Chapter 2 Problems

2.2 A car travels up a hill at a constant speed of 40km/h and returns down the hill at a constant speed of 60 km/h. Calculate the average speed for the trip.

This problem is a bit more subtle than it might seem. It is tempting to simply average the two velocities that are given. The problem is that the car does not spend the same amount of time at each velocity, so this approach will give the wrong answer. We need to use the definition of average speed to get the correct answer.

$$s = \frac{total\,dist}{total\,time}$$

For this problem, we'll consider the distance from the bottom to the top of the hill to be d and calculate the time up and the time down the hill to find the total time.

$$t_{up} = \frac{d}{40km/hr}$$

$$t_{down} = \frac{d}{60km/h}$$

$$t_{total} = t_{up} + t_{down} = \frac{d}{40km/hr} + \frac{d}{60km/hr}$$

$$= \frac{60km/hr + 40km/hr}{40km/hr \cdot 60km/hr} d$$

$$s = \frac{2d}{t_{total}} = \frac{2d}{\frac{60km/hr + 40km/hr}{40km/hr \cdot 60km/hr} d} = \frac{2 \cdot 40km/hr \cdot 60km/hr}{60km/hr + 40km/hr}$$

$$= 48km/hr$$

2.3 During a hard sneeze, your eyes might shut for 0.5s. If you are driving a car at 90km/h during such a sneeze, how far does the car move during that time

$$s = \frac{90km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = 25m/s$$

$$d = s \cdot t = 25m/s \cdot 0.5s = 12.5m/s$$

2.5 The position of an object moving in a straight line is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t in seconds (a). What is the position of the object at t=1,2,3, and 4s? (b) What is the object's displacement between t = 0 and t = 4s. (c) What is the average velocity for the time interval from t=2 s to t = 4s? (d) Graph x vs t for $0 \le t \le 4s$ and indicate how the answer for c can be found from the graph.

(a-d) We plug in to calculate positions.

$$x(1) = 0m$$
$$x(2) = -2.0m$$
$$x(3) = 0m$$
$$x(4) = 12m$$

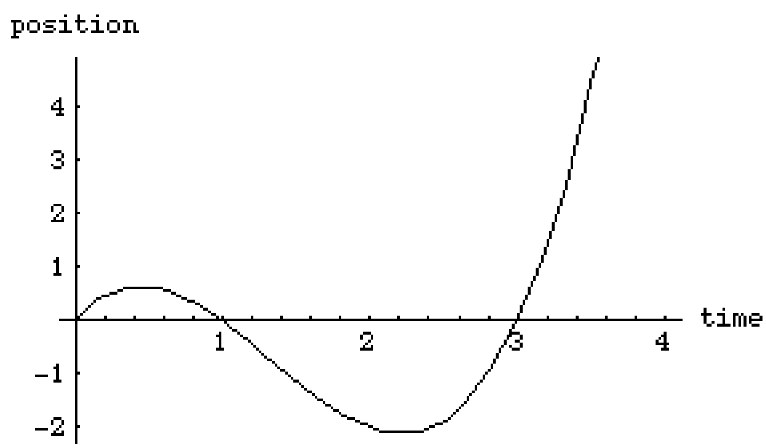
(e) We can calculate the displacement from the positions.

$$\begin{aligned}\Delta x &= x(4) - x(0) \\ &= 12m - 0m \\ &= 12m\end{aligned}$$

(f) We calculate the average velocity using the displacements and time interval.

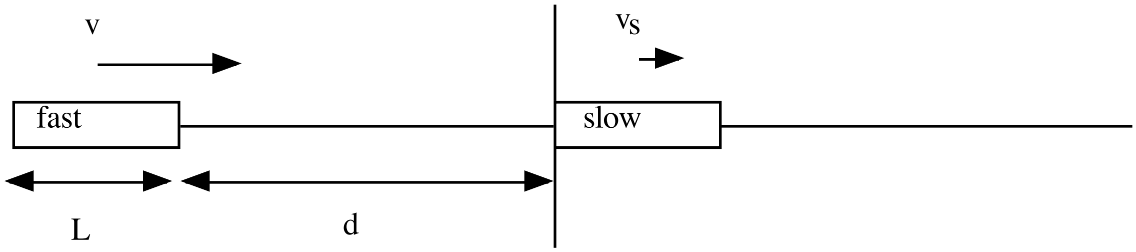
$$v = \frac{x(4) - x(2)}{4s - 2s} = \frac{12m - (-2m)}{2s} = 7m/s$$

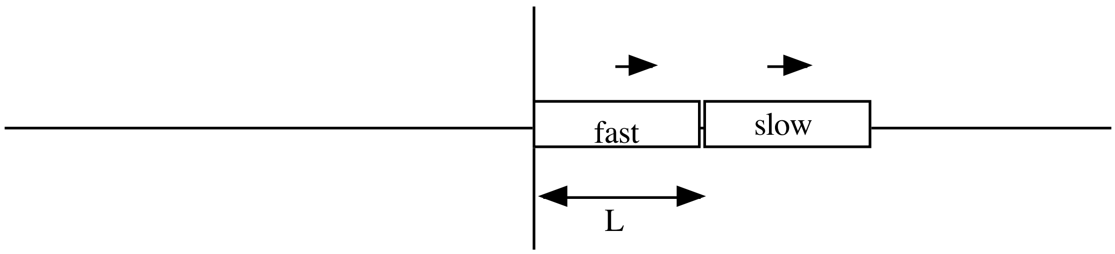
(g) A graph of x vs t.. The average velocity can be computed by connecting x(4) and x(2) with a straight line and computing the slope. Note: Graph done with Mathematica.



2.12 Traffic Shock wave. An abrupt slowdown in concentrated traffic can travel as a pulse, termed a shock wave, along athe line of cars, either downstreame (in the traffic direction) or upstream, or it can be stationary. Figure 2-23 shows a uniformly spaced line of cars moving at speed $v = 25m/s$ toward a uniformly spaced line of slow cars, moving at speed $v_s = 5m/s$. Assume that each faster car adds length $L = 12m$ (car length plus buffer zone) to the line of cars when it joins the line, and assume it slows abruptly at the last instant. (a) For what separation distance d between the faster cars does the shock wave remain stationary? If the separation is twice that amount, what are the (b) speed and (c) direction (upstream or downstream) of the shock wave.

To do this problem, its useful to consider the last slow car and the first fast car. We'll count the rear of the car as the place whre we measure its position. If the pulse is to remain stationary, the picture looks like





In a time t, the slow car goes a distance L and the fast car goes a distance $d+L$. We can write constant acceleration equations for both cars. We then solve the equation for the slow car for t and then use that time in the equation

Fast Car

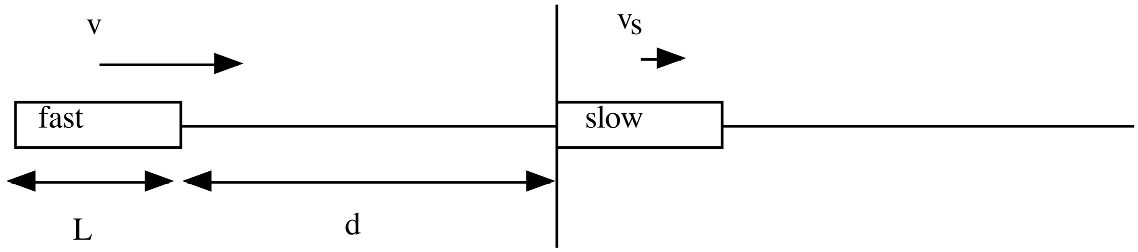
$$x_{if} = -L - d$$
$$x_{ff} = 0$$
$$v = 25m/s$$
$$x_{ff} = x_{if} + vt$$
$$0 = -L - d + vt$$

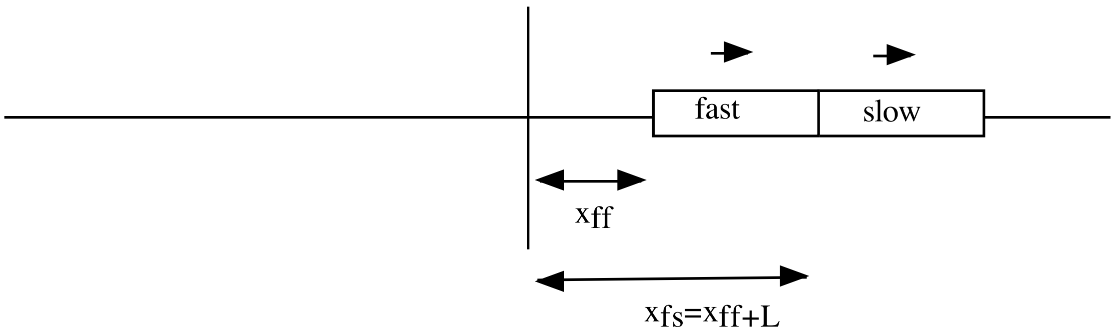
Slow Car

$$x_{is} = 0$$
$$x_{fs} = L$$
$$v_s = 5m/s$$
$$x_{fs} = x_{is} + v_f t$$
$$L = 0 + v_s t$$

$$L = 0 + v_s t$$
$$t = \frac{L}{v_s}$$
$$0 = -L - d + v\,t$$
$$0 = -L - d + v \cdot \frac{L}{v_s}$$
$$d = L(\frac{v}{v_s} - 1)$$
$$= L(\frac{v - v_s}{v_s}) = 12m \cdot (\frac{25m/s - 5m/s}{5m/s})$$
$$d = 48m$$

We can find the movement of the wave by finding the position of the car when it reaches the wave. We do this by writing the position of each car, recognizing that the slow car is a distance L in front of the fast car when the fast car joins the line. We use this condition to find the time when the cars meet. Using the final position of the fast car and the time, we can find the velocity of the pulse.





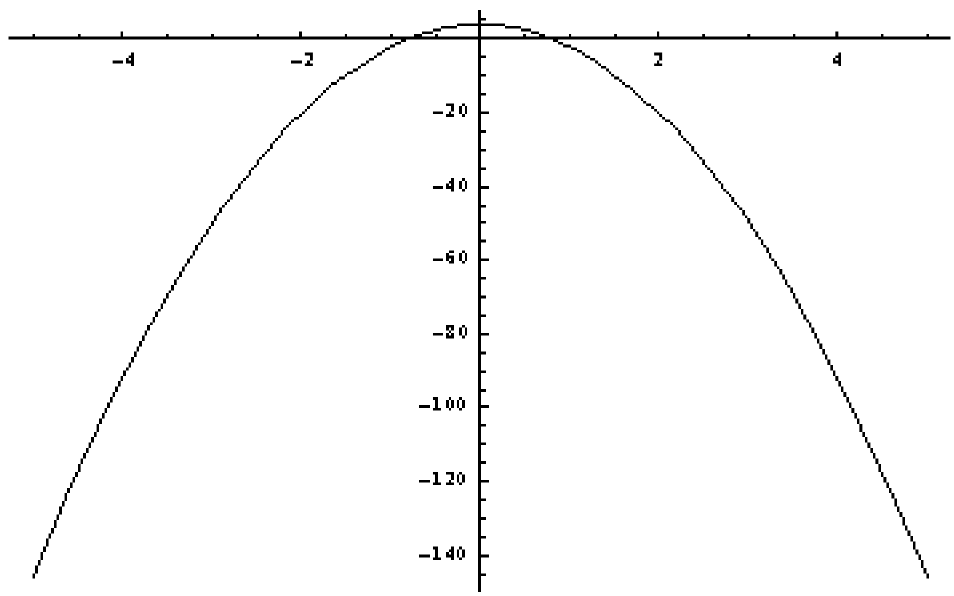
$$x_{ff} = -L - d + vt$$
$$x_{ff} + L = 0 + v_s t$$
$$t = \frac{x_{ff} + L}{v_s}$$

Fast Car	*Slow Car*
$x_{if} = -L - d$	$x_{is} = 0$
$x_{ff} = ?$	$x_{fs} = x_{ff} + L$
$v = 25m/s$	$v_s = 5m/s$
$x_{ff} = x_{if} + vt$	$x_{fs} = x_{is} + v_f t$
$x_{ff} = -L - d + vt$	$x_{ff} + L = 0 + v_s t$

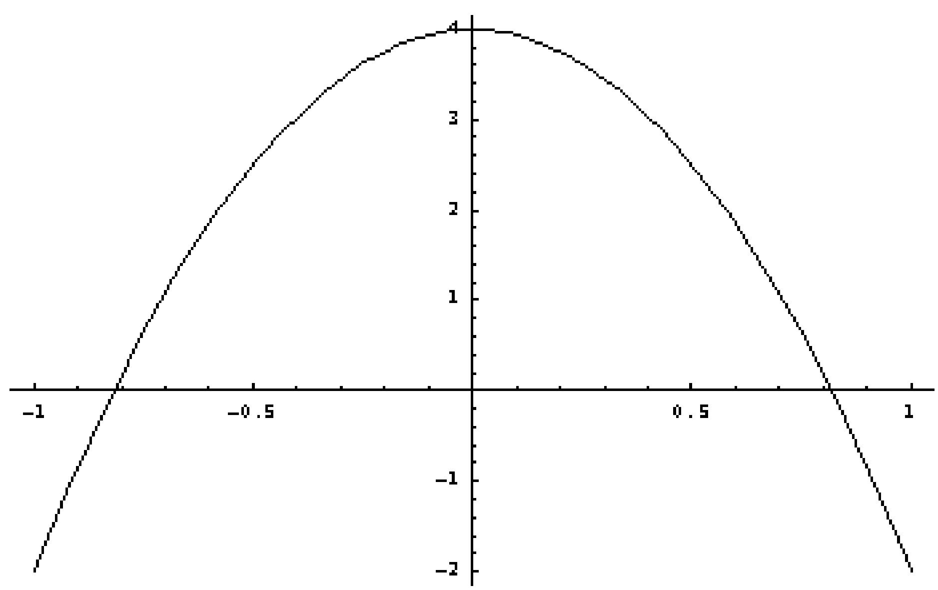
$$x_{ff} = -L - d + v \cdot \frac{x_{ff} + L}{v_s}$$
$$x_{ff}(1 - \frac{v}{v_s}) = L(\frac{v}{v_s} - 1) - d$$
$$-x_{ff}(\frac{v - v_s}{v_s}) = L(\frac{v - v_s}{v_s}) - d$$
$$x_{ff} = (\frac{v_s}{v - v_s})d - L$$
$$= (\frac{5m/s}{25m/s - 5m/s}) \cdot 96m - 12m$$
$$= 12m$$
$$t = \frac{x_{ff} + L}{v_s} = \frac{12m + 12m}{5m/s} = 4.8s$$
$$v_{pulse} = \frac{x_{ff}}{t} = \frac{12m/s}{4.8} = 2.5m/s$$

2.14 The position function $x(t)$ of a particle moving along an x axis is $x(t) = 4.0 - 6.0t^2$ where $x(t)$ is in m and t in seconds. (a) At what time and (b) where does the particle (momantarily) stop? At what (c) negative time and (d) positive time does the particle pass through the origin? (e) Graph $x(t)$ vs. t for the range -5s to +5s. (f) To shift the curve rightward on the graph should we include the term $+20t$ or the term $-20t$ in the $x(t)$? (g) Does that incluseion increase or decrease the value of x at which the particle momentarily stops.

Let's begin by plotting the position as a function of time between -5 and 5



Here is a close up--plotted -1 to +1.



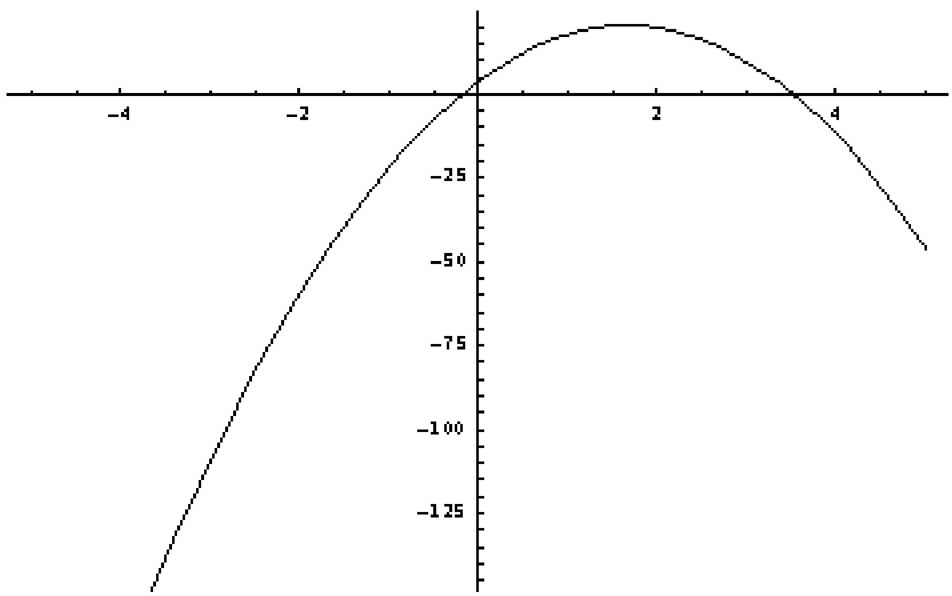
We can see from the graph that the particle stops at $x = 4$, $t = 0$. We can get this from our equation by setting the velocity to zero and solving for the time.

$$x(t) = 4.0 - 6.0t^2$$
$$v = \frac{dx}{dt} = -12t$$
$$0 = -12t$$
$$t = 0$$
$$x(0) = 4.0$$

We can find when the particle passes through zero...

$$x(t) = 4.0 - 6.0t^2$$
$$0 = 4.0 - 6.0t^2$$
$$t = \pm\sqrt{\frac{2}{3}}\ s = \pm 0.8165\ s$$

To shift the curve to the right, we need to include $+20t$. If we plot with this term included we can see the shift we want.



Where and when does the stop occur? We can calculate this

$$x(t) = 4.0 - 6.0t^2 + 20t$$
$$v = \frac{dx}{dt} = -12t + 20$$
$$0 = -12t + 20$$
$$t = \frac{20}{12}s = \frac{4}{3}s$$
$$x(\frac{4}{3}) = 20m$$

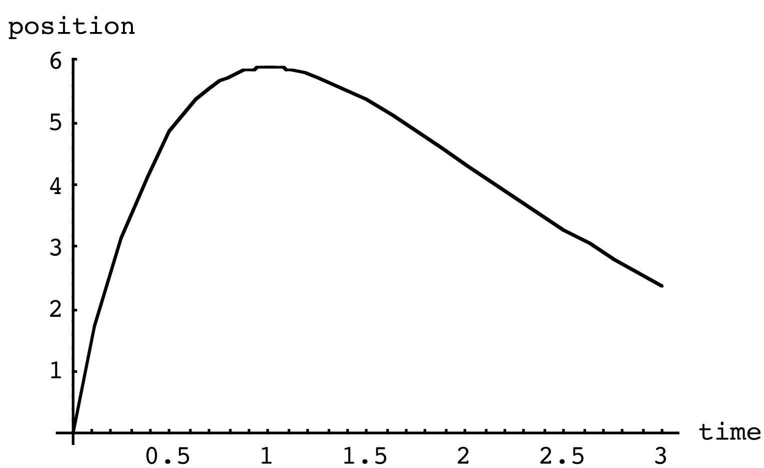
The graph is misleading (it mislead me!) in that the curve looks the same and Mathematica has rescaled the vertical axis--so it looks as if the x position is the same as before . If you look carefully at the new scale, you can see that the stopping point is at 20m.

2.16 An electron moving along the x axis has a position given by $x = 16t\,e^{-t}m$ **w**here t is in seconds. How far is the electron from the origin when it momentarily stops.

We need to find out when the electron stops. If we know when it stops, we can find out where it is. To find out when it stops, we find the instantaneous

$$v = \frac{dx}{dt} = \frac{d}{dt}(16\,t\,e^{-t}) = 16\;e^{-t} - 16t\,e^{-t}$$
$$= 16(1-t)e^{-t}$$
$$0 = 16(1-t)e^{-t} \Rightarrow t = 1s$$

$$x(1) = 16 \cdot 1 \cdot e^{-1}m = 5.886m$$



2.18 (a) If the position of a particle is given by $x(t)=20t-5t^3$ where $x(t)$ is in m and t is in seconds, when if ever is the particle's velocity zero? (b) When is its acceleration a zero? (c) For what time range (positive or negative) is a negative? (c) Positive (e) Graph $x(t)$, $v(t)$ and $a(t)$

First we write expressions for $x(t)$, $v(t)$ and $a(t)$.

$$x(t)=20t-5t^3$$
$$v(t)=\frac{dx}{dt}=20-15t^2$$
$$a(t)=\frac{dv}{dt}=-30\,t$$

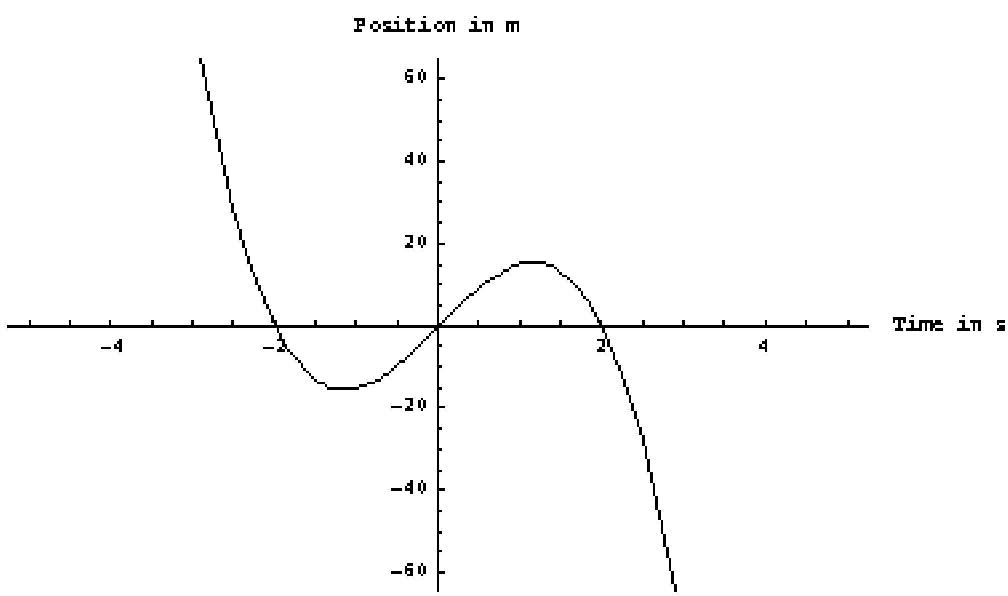
(a) We can solv for when the velocity is zero:

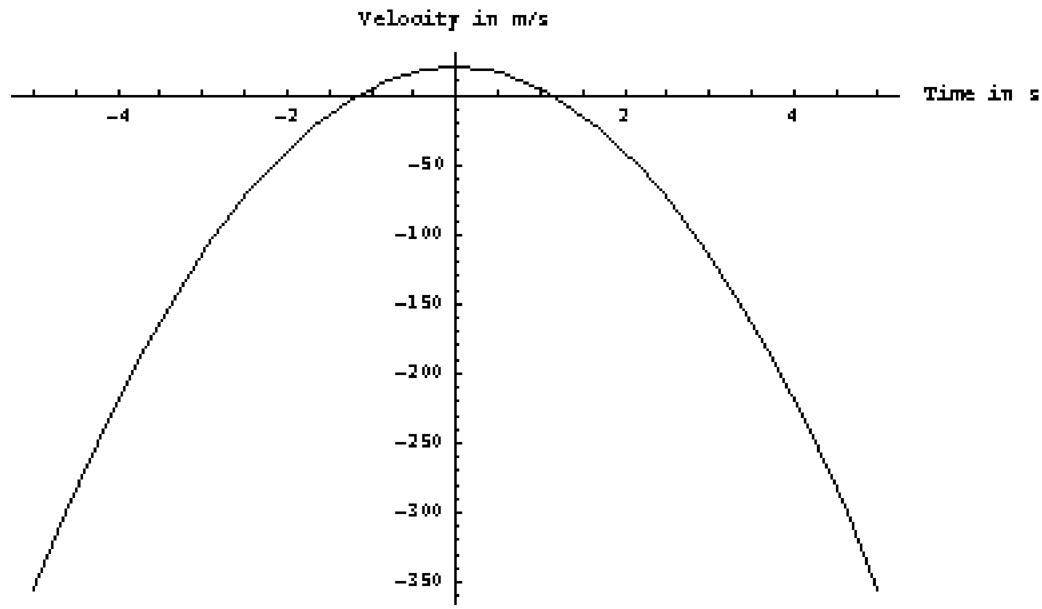
$$v(t)=20-15t^2$$
$$0=20-15t^2$$
$$t=\pm\sqrt{\frac{20}{15}}s$$
$$=\pm 1.155s$$

(b) The acceleration is zero at $t=0s$

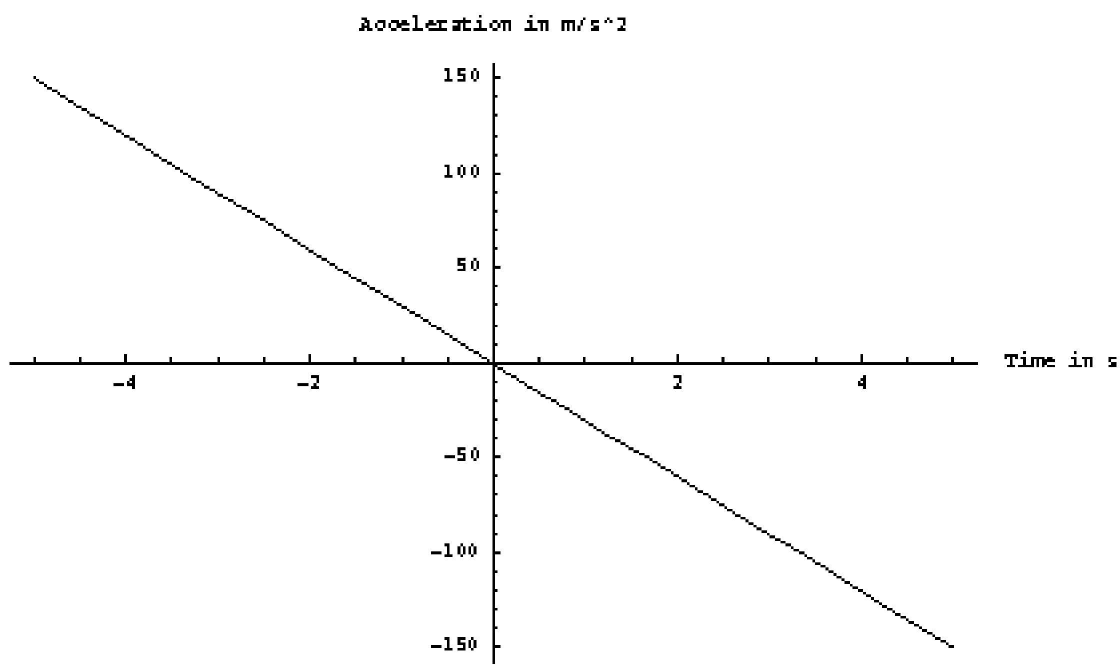
(c) Acceleration is positive when $t<0s$

(d) Acceleration is negative when $t>0s$





Velocity in m/s
Time in s
-4
-2
2
4
-50
-100
-150
-200
-250
-300
-350



Acceleration in m/s^2
150
100
50
Time in s
-4
-2
2
4
-50
-100
-150

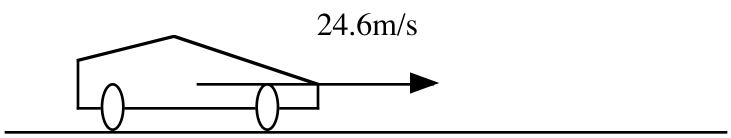
2.23 An electron has a constant acceleration of $+3.2 m/s^2$. At a certain instant its velocity is +9.6m/s. What is its velocity 2.5 s earlier and (b) 2.5s later?

Take the time to be zero when the velocity is 9.6 m/s.

$$a = 3.2 m/s^2$$
$$v_0 = 9.6 m/s$$
$$v = v_0 + at$$

a. At t=-2.5s, $v = 1.6 m/s$
b. At t=+2.5s, $v = 17.6 m/s$.

2.26 On a dry road, a car with good tires may be able to brake with a constant deceleration of $4.92\ m/s^2$ (a) How long does such a car, initially traveling at 24.6 m/s take to stop? (b) How far does it travel in this time? Graph x verses t and v versus t for the deceleration.



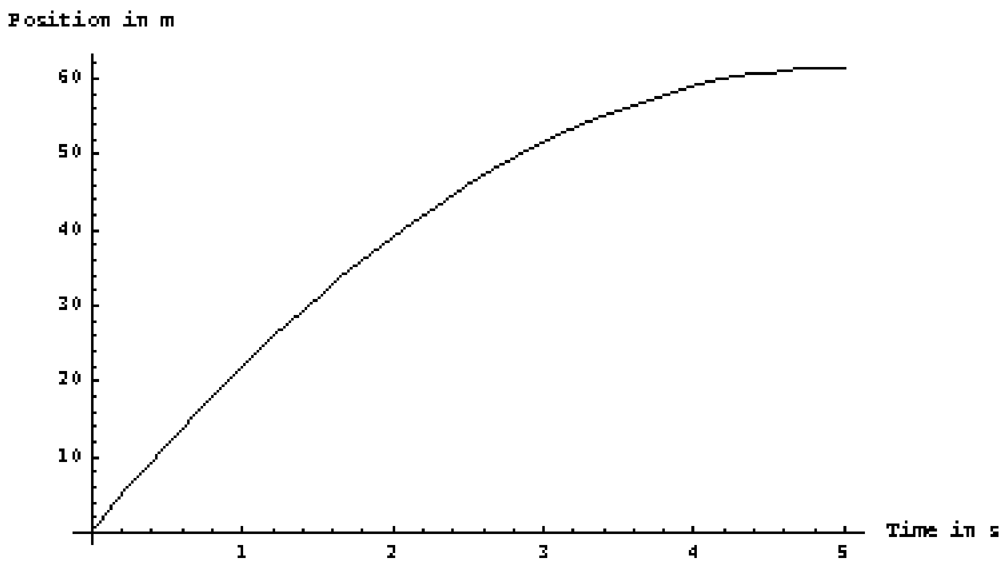
This is a classic constant acceleration problem. We begin by writing out what we know and then solving for time.

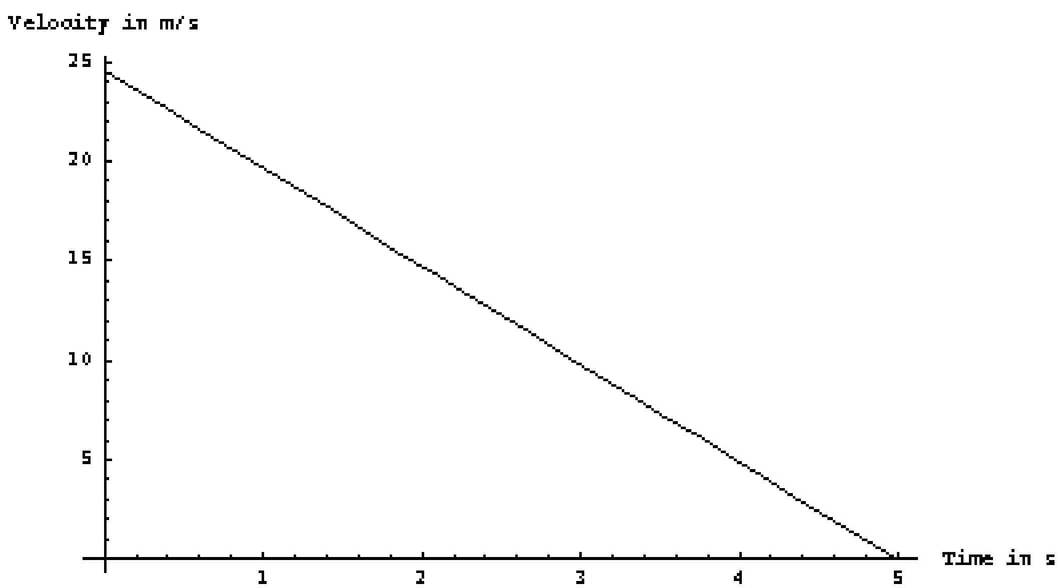
$$x_i = 0$$
$$x_f = ?$$
$$v_i = 24.6 m/s$$
$$v_f = 0$$
$$a = -4.92 m/s^2$$
$$t = ?$$

$$v_f = v_i + at$$
$$0 = v_i + at$$
$$t = -\frac{v_i}{a} = -\frac{24.6\, m/s}{-4.92\, m/s^2} = 5s$$

b. Now we can find the stopping point

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$
$$x_f = 0 + (24.6 m/s) \cdot 5s + \frac{1}{2} \cdot (-4.92 m/s^2) \cdot (5s)^2$$
$$= 61.5\, m$$


Position in m
60
50
40
30
20
10
1
2
3
4
5
Time in s


Velocity in m/s
25
20
15
10
5
1
2
3
4
5
Time in s

2.30 A world's land speed record was set by Colonel John P. Stapp when in March 1954 he rode a rocket-propelled sled that moved along a track at 1020 km/h. He and the sled were brought to a stop in 1.4 s. In terms of g, what acceleration did he experience while stopping?

$$v_i = \frac{1020km}{hr} \cdot \frac{1000m}{1km} \cdot \frac{1hr}{3600s} = 283.3m/s$$

$$v_f = 0$$

$$t = 1.4s$$

$$v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t} = \frac{0 - 283.3m/s}{1.4s} = -202.381m/s^2$$

$$\#g's = \frac{202.381m/s^2}{9.8m/s^2} = 20.65$$

2.37 When a high-speed passenger train traveling at 161 km/h rounds a bend, the engineer is shocked to see that a locomotive has improperly entered onto the track from the siding is a distance D=676 m ahead (Fig 2-24). The locomotive is moving at 29 km/h. the engineer of the high speed train immediately applies the brakes (a) What must be the magnitude of the resulting constant deceleration if a collision is to be just avoided? (b) Assume the engineer is at x=0 when, at t=0, he first spots the locomotive. Sketch curves for the e locomotive and the high speed train for the case in which a collision is just avoided and is not quite avoided.

We begin by writing what we know about each train

High Speed Train

$$x_{ih} = 0$$

$$x_{fh} = ?$$

$$v_{ih} = \frac{161km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = 44.72m/s$$

$$v_{fh} = ?$$

$$a_h = ?$$

Low Speed Train

$$x_{il} = 676m$$

$$x_{fl} = ?$$

$$v_{il} = \frac{29km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = 8.06m/s$$

$$v_f = v_i$$

$$a_l = 0$$

The correct condition is that the velocity of the fast train matches the velocity of the slow train. If the trains have not collided by the time this happens, they never will. After this occurs, the fast train will fall further and further behind. We need to set the two positions equal to each other to find out when the collision will occur.

$$v_{il} = v_{ih} + a_h t$$

$$t = \frac{v_{il} - v_{ih}}{a_h}$$

$$x_{ih} + v_{ih}\, t + \frac{1}{2} a_h\, t^2 = x_{il} + v_{il}\, t + \frac{1}{2} a_l\, t^2$$

$$0 + v_{ih}\, t + \frac{1}{2} a_h\, t^2 = x_{il} + v_{il}\, t + 0$$

$$v_{ih} \cdot \left(\frac{v_{il} - v_{ih}}{a_h} \right) + \frac{1}{2} a_h \left(\frac{v_{il} - v_{ih}}{a_h} \right)^2 = x_{il} + v_{il} \cdot \left(\frac{v_{il} - v_{ih}}{a_h} \right)$$

$$\left(\frac{v_{ih}(v_{il} - v_{ih})}{a_h} \right) + \frac{1}{2} \left(\frac{(v_{il} - v_{ih})^2}{a_h} \right) = x_{il} + \left(\frac{v_{il}(v_{il} - v_{ih})}{a_h} \right)$$

$$v_{ih}(v_{il} - v_{ih}) + \frac{1}{2}(v_{il} - v_{ih})^2 = a_h x_{il} + v_{il}(v_{il} - v_{ih})$$

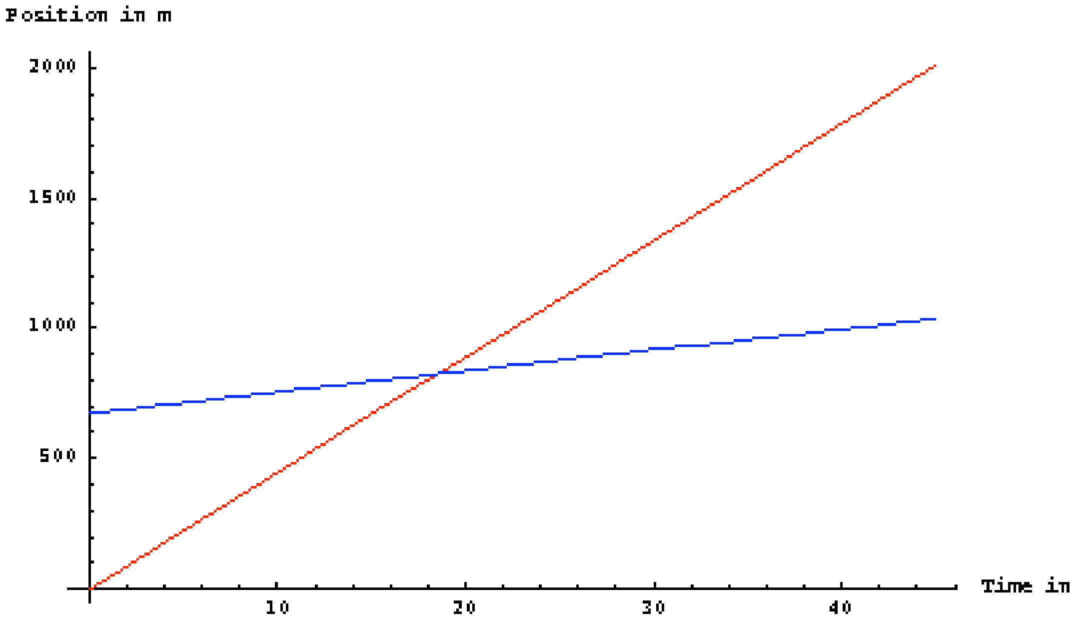
$$v_{ih}(v_{il} - v_{ih}) - v_{il}(v_{il} - v_{ih}) + \frac{1}{2}(v_{il} - v_{ih})^2 = a_h x_{il}$$

$$(v_{ih} - v_{il})(v_{il} - v_{ih}) + \frac{1}{2}(v_{il} - v_{ih})^2 = a_h x_{il}$$

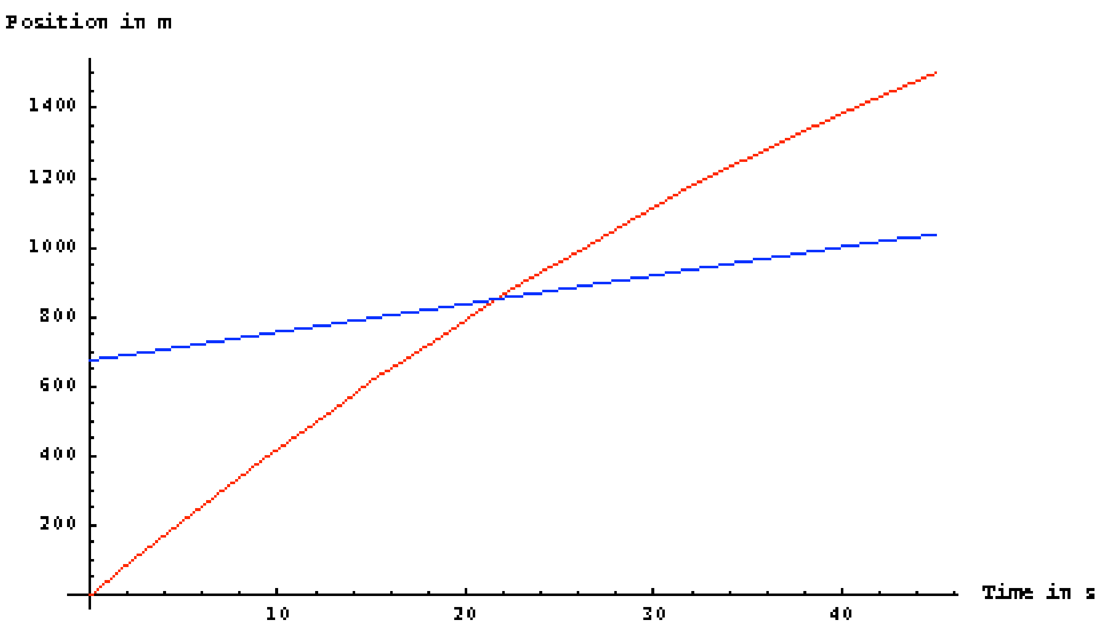
$$a_h = \frac{(v_{ih} - v_{il})(v_{il} - v_{ih}) + \frac{1}{2}(v_{il} - v_{ih})^2}{x_{il}}$$

$$a_h = -0.994$$

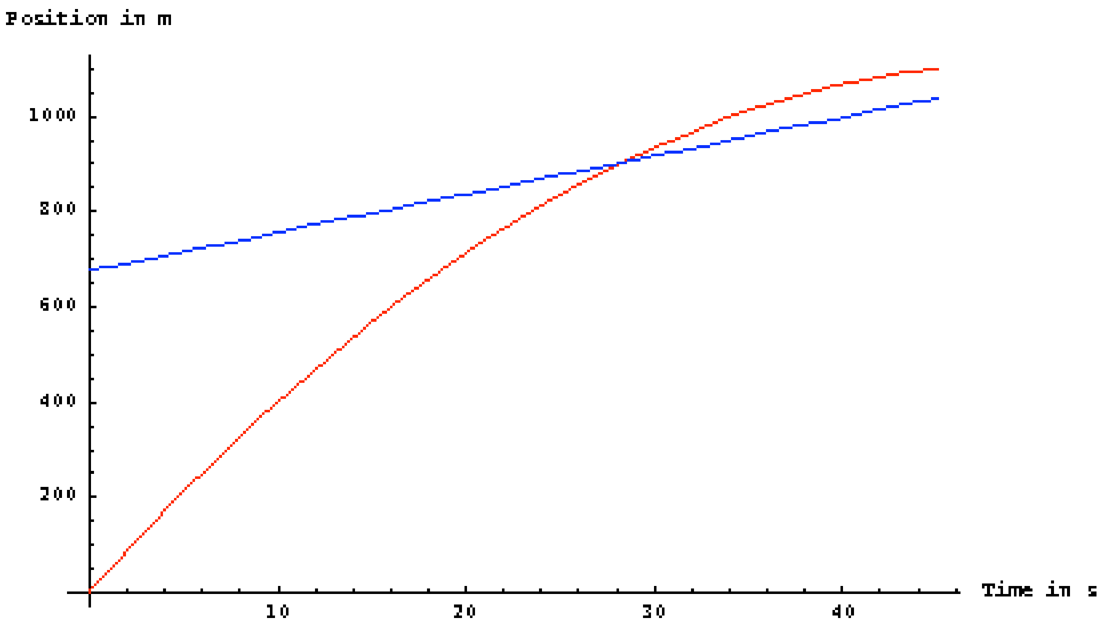
$$a = 0\,m/s^2$$



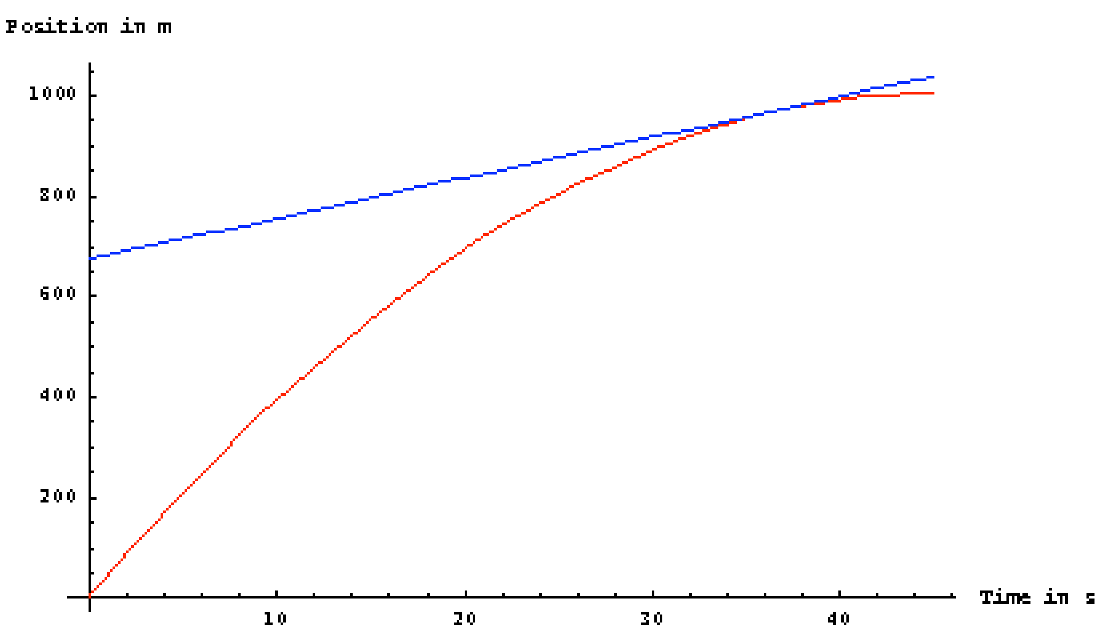
$$a = -0.5\,m/s^2$$



$$a = -0.9\ m/s^2$$



$$a = -0.994\ m/s^2$$



Notice that in the last picture, the slopes of the curves match just where they meet.

2.47 (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50m. (b) How long will it be in the air. Sketch y, v, a, vs t.

$$y_i = 0m$$
$$y_f = 50m$$
$$v_i = ?$$
$$v_f = 0m/s$$
$$t = ?$$
$$a = -g$$

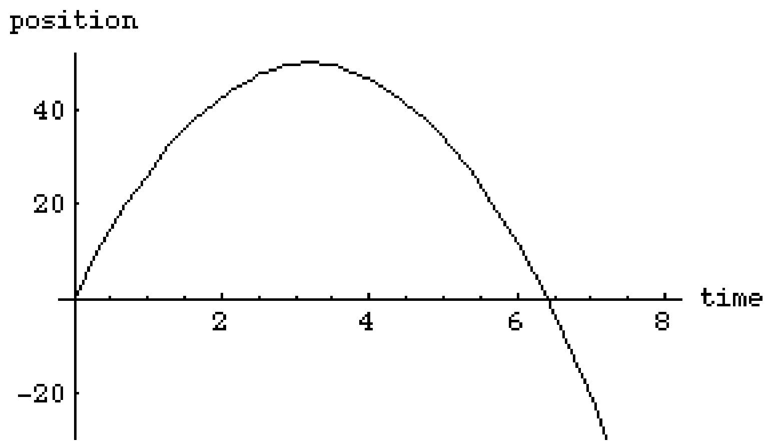
(a). We compute the initial velocity first.

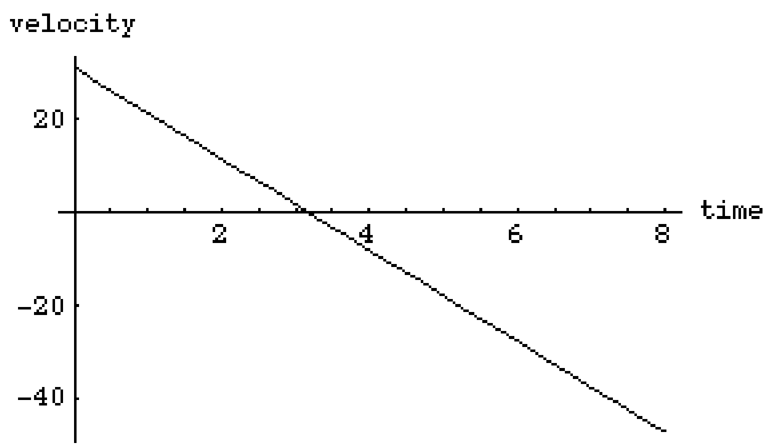
$$v_f^{\,2} = v_i^2 + 2a(y_f - y_i)$$
$$0 = v_i^2 - 2g(y_f - 0)$$
$$v_i = \sqrt{2g(50m)} = 31.3m/s$$

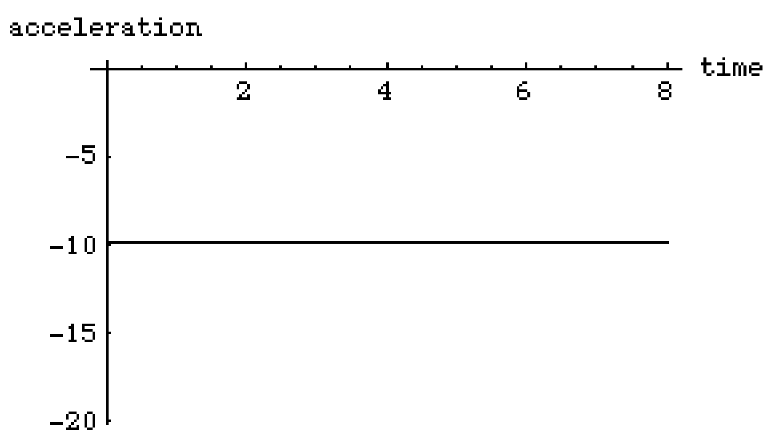
(b) Now that we know initial velocity, we can find the time to reach the highest point.

$$v_f = v_i + at$$
$$0 = v_i - gt$$
$$t = \frac{v_i}{g} = \frac{31.3m/s}{9.8m/s^2} = 3.19s$$

The path is symmetric, so the entire time of flight is 6.38s.


position
40
20
-20
time
2
4
6
8


velocity
20
-20
-40
time
2
4
6
8


acceleration
time
2
4
6
8
-5
-10
-15
-20