Chapter 2 Problems

2.3 During a hard sneeze, your eyes might shut for 0.5s. If you are driving a car at 90km/h during such a sneeze, how far does the car move during that time

$$s = \frac{90km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = 25m/s$$
$$d = s \cdot t = 25m/s \cdot 0.5s = 12.5m/s$$

2.5 The position of an object moving in a straight line is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t in seconds (a). What is the position of the object at t=1,2,3, and 4s? (b) What is the object's displacement between t = 0 and t = 4s. (c) What is the average velocity for the time interval from t=2 s to t = 4s? (d) Graph x vs t for $0 \le t \le 4s$ and indicate how the answer for c can be found from the graph.

(a-d) We plug in to calculate positions.

$$x(1) = 0m$$
$$x(2) = -2.0m$$
$$x(3) = 0m$$
$$x(4) = 12m$$

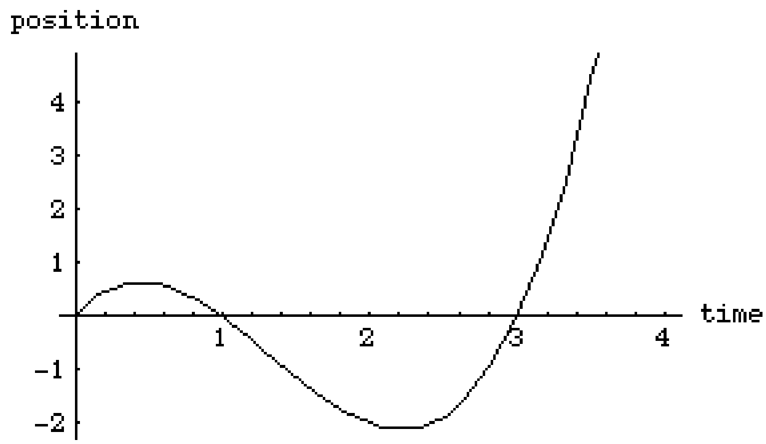
e) We can calculate the displacement from the positions.

$$\begin{aligned}\Delta x &= x(4) - x(0)\\ &= 12m - 0m\\ &= 12m\end{aligned}$$

(f) We calculate the average velocity using the displacements and time interval.

$$v = \frac{x(4) - x(2)}{4s - 2s} = \frac{12m - (-2m)}{2s} = 7m/s$$

(g) A graph of x vs t.. The average velocity can be computed by connecting x(4) and x(2) with a straight line and computing the slope. Note: Graph done with Mathematica.



2.9 You are to drive to an interview in another town at a distance of 300 km on an expressway. The interview is at 11:15 AM. You plan to drive 100 km/h, so you eave at 8:00 AM to allow some extra time. You drive at that speed for the first 100 km, but then construction work forces you to slow to 40 km/h for 40 km. What would be the least speed needed for the rest of the trip to arrive in time for the interview?

We need to compute how much time has passed and how far you have gone. If you drive at 100 km/h for 100 km, you spend 1 hour driving that segment. The next segment is 40 km at 40 km/h. This takes 1 hour as well. So you have driven 2 hours and covered 140 km. You allowed 3.25 hours for the entire 300km trip and you now have 1.25 hours left to cover the remaining 160km. This means that you need to drive at

$$v = \frac{160km}{1.25h} = 128\,\frac{km}{h}$$

to complete the trip on time.

2.11 Two trains, each having a speed of 30km/h, are headed at each other on the same straight track. A bird that can fly 60km/h flies off the front of one train when they are 60 km apart and heads directly for the other train. On reaching the other train, the bird flies directly bact to the first train, and so forth. (We have no idea why a bird would behave in this way.) What is the total distance the bird travels before the trains collide?

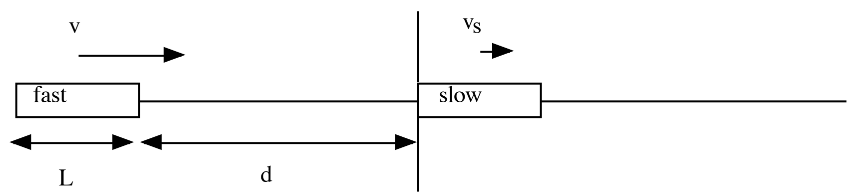
The easiest way to think about and do this problem is to note that the bird flies at a constant speed for the entire time that the trains travel until colliding. The distance the bird travels is thus

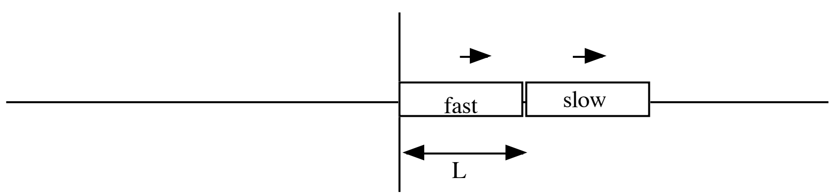
$$d = v\,t = 60\frac{km}{h}\,t$$

We need to compute the time that the trains run before colliding. By symmetry, we can argue that the trains will collide in the middle, at 30 km. The time to collision is easy to compute--it's just the time for either time to travel 30 km, which is 1 hour.   Now that we know how long the trains will run, we can see that the distance the bird will travel is 60 km.

2.12 Traffic Shock wave. An abrupt slowdown in concentrated traffic can travel as a pulse, termed a shock wave, along the line of cars, either downstream (in the traffic direction) or upstream, or it can be stationary. Figure 2-23 shows a uniformly spaced line of cars moving at speed $v = 25 m/s$ toward a uniformly spaced line of slow cars, moving at speed $v_s = 5 m/s$. Assume that each faster car adds length $L = 12m$ (car length plus buffer zone) to the line of cars when it joins the line, and assume it slows abruptly at the last instant. (a) For what separation distance d between the faster cars does the shock wave remain stationary? If the separation is twice that amount, what are the (b) speed and (c) direction (upstream or downstream) of the shock wave.

To do this problem, its useful to consider the last slow car and the first fast car. We'll count the rear of the car as the place where we measure its position. If the pulse is to remain stationary, the picture looks like





In this treatment, we'll consider the position of the back of each car. In a time t, the slow car goes a distance L and the fast car goes a distance d+L. We can write constant acceleration equations for both cars. We then solve the equation for the slow car for t and then use that time in the equation

Fast Car

$x_{if} = -L - d$

$x_{ff} = 0$

$v = 25 m/s$

$x_{ff} = x_{if} + vt$

$0 = -L - d + vt$

Slow Car

$x_{is} = 0$

$x_{fs} = L$

$v_s = 5 m/s$

$x_{fs} = x_{is} + v_f t$

$L = 0 + v_s t$

$$L = 0 + v_s t$$

$$t = \frac{L}{v_s}$$

$$0 = -L - d + vt$$

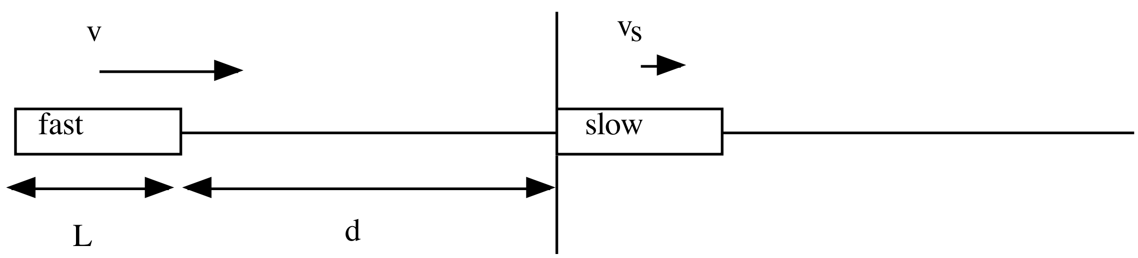
$$0 = -L - d + v \cdot \frac{L}{v_s}$$

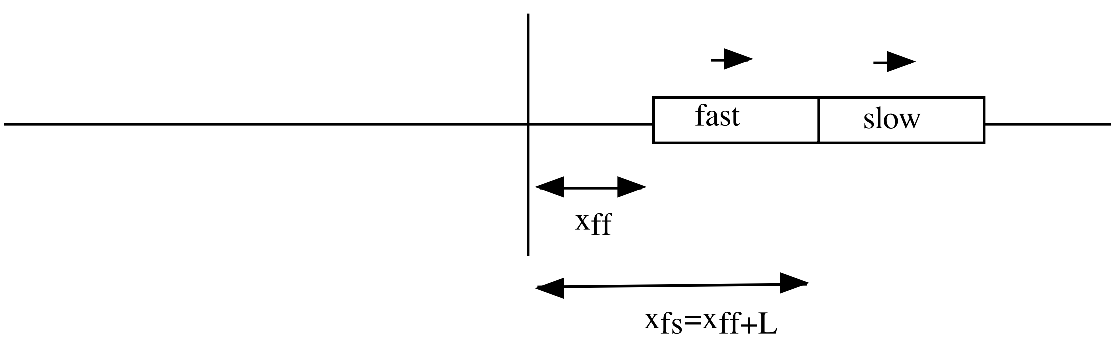
$$d = L(\frac{v}{v_s} - 1)$$

$$= L(\frac{v - v_s}{v_s}) = 12m \cdot (\frac{25m/s - 5m/s}{5m/s})$$

$$d = 48m$$

We can find the movement of the wave by finding the position of the car when it reaches the wave. We do this by writing the position of each car, recognizing that the slow car is a distance L in front of the fast car when the fast car joins the line. We use this condition to find the time when the cars meet. Using the final position of the fast car and the time, we can find the velocity of the pulse.





$$x_{ff} = -L - d + v\,t$$
$$x_{ff} + L = 0 + v_s t$$
$$t = \frac{x_{ff} + L}{v_s}$$

Fast Car	*Slow Car*
$x_{if} = -L - d$	$x_{is} = 0$
$x_{ff} = ?$	$x_{fs} = x_{ff} + L$
$v = 25m/s$	$v_s = 5m/s$
$x_{ff} = x_{if} + vt$	$x_{fs} = x_{is} + v_f t$
$x_{ff} = -L - d + vt$	$x_{ff} + L = 0 + v_s t$

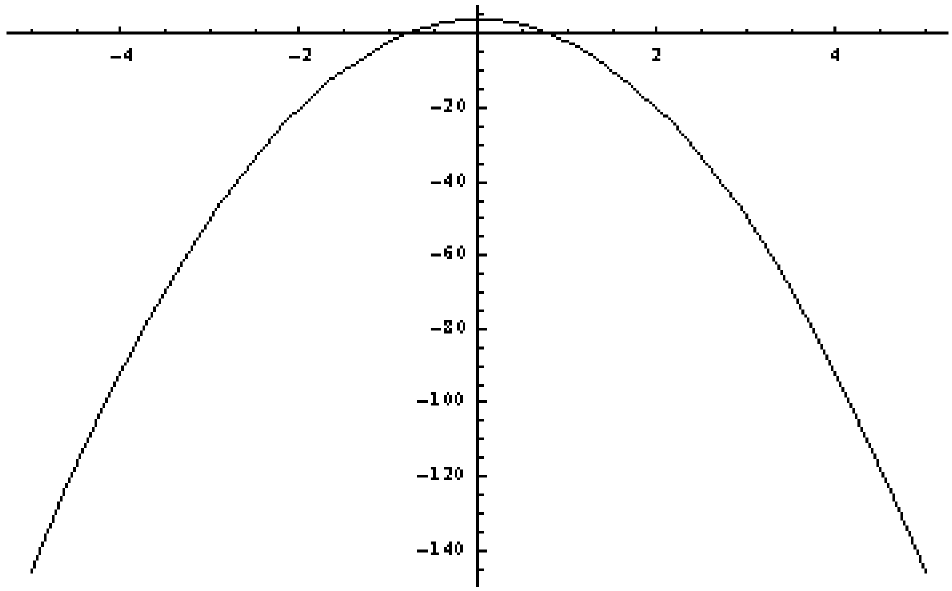
$$x_{ff} = -L - d + v \cdot \frac{x_{ff} + L}{v_s}$$
$$x_{ff}(1 - \frac{v}{v_s}) = L(\frac{v}{v_s} - 1) - d$$
$$-x_{ff}(\frac{v - v_s}{v_s}) = L(\frac{v - v_s}{v_s}) - d$$
$$x_{ff} = (\frac{v_s}{v - v_s})d - L$$
$$= (\frac{5m/s}{25m/s - 5m/s}) \cdot 96m - 12m$$
$$= 12m$$

$$t = \frac{x_{ff} + L}{v_s} = \frac{12m + 12m}{5m/s} = 4.8s$$

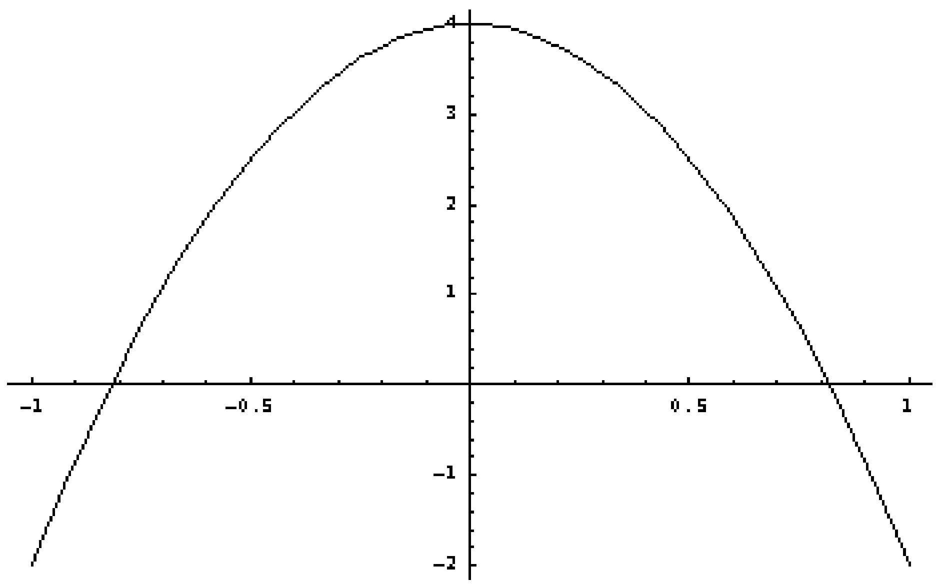
$$v_{pulse} = \frac{x_{ff}}{t} = \frac{12m/s}{4.8} = 2.5m/s$$

2.14 The position function $x(t)$ of a particle moving along an x axis is $x(t) = 4.0 - 6.0t^2$ where $x(t)$ is in m and t in seconds. (a) At what time and (b) where does the particle (momentarily) stop? At what (c) negative time and (d) positive time does the particle pass through the origin? (e) Graph $x(t)$ vs. t for the range -5s to +5s. (f) To shift the curve rightward on the graph should we include the term $+20t$ or the term $-20t$ in the $x(t)$? (g) Does that inclusion increase or decrease the value of x at which the particle momentarily stops.

Let's begin by plotting the position as a function of time between -5 and 5



Here is a close up--plotted -1 to +1.



We can see from the graph that the particle stops at $x = 4$, $t = 0$. We can get this from our equation by setting the velocity to zero and solving for the time.

$$x(t) = 4.0 - 6.0t^2$$

$$v = \frac{dx}{dt} = -12t$$

$$0 = -12t$$

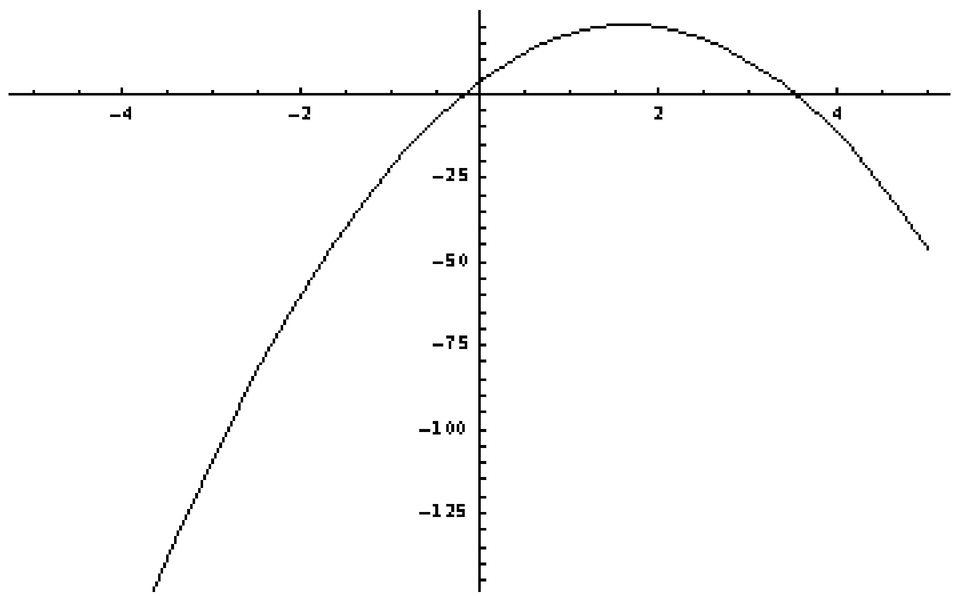
$$t = 0$$

$$x(0) = 4.0$$

We can find when the particle passes through zero $(x = 0)$,

$$x(t) = 4.0 - 6.0t^2$$
$$0 = 4.0 - 6.0t^2$$
$$t = \pm\sqrt{\frac{2}{3}}\ s = \pm 0.8165 s$$

To shift the curve to the right, we need to include $+20t$. If we plot with this term included we can see the shift we want.



Where and when does the stop occur? We can calculate this

$$x(t) = 4.0 - 6.0t^2 + 20t$$
$$v = \frac{dx}{dt} = -12t + 20$$
$$0 = -12t + 20$$
$$t = \frac{20}{12}s = \frac{4}{3}s$$
$$x(\frac{4}{3}) = 20m$$

The graph is misleading (it mislead me!) in that the curve looks the same and *Mathematica* has rescaled the vertical axis--so it looks as if the x position is the same as before . If you look carefully at the new scale, you can see that the stopping point is at 20m.

2.15 (a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where t is in seconds and x is in meters), what is its velocity at t=1 s? (b) Is it moving toward increasing or decreasing x just then? (c) What is its speed just then? (d) Is the speed larger or smaller at later times? (Try answering the next two question without further calculation.) (e) Is there ever an instant when the velocity is zero? (f) Is there a time after t=3s when the particle is moving toward decreasing x?

To proceed, we begin by taking the derivative to find v.

$$x = 4 - 12t + 3t^2$$
$$v = \frac{dx}{dt} = 6t - 12$$

a. At 1s, $v(1) = -6m/s$

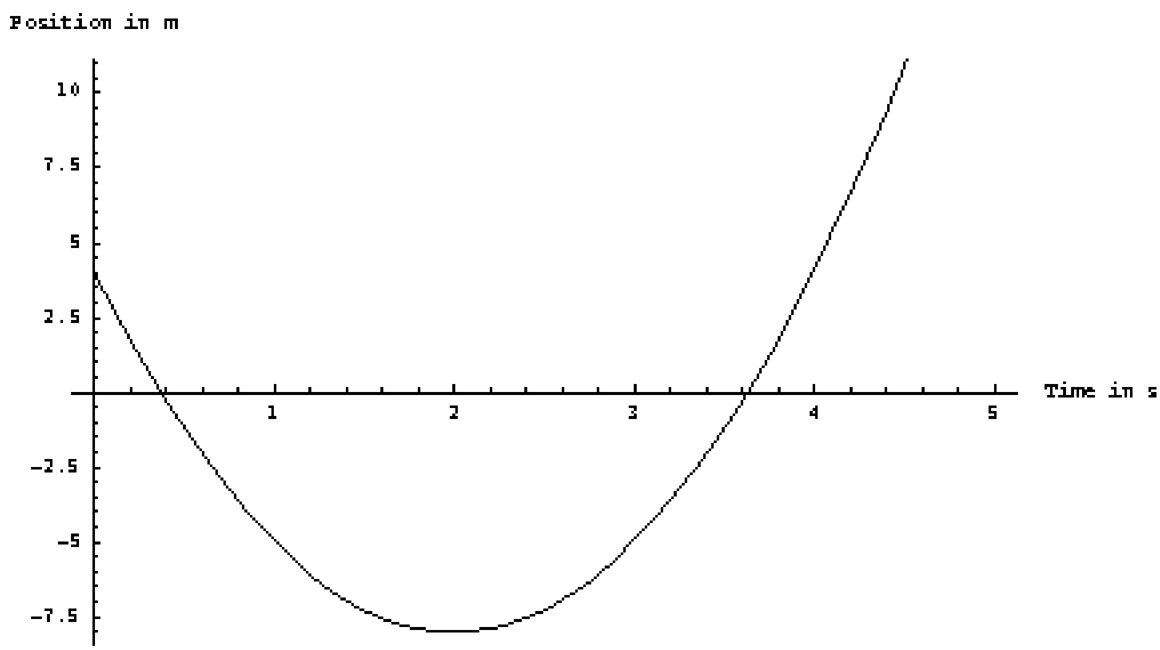
b. Direction is negative--toward the left and more negative since $x(1) = -5m$.

c. Speed is $6m/s$

d. Speed gets smaller (zero at t=2) and then larger.

e. Speed is zero at t=2s.

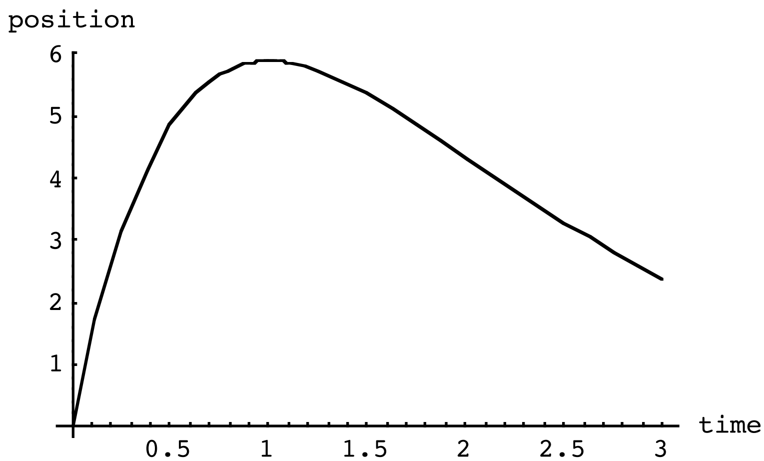
f. After t=3s, the velocity is always positive. See plot of position below.



2.16 An electron moving along the x axis has a position given by $x = 16t\,e^{-t}m$ **w**here t is in seconds. How far is the electron from the origin when it momentarily stops?

We need to find out when the electron stops. If we know when it stops, we can find out where it is. To find out when it stops, we find the instantaneous

$$v = \frac{dx}{dt} = \frac{d}{dt}(16\,t\,e^{-t}) = 16\;e^{-t} - 16t\,e^{-t}$$
$$= 16(1-t)e^{-t} \qquad\qquad x(1) = 16 \cdot 1 \cdot e^{-1} m = 5.886m$$
$$0 = 16(1-t)e^{-t} \;\Rightarrow t = 1s$$



2.18 (a) If the position of a particle is given by $x(t) = 20t - 5t^3$ where $x(t)$ is in m and t is in seconds, when if ever is the particle's velocity zero? (b) When is its acceleration a zero? (c) For what time range (positive or negative) is a negative? (c) Positive (e) Graph $x(t)$, $v(t)$ and $a(t)$.

First we write expressions for $x(t)$, $v(t)$ and $a(t)$.

$$x(t) = 20t - 5t^3$$
$$v(t) = \frac{dx}{dt} = 20 - 15t^2$$
$$a(t) = \frac{dv}{dt} = -30\,t$$

(a) We can solve for when the velocity is zero:

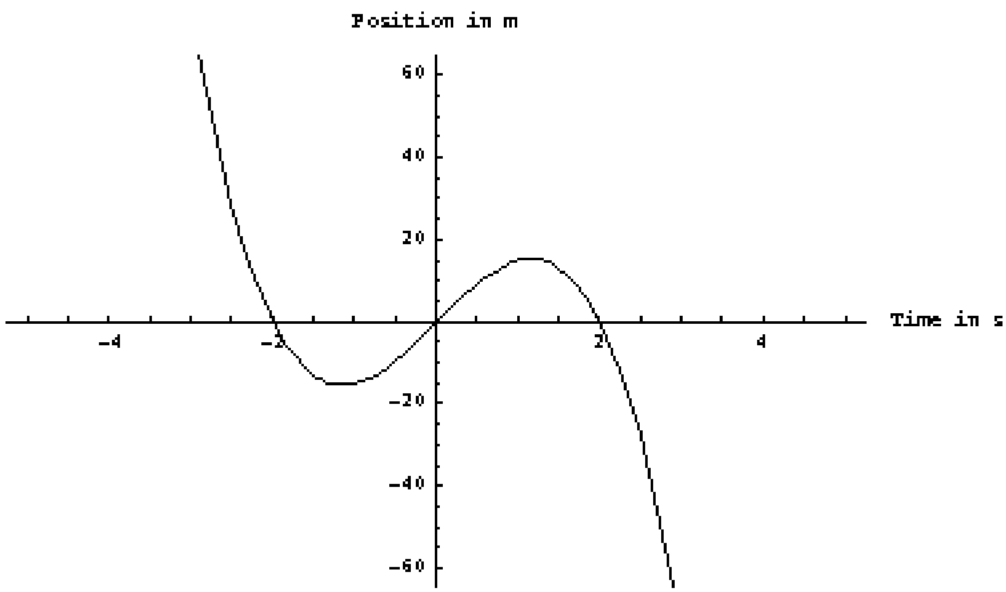
$$v(t) = 20 - 15t^2$$
$$0 = 20 - 15t^2$$
$$t = \pm\sqrt{\frac{20}{15}}\,s$$
$$= \pm 1.155s$$

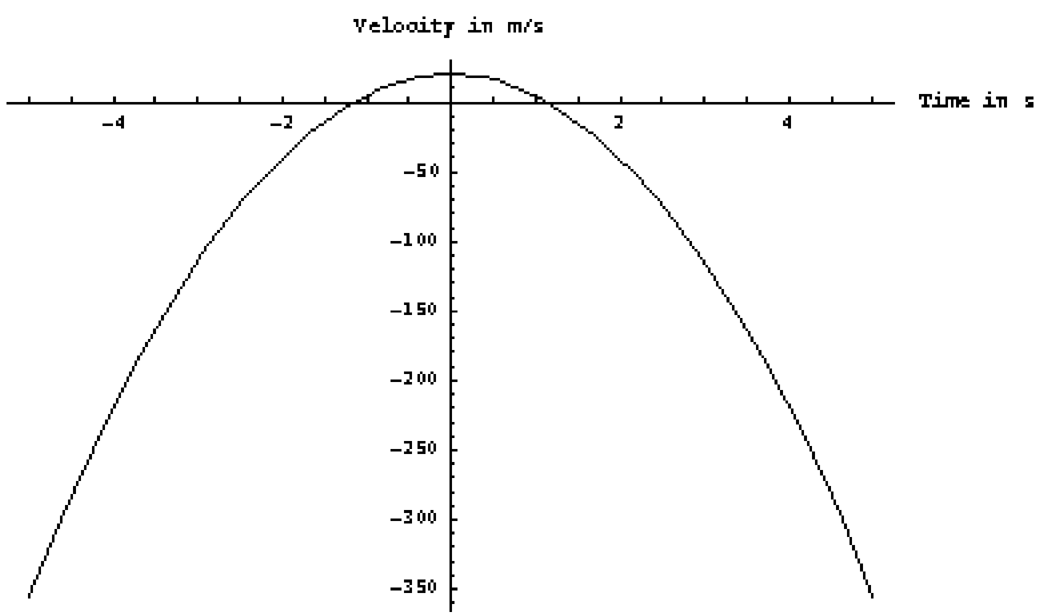
(b) The acceleration is zero at $t = 0s$

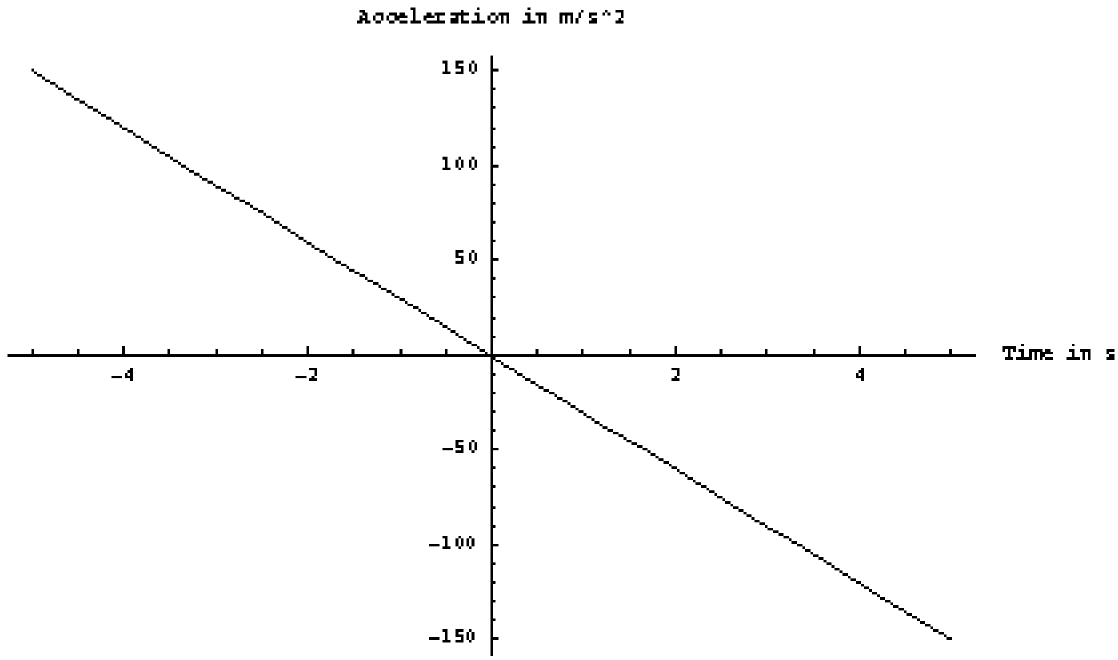
(c) Acceleration is positive when $t < 0s$

(d) Acceleration is negative when $t > 0s$

Graphs







2.23 An electron has a constant acceleration of $+3.2m/s^2$. At a certain instant its velocity is +9.6m/s. What is its velocity 2.5 s earlier and (b) 2.5s later?

Take the time to be zero when the velocity is 9.6 m/s.

$$a = 3.2m/s^2$$
$$v_0 = 9.6m/s$$
$$v = v_0 + at$$

a. At t=-2.5s, $v = 1.6m/s$
b. At t=+2.5s, $v = 17.6m/s$.

2.25 Suppose a rocket ship in deep space moves with a constant acceleration equal to $9.8m/s^2$, which gives the illusion of normal gravity during the flight. (a) If it starts from rest, how long will it take to acquire a speed one-tenth that of the speed of light, which travels at $3.0\times10^8 m/s$. (b) How fare will it travel in so doing.

(a) How long to reach one-tenth the speed of light?

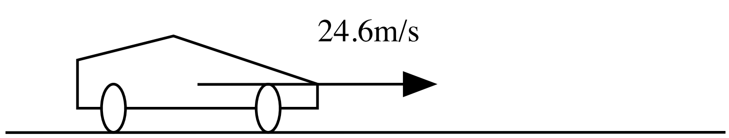
$$x_i = 0$$
$$x_f = ?$$
$$v_i = 0m/s$$
$$v_f = 3.0\times 10^7 m/s$$
$$a = 9.8\, m/s^2$$
$$t = ?$$

$$v_f = v_i + at$$
$$v_f = 0 + at$$
$$t = -\frac{v_f}{a} = -\frac{3.0\times 10^7 m/s}{9.8\, m/s^2} = 3.06\times 10^6 s$$

(b) How far does it go in that time?

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$
$$x_f = 0 + 0 + \frac{1}{2}\cdot(9.8m/s^2)\cdot(3.06\times 10s)^2$$
$$= 4.6\times 10^{13} m$$

2.26 On a dry road, a car with good tires may be able to brake with a constant deceleration of $4.92\ m/s^2$ (a) How long does such a car, initially traveling at 24.6 m/s take to stop? (b) How far does it travel in this time? Graph x verses t and v versus t for the deceleration.



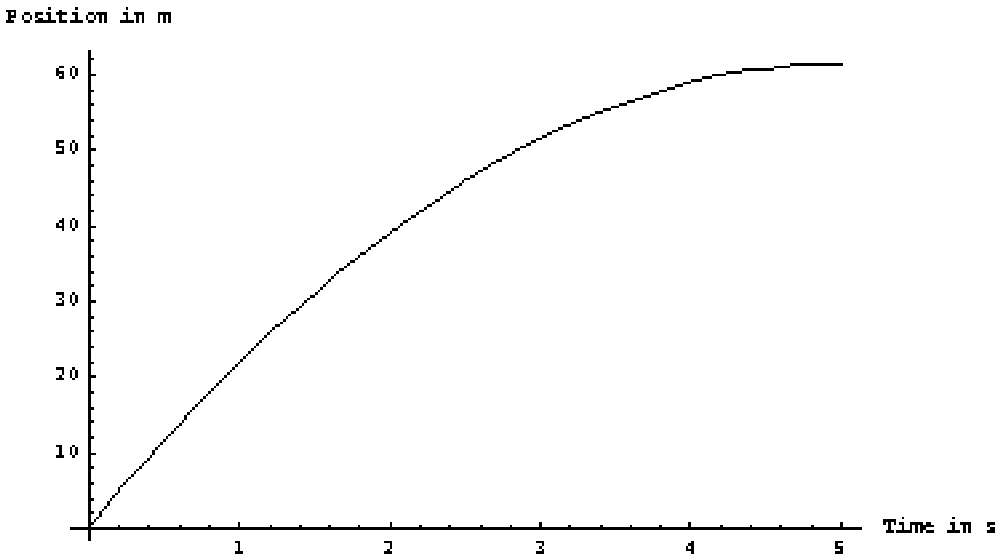
This is a classic constant acceleration problem. We begin by writing out what we know and then solving for time.

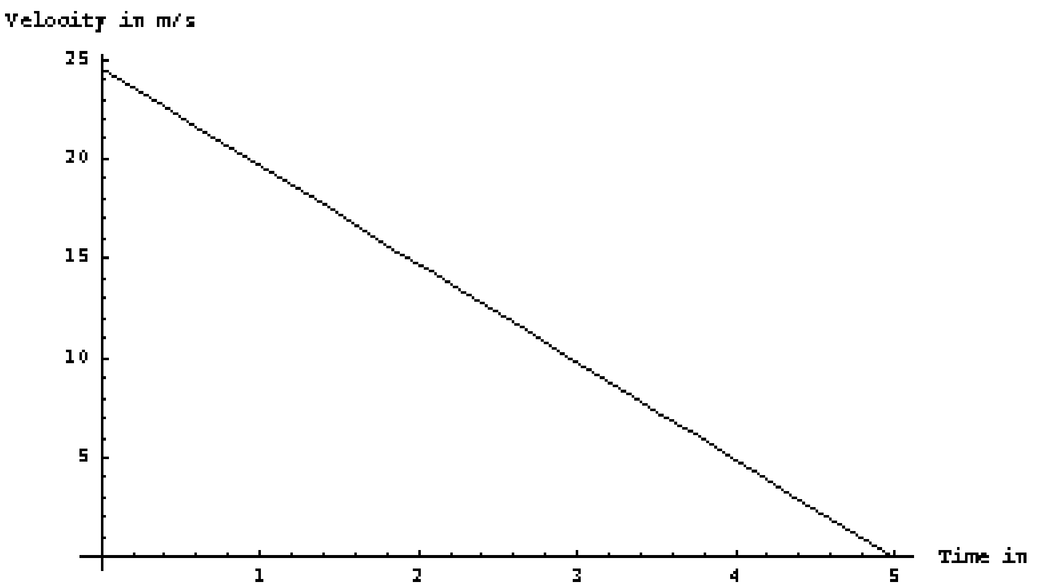
$$x_i = 0$$
$$x_f = ?$$
$$v_i = 24.6m/s$$
$$v_f = 0$$
$$a = -4.92m/s^2$$
$$t = ?$$

$$v_f = v_i + at$$
$$0 = v_i + at$$
$$t = -\frac{v_i}{a} = -\frac{24.6\, m/s}{-4.92\, m/s^2} = 5s$$

b. Now we can find the stopping point

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$
$$x_f = 0 + (24.6\, m/s)\cdot 5s + \frac{1}{2}\cdot(-4.92m/s^2)\cdot(5s)^2$$
$$= 61.5\, m$$





2.30 A world's land speed record was set by Colonel John P. Stapp when in March 1954 he rode a rocket-propelled sled that moved along a track at 1020 km/h. He and the sled were brought to a stop in 1.4 s. In terms of g, what acceleration did he experience while stopping?

$$v_i = \frac{1020\,km}{hr} \cdot \frac{1000\,m}{1\,km} \cdot \frac{1\,hr}{3600\,s} = 283.3\,m/s$$

$$v_f = 0$$

$$t = 1.4\,s$$

$$v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t} = \frac{0 - 283.3\,m/s}{1.4\,s} = -202.381\,m/s^2$$

$$\#\,g's = \frac{202.381\,m/s^2}{9.8\,m/s^2} = 20.65$$

2.44 Raindrops fall to Earth from a cloud 1700m above the Earth's surface. If they were not slowed by air resistance, how fast would the drops be moving when they struck the ground? Would it be safe to walk outside during a rainstorm?

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$

$$v_f^2 = 0^2 - 2 \cdot 9.8\,m/s \cdot (-1700\,m)$$

$$v_f = 182.5\,m/s!$$

It's not safe.

2.47 (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50m. (b) How long will it be in the air. Sketch y, v, a, vs t.

$$y_i = 0\,m$$

$$y_f = 50\,m$$

$$v_i = ?$$

$$v_f = 0\,m/s$$

$$t = ?$$

$$a = -g$$

(a). We compute the initial velocity first.

$$v_f^2 = v_i^2 + 2a\,(y_f - y_i)$$

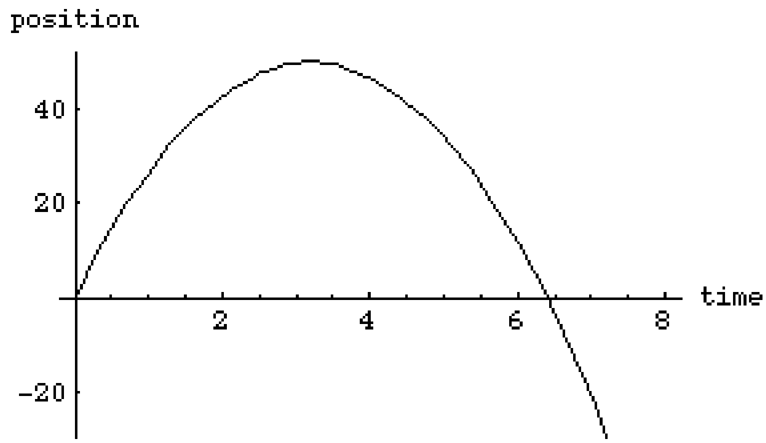
$$0 = v_i^2 - 2g\,(y_f - 0)$$

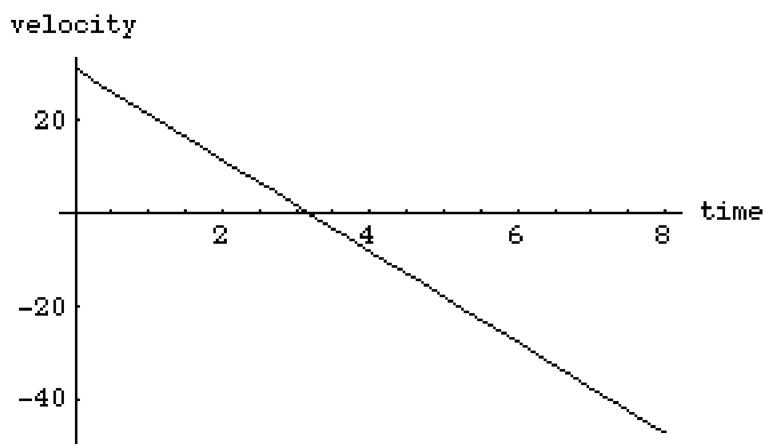
$$v_i = \sqrt{2g\,(50m)} = 31.3\,m/s$$

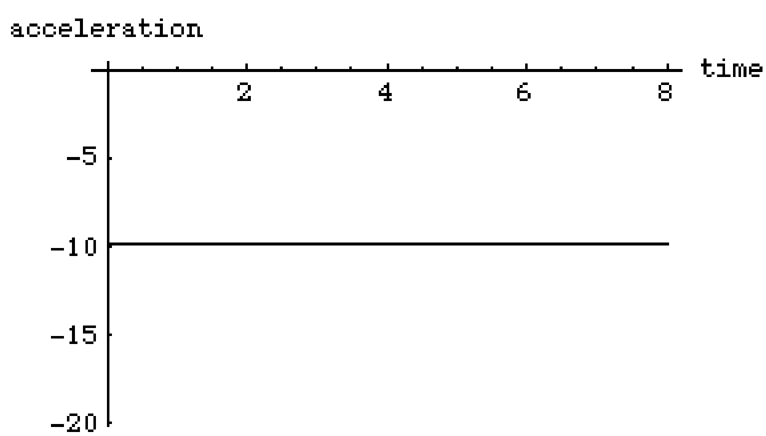
(b) Now that we know initial velocity, we can find the time to reach the highest point.

$$v_f = v_i + at$$
$$0 = v_i - gt$$
$$t = \frac{v_i}{g} = \frac{31.3m/s}{9.8m/s^2} = 3.19s$$

The path is symmetric, so the entire time of flight is 6.38s.







2.49 A hot-air balloon is ascending at the rate of 12m/s and is 80m above the ground when a package is dropped over the side. (a) How long does the package take to reach the ground? With what speed does it hit the ground?

$$y_i = 80m$$
$$y_f = 0$$
$$v_i = +12m/s$$
$$v_f = ?$$
$$a = -g$$
$$t = ?$$

(a) We determine the time

$$y_f = y_i + v_i t + \frac{1}{2} a t^2$$
$$0 = y_i + v_i t - \frac{1}{2} g t^2$$
$$0 = 80 + 12t - \frac{1}{2} \cdot 9.8 t^2 = 80 + 12t - 4.9t^2$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{12^2 - 4 \cdot (-4.9) \cdot 80}}{2 \cdot (-4.9)}$$
$$= 5.44s$$

(choose the positive time).

Now that we know the time, find the speed

$$v_f = v_i + at$$
$$= v_i - gt$$
$$= 12m/s - 9.8m/s^2 \cdot 5.44s$$
$$= -41.3m/s$$
$$|v_f| = 41.3m/s$$