos 30 = 12.99m$$
$$r_y = 15\sin 30 = 7.50m$$

3.9 (a) In unit-vector notation , what is the sum of

$$\vec{a} = (4.0m)\,\hat{i} + (3.0m)\,\hat{j}$$
$$\vec{b} = (-13.0m)\,\hat{i} + (7.0m)\,\hat{j}$$

What are (b) the magnitude and (c) the direction of $\vec{a}+\vec{b}$ relative to $\hat{i}$.

To compute the sum in vector noation, we just add the components.

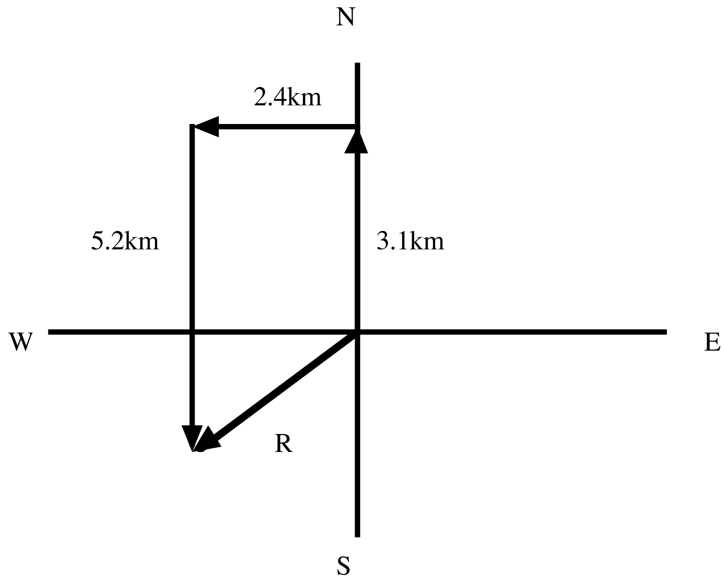
$$\vec{a}+\vec{b} = (4.0-13.0)\hat{i} + (3.0+7.0)\,\hat{j}$$
$$= (-9.0m)\,\hat{i} + (10m)\,\hat{j}$$

We find the magnitude and direction...

$$\left|\vec{a}+\vec{b}\right| = \sqrt{9^2+10^2} = 13.45m$$
$$\tan\theta = \frac{10}{-9} \Rightarrow \theta = \tan^{-1}(\frac{10}{-9}) = -48.01°$$
$$\theta = -48.01° + 180° = 131.99°$$

Note that the calculator returns an angle of -48 degrees--but when we look at the components, we can see that the angle is actually in another quadrant that is 180 degrees away.

3.10 A person walks in the following pattern: 3.1km north, then 2.4km west, and finally 5.2km south. (a) Sketch the vector diagram that represents this motion. (b) How far and (c) and in what direction would a bird fly in a straight line from the same starting point to the same final point.



We are trying to find the length and direction of R. We proceed by writing the steps of the trip in component form

$$\vec{a} = 3.1\ km\ \hat{j}$$
$$\vec{b} = -2.4\ km\hat{i}$$
$$\vec{c} = -5.2\ km\ \hat{j}$$

$$\vec{R} = \vec{a} + \vec{b} + \vec{c}$$
$$= -2.4\ km\hat{i} - 2.1\ km\ \hat{j}$$
$$\left|\vec{R}\right| = \sqrt{2.4^2 + 2.1^2} = 3.19\ km$$
$$\tan\theta = \frac{2.1}{2.4} \Rightarrow \theta = 41.2° \textit{ south of west}$$

3.13 Two vectors are given by $\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 4\hat{k}$. Find $\vec{a} + \vec{b}$, $\vec{a} - \vec{b}$ and a vector $\vec{c}$ such that $\check{a} - \dot{b} + \check{c} = 0$ $\vec{a} - \vec{b} + \vec{c} = 0$

$$\vec{a} + \vec{b} = 3\hat{i} - 2\hat{j} + 5\hat{k}$$

$$\vec{a} - \vec{b} = 5\hat{i} - 4\hat{j} - 3\hat{k}$$

$$0 = \vec{a} - \vec{b} + \vec{c}$$
$$\vec{c} = -(\vec{a} - \vec{b})$$
$$= -(5\hat{i} - 4\hat{j} - 3\hat{k})$$
$$= -5\hat{i} + 4\hat{j} + 3\hat{k}$$

3.20 What is the sum of the follwoing four vecrtors in unit-vector notation? For that sum, what are (b) the magnitude, (c) the angle in degrees, and (d) the angle in radians?

$$\vec{E}: 6.00\, m\ at\ +0.900\, rad \qquad \vec{F}: 5.00\, m\ at\ +-75.0°$$

$$\vec{G}: 4.00\, m\ at\ +1.200\, rad \qquad \vec{H}: 6.00\, m\ at\ -210.0°$$

We can write out each of these vectors in vector notation.

$$\vec{E}: 6.00\, m\ at\ +0.900\, rad \Rightarrow \vec{E} = 6\cos(0.9 rad)\ \hat{i} + 6\sin(0.9 rad)\ \hat{j} = 3.73\ \hat{i} + 4.70\ \hat{j}$$

$$\vec{G}: 4.00\, m\ at\ +1.200\, rad \Rightarrow \vec{G} = 4\cos(1.2 rad)\ \hat{i} + 4\sin(1.2 rad)\ \hat{j} = 1.45\ \hat{i} + 3.73\ \hat{j}$$

$$\vec{F}: 5.00\, m\ at\ +-75.0° \Rightarrow \vec{F} = 5\cos(-75.0°)\ \hat{i} + 5\sin(-75.0°)\ \hat{j} = 1.29\ \hat{i} + -4.83\ \hat{j}$$

$$\vec{H}: 6.00\, m\ at - 210.0° \Rightarrow \vec{H} = 6\cos(-210.0°)\ \hat{i} + 6\sin(-210.0°)\ \hat{j} = -5.19\ \hat{i} + 3.00\ \hat{j}$$

$$\vec{R} = (3.73 + 1.45 + 1.29 - 5.19)\hat{i} + (4.70 + 3.73 - 4.83 + 3.00)\hat{j}$$

$$= 1.28\hat{i} + 6.60\hat{j}$$

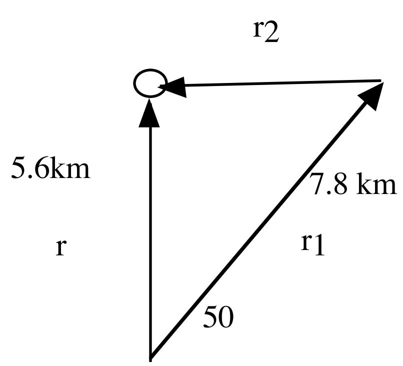
$$\left|\vec{R}\right| = \sqrt{1.28^2 + 6.60^2} = 6.73m$$

$$\theta = \tan^{-1}\left(\frac{6.60}{1.28}\right) = 79.02°$$

$$= 1.379\, rad$$

3.22 An explorer is caught i a whiteout (in which the snowfall is so thick that the ground cannot be distinguished from the sky) while returning to base camp. He was supposed to travel due north for 5.6 km, but when the snow clears he discovers that he actually traveled 7.8 km at 50 degrees north of due east. (a) How far and (b) in what direction must he now travel to reach base camp.

We begin by drawing the trip. The trip that the explorer made is **r1**. The trip that he was supposed to make was **r**. The trip back to camp is **r2**. We can proceed by writing the vector equation



$$\vec{r} = \vec{r}_1 + \vec{r}_2$$

We can see that to solve for **r2**

$$\vec{r}_2 = \vec{r} - \vec{r}_1$$

Now let's write the vectors in component form

$$\begin{aligned}
\vec{r}_1 &= 7.8\cos 50\ \hat{i} + 7.8\sin 50\ \hat{j} \\
&= 5.01\,\hat{i} + 5.98\ \hat{j} \\
\vec{r} &= 0\ \hat{i} + 5.6\,\hat{j} \\
\vec{r}_2 &= \vec{r} - \vec{r}_1 \\
&= (0 - 5.01)\ \hat{i} + (5.6 - 5.98\,)\hat{j} \\
&= -5.01\,\hat{i} - 0.38)\,\hat{j}
\end{aligned}$$

We now compute magnitude and direction

$$\left|\vec{r}_2\right| = \sqrt{(-5.01)^2 + (-0.38)^2} = 5.024\,km$$

$$\tan\theta = \frac{-0.38}{-5.01} \Rightarrow \theta = 4.33\ \textit{South of West}$$

3.30 What is the sum of the following four vectors in (a) unit-vector notation and as (b)magnitude and (c) angle.

$$\vec{A} = 2.00\hat{i} + 3.00\,\hat{j} \qquad \vec{B}\!: 4.00\ \ at 65°$$

$$\vec{C} = -4.00\,\hat{i} - 6.00\,\hat{j} \qquad \vec{D}\!: 5.00 at - 235°$$

$$\begin{aligned}
&\vec{B} = 4\cos 65\,\hat{i} + 4\sin 65\,\hat{j} = 1.69\,i + 3.63\,j \\
&\vec{D} = 5\cos(-235)\,\hat{i} + 5\sin(-235)\,\hat{j} = -2.87\,\hat{i} + 4.096\,\hat{j} \\
&\vec{A} + \vec{B} + \vec{C} + \vec{D} = -3.18\hat{i} + 4.73\,\hat{j} \\
&\vec{A} + \vec{B} + \vec{C} + \vec{D}\!: \sqrt{3.18^2 + 4.73^2} = 5.7\,m \\
&\theta = \tan^{-1}\!\left(\frac{4.73}{-3.18}\right) = 123.9°
\end{aligned}$$

3.34

3.39 Use the definition of the scalar product $\vec{a}\cdot\vec{b} = ab\cos\theta$, and the fact that $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$ to calculate the angle between the two vectors given by $\vec{a} = 3.0\hat{i} + 3.0\,\hat{j} + 3.0\hat{k}$ and $\vec{b} = 2.0\hat{i} + 1.0\,\hat{j} + 3.0\hat{k}$

$$
\begin{aligned}
\vec{a}\cdot\vec{b} &= ab\cos\theta \\
\vec{a}\cdot\vec{b} &= a_x b_x + a_y b_y + a_z b_z \\
ab\cos\theta &= a_x b_x + a_y b_y + a_z b_z \\
\cos\theta &= \frac{a_x b_x + a_y b_y + a_z b_z}{ab} \\
&= \frac{3\cdot 2 + 3\cdot 1 + 3\cdot 3}{\sqrt{3^2+3^2+3^2}\sqrt{2^2+1^2+3^2}} \\
&= \frac{18}{\sqrt{27}\sqrt{14}} \\
&= 0.92582 \\
\theta &= \cos^{-1}(0.92582) \\
&= 22.2^\circ
\end{aligned}
$$

3.35 Three vectors are given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k}$,$\vec{b} = -1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$ and $\vec{c} = 2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k}$. Find (a) $\vec{a}\cdot(\vec{b}\times\vec{c})$ (b) $\vec{a}\cdot(\vec{b}+\vec{c})$ and (c) $\vec{a}\times(\vec{b}+\vec{c})$.

In each case, we need to compute the value inside the parentheses first. We begin with (a). We compute the cross product first.

$$
\begin{aligned}
\vec{b}\times\vec{c} &= \det\begin{Bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1.0 & -4.0 & 2.0 \\ 2.0 & 2.0 & 1.0 \end{Bmatrix} \\
&= \hat{i}(-4.0\cdot 1.0 - 2.0\cdot 2.0) - \hat{j}(-1.0\cdot 1.0 - 2.0\cdot 2.0) + \hat{k}(-1.0\cdot 2.0 - (-4.0)\cdot 2.0) \\
&= -8.0\hat{i} + 5.0\hat{j} + 6.0\hat{k}
\end{aligned}
$$

Now that we have the cross product, we can compute the dot product.

$$
\begin{aligned}
\vec{a}\cdot(\vec{b}\times\vec{c}) &= (3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k})\cdot(-8.0\hat{i} + 5.0\hat{j} + 6.0\hat{k}) \\
&= (3.0\cdot -8.0) + (3.0\cdot 5.0) + (-2.0\cdot 6.0) \\
&= -21.0
\end{aligned}
$$

Now we move to (b). Since it does not involve a cross product, its easier.

$$\begin{aligned}\vec{b} &= -1.0\,\hat{i} - 4.0\,\hat{j} + 2.0\,\hat{k}\\ \vec{c} &= 2.0\hat{i} + 2.0\,\hat{j} + 1.0\,\hat{k}\\ \vec{b}+\vec{c} &= (-1.0+2.0)\,\hat{i} + (-4.0+2.0)\,\hat{j} + (2.0+1.0)\,\hat{k}\\ &= 1.0\hat{i} + (-2.0)\,\hat{j} + 3.0\hat{k}\\ \vec{a}\cdot(\vec{b}+\vec{c}) &= (3.0\,\hat{i} + 3.0\,\hat{j} - 2.0\hat{k})\cdot(1.0\hat{i} + (-2.0)\,\hat{j} + 3.0\,\hat{k})\\ &= (3.0\cdot 1.0) + (3.0\cdot -2.0) + (-2.0\cdot 3.0)\\ &= -9.0\end{aligned}$$

We finally consider (c). We can use the result from (b) for the sum of the vectors.

$$\vec{b}+\vec{c} = 1.0\hat{i} + (-2.0)\,\hat{j} + 3.0\hat{k}$$

$$\vec{a}\times(\vec{b}+\vec{c}) = \det\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3.0 & 3.0 & -2.0\\ 1.0 & -2.0 & 3.0\end{bmatrix}$$

$$= \hat{i}(3.0\cdot 3.0 - (-2.0)\cdot(-2.0)) - \hat{j}(3.0\cdot 3.0 - (-2.0)\cdot(1.0)) + \hat{k}(3.0\cdot(-2.0) - 3.0\cdot 1.0)$$

$$= 5.0\hat{i} - 11.0\hat{j} - 9.0\,\hat{k}$$

3.40 For the following three vectors, what is $3\vec{C}\cdot(2\vec{A}\times\vec{B})$

$$\begin{aligned}\vec{A} &= 2.00\hat{i} + 3.00\,\hat{j} - 4.00\hat{k}\\ \vec{B} &= -3.00\hat{i} + 4.00\,\hat{j} + 2.00\hat{k}\\ \vec{C} &= 7.00\hat{i} - 8.00\,\hat{j}\end{aligned}$$

$$2\vec{A} = 4.00\,\hat{i} + 6.00\,\hat{j} - 8.00\,\hat{k}$$

$$3\vec{C} = 21.00\,\hat{i} - 24.00\,\hat{j}$$

$$2\vec{A}\times\vec{B} = \det\begin{Bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & -8 \\ -3 & 4 & 2 \end{Bmatrix}$$

$$= \hat{i}\,(6\cdot 2 - (-8)\cdot 4) - \hat{j}\,(4\cdot 2 - (-8)\cdot(-3)) + \hat{k}\,(4\cdot 4 - 6\cdot(-3))$$

$$= 44\,\hat{i} + 16\,\hat{j} + 34\,\hat{k}$$

$$3\vec{C}\cdot(2\vec{A}\times\vec{B}) = (21)\cdot 44 + (-24)\cdot 16 = 54$$