Chapter 2

2.1 During a hard sneeze, your eyes might shut for 0.5s. If you are driving a car at 90km/h during such a sneeze, how far does the car move during that time

$$s = \frac{90km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = 25m/s$$
$$d = s \cdot t = 25m/s \cdot 0.5s = 12.5m/s$$

2.4 A car travels up a hill at a constant speed of 40km/h and returns down the hill at a constant speed of 60 km/h. Calculate the average speed for the trip.

This problem is a bit more subtle than it might seem. It is tempting to simply average the two velocities that are given. The problem is that the car does not spend the same amount of time at each velocity, so this approach will give the wrong answer. We need to use the definition of average speed to get the correct answer.

$$s = \frac{total\,dist}{total\,time}$$

For this problem, we'll consider the distance from the bottom to the top of the hill to be d and calculate the time up and the time down the hill to find the total time.

$$t_{up} = \frac{d}{40km/hr}$$
$$t_{down} = \frac{d}{60km/h}$$
$$t_{total} = t_{up} + t_{down} = \frac{d}{40km/hr} + \frac{d}{60km/hr}$$
$$= \frac{60km/hr + 40km/hr}{40km/hr \cdot 60km/hr} d$$
$$s = \frac{2d}{t_{total}} = \frac{2d}{\frac{60km/hr + 40km/hr}{40km/hr \cdot 60km/hr} d} = \frac{2 \cdot 40km/hr \cdot 60km/hr}{60km/hr + 40km/hr}$$
$$= 48km/hr$$

2.5 The position of an abject moving in a straight line is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t in seconds (a). What is the position of the object at t=1,2,3, and 4s? (b) What is the object's displacement between t = 0 and t = 4s. (c) What is the average velocity for the time interval from t=2 s to t = 4s? (d) Graph x vs t for $0 \le t \le 4s$ and indicate how the answer for c can be found from the graph.

(a-d) We plug in to calculate positions.

$$x(1) = 0m$$
$$x(2) = -2.0m$$
$$x(3) = 0m$$
$$x(4) = 12m$$

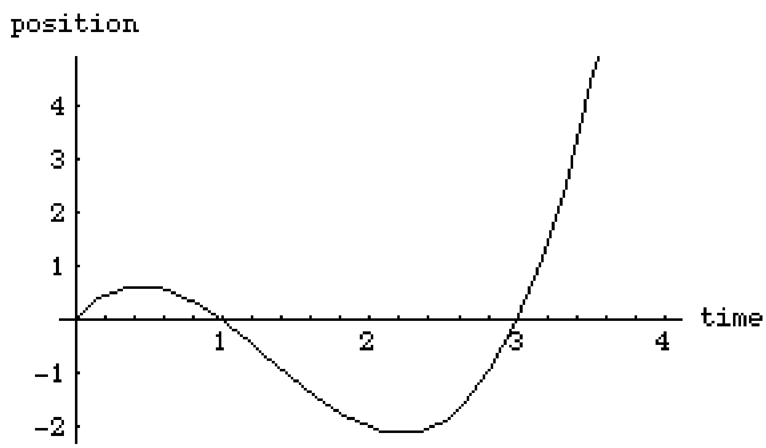
(e) We can calculate the displacement from the positions.

$$\begin{aligned}\Delta x &= x(4) - x(0)\\ &= 12m - 0m\\ &= 12m\end{aligned}$$

(f) We calculate the average velocity using the displacements and time interval.

$$v = \frac{x(4) - x(2)}{4s - 2s} = \frac{12m - (-2m)}{2s} = 7m/s$$

(g) A graph of x vs t.. The average velocity can be computed by connecting x(4) and x(2) with a straight line and computing the slope. Note: Graph done with Mathematica.



2.12 (a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where t is in seconds and x is in meters), what is its velocity at t=1 s? (b) Is it moving toward increasing or decreasing x just then? (c) What is its speed just then? (d) Is the speed larger or smaller at later times? (Try answering the next two question without further calculation.) (e) Is there ever an instant when the velocity is zero? (f) Is there a time after t=3s when the particle is moving toward decreasing x?

To proceed, we begin by taking the derivative to find v.

$$x = 4 - 12t + 3t^2$$

$$v = \frac{dx}{dt} = 6t - 12$$

a. At 1s, $v(1) = -6m/s$

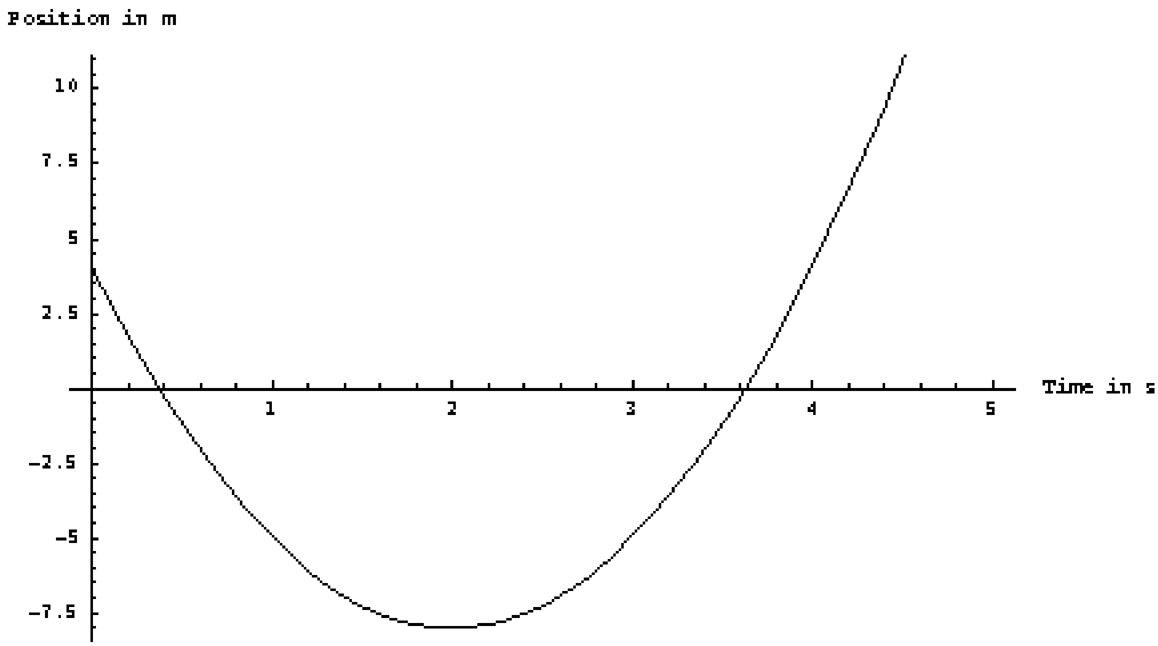
b. Direction is negative--toward the left and more negative since $x(1) = -5m$.

c. Speed is $6m/s$

d. Speed gets smaller (zero at t=2) and then larger.

e. Speed is zero at t=2s.

f. After t=3s, the velocity is always positive. See plot of position below.



2.16 An electron moving along the x axis has a position given by $x = 16t\,e^{-t}m$ **w**here t is in seconds. How far is the electron from the origin when it momentarily stops.

We need to find out when the electron stops. If we know when it stops, we can find out where it is. To find out when it stops, we find the instantaneous

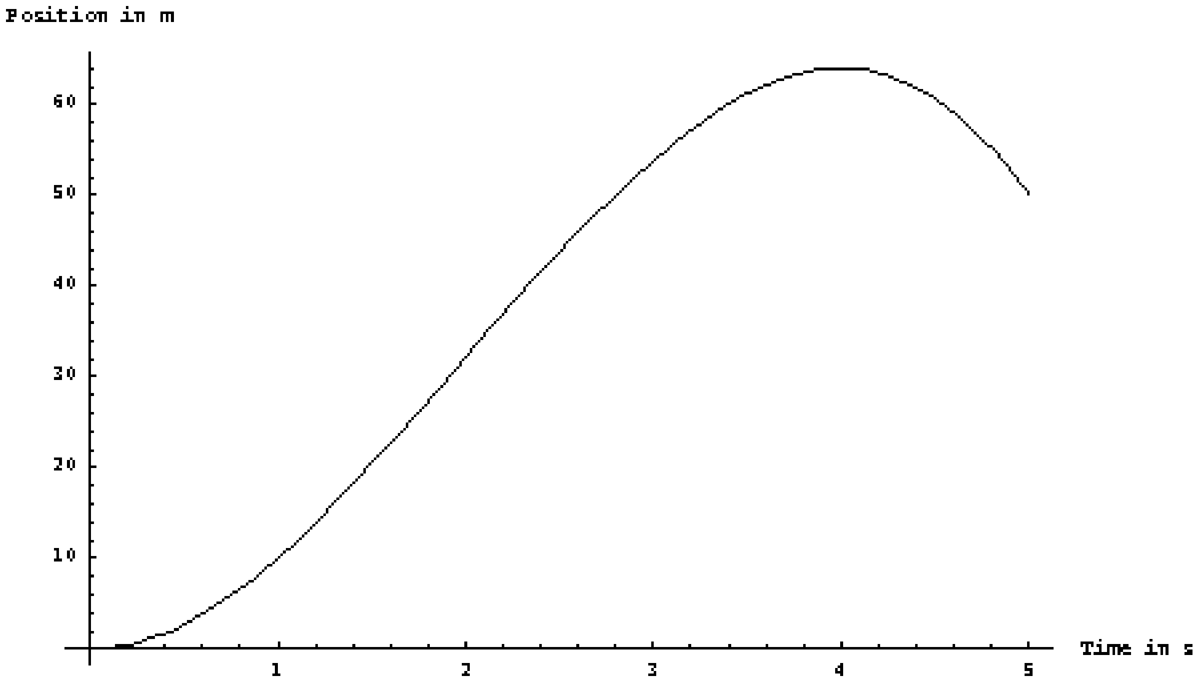
$$v = \frac{dx}{dt} = \frac{d}{dt}(16\,t\,e^{-t}) = 16\;e^{-t} - 16t\,e^{-t}$$

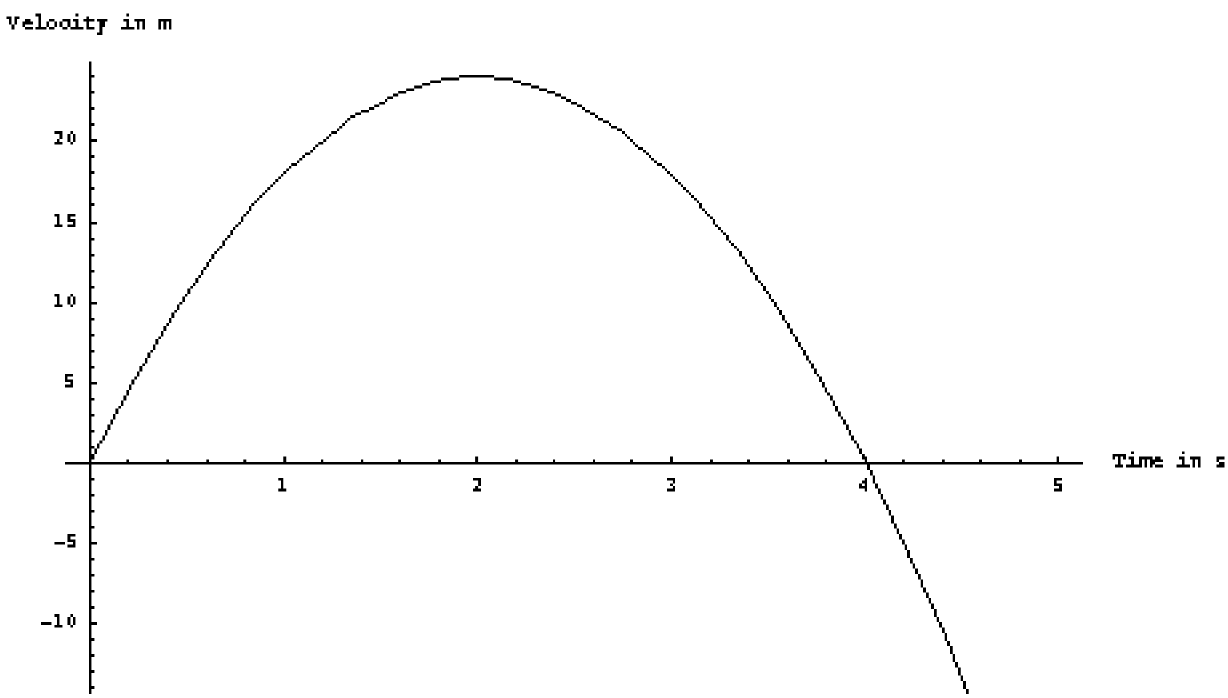
$$= 16(1-t)e^{-t} \qquad\qquad x(1) = 16\cdot 1\cdot e^{-1}m = 5.886m$$

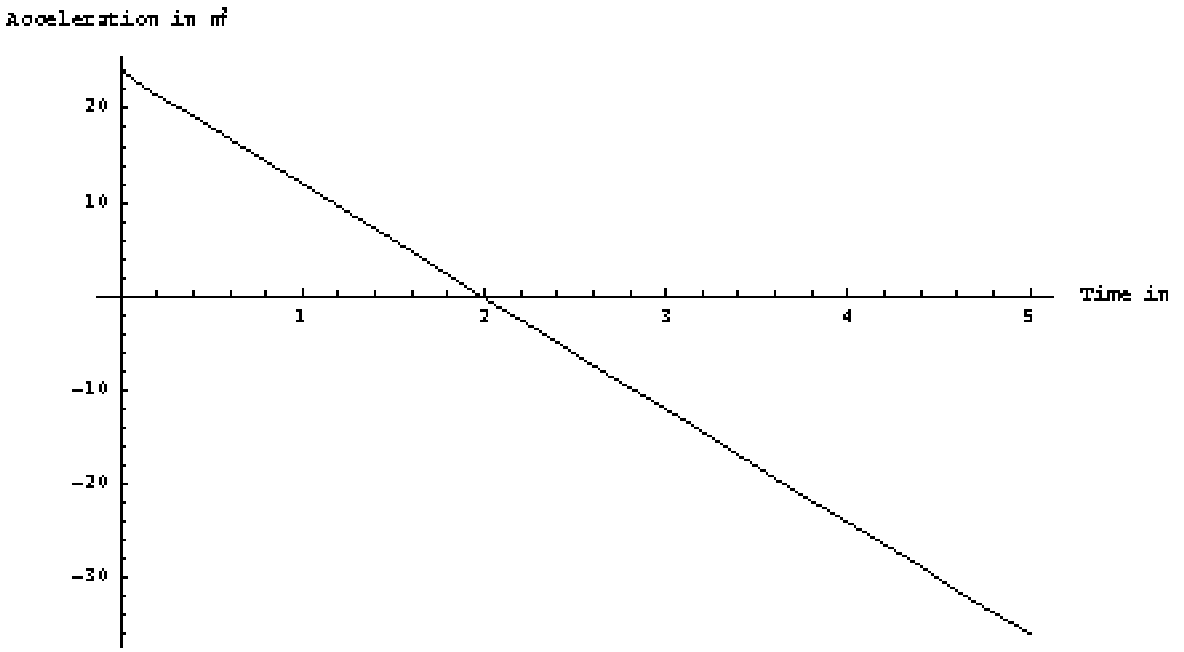
$$0 = 16(1-t)e^{-t} \;\Rightarrow t = 1s$$

2.17 The position of a particle moving along an axis is given by $x = 12t^2 - 2t^3$ where x is in meters and t is in seconds. Determine (a) the position , (b) the velocity , and (c) the acceleration of the particle at t=3.0s (d) What is the maximum positive coordinate reached by the particle and (e) at what time is it reached? (f) What is the maximum positive velocity reached by the particle and (g) at what time is it reached? (h) What is the acceleration of the particle at the instant the particle is not moving (other than at t=0)? (i) Determine the average velocity of the particle between t=0 and t=3.

Its nice to plot the position, velocity and acceleration first.


Position in m
60
50
40
30
20
10
1
2
3
4
5
Time in s


Velocity in m
20
15
10
5
1
2
3
4
5
-5
-10
Time in s



(a-c) We compute the position, velocity and acceleration at t=3s

$$x(t) = 12t^2 - 2t^3 \qquad x(3) = 12\cdot(3)^2 - 2\cdot(3)^3 = 54m$$

$$v(t) = \frac{dx}{dt} = 24t - 6t^2 \qquad v(3) = \frac{dx}{dt} = 24\cdot(3) - 6\cdot(3)^2 = 18m/s$$

$$a(t) = \frac{dv}{dt} = 24 - 12t \qquad a(3) = \frac{dv}{dt} = 24 - 12\cdot(3) = -12m/s^2$$

(d,e) The maximum x occurs when the velocity (first derivative) equals zero.

$$v(t) = 24t - 6t^2$$

$$0 = 24t - 6t^2$$

$$t = \begin{cases} 0s \\ 4s \end{cases}$$

$$x(0) = 0$$

$$x(4) = 54m$$

(f, g) The maximum positive velocity occurs when the acceleration equals zero

$$a(t) = 24 - 12t$$

$$0 = 24 - 12t$$

$$t = 2s$$

$$v(2) = 24m/s$$

(h) The acceleration of the particle when it is not moving can be computed using the time we found in parts (d,e) above.

$$a(t) = 24 - 12t$$
$$a(4) = 24 - 12t$$
$$a(4) = -24m/s^2$$

(i) Determine the average velocity of the particle between t=0 and t=3s.

$$a(t) = 24 - 12t$$
$$v_{avg} = \frac{a(3) - a(0)}{3 - 0} = -12m/s$$

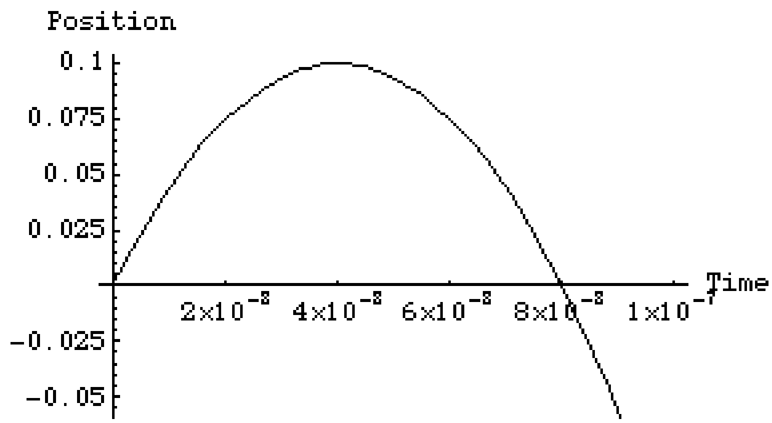
2.20 A muon (an elementary particle) enters an electric field with a speed of $5.00 \times 10^6 m/s$, whereupon the field slows it at the rate of 1.25×10^{14} m/s^2. (a) How far does the muon take to stop? (b) Graph x vs. t and v vs t for the muon.

$$v_0 = 5.00 \times 10^6 m/s$$
$$a = -1.25 \times 10^{14} \text{ m/s}^2$$
$$v = v_0 + at$$

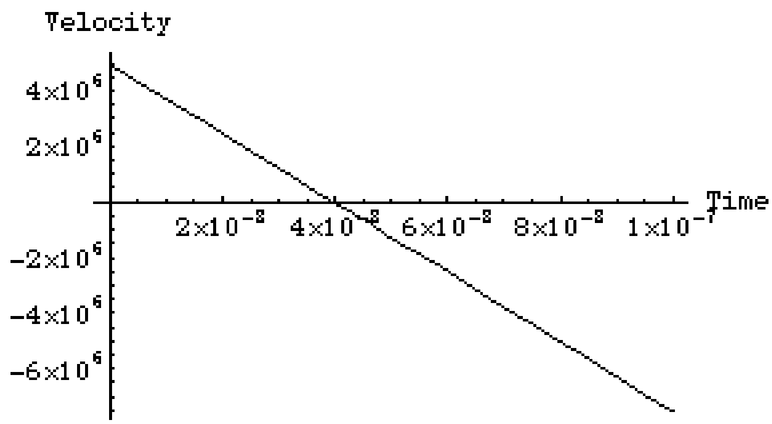
a. We solve for the time to stop

$$v = 0$$
$$v = v_0 + at$$
$$0 = v_0 + at$$
$$t = -\frac{v_0}{a} = -\frac{5.00 \times 10^6 m/s}{-1.25 \times 10^{14} \text{ m/s}^2} = 4.00 \times 10^{-8} s$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$x = 0 + 5.00 \times 10^6 m/s \cdot 4.00 \times 10^{-8} s - \frac{1}{2} \cdot 1.25 \times 10^{14} \text{ m/s}^2 \cdot (4.00 \times 10^{-8} s)^2$$
$$= 0.1m$$

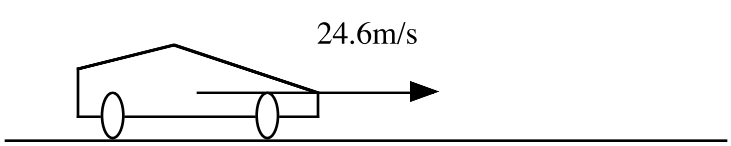
Position vs. time.



Velocity vs. time



2.22 On a dry road, a car with good tires may be able to brake with a constant deceleration of $4.92\ m/s^2$ (a) How long does such a car, initially traveling at 24.6 m/s take to stop? (b) How far does it travel in this time? Graph x verses t and v versus t for the deceleration.



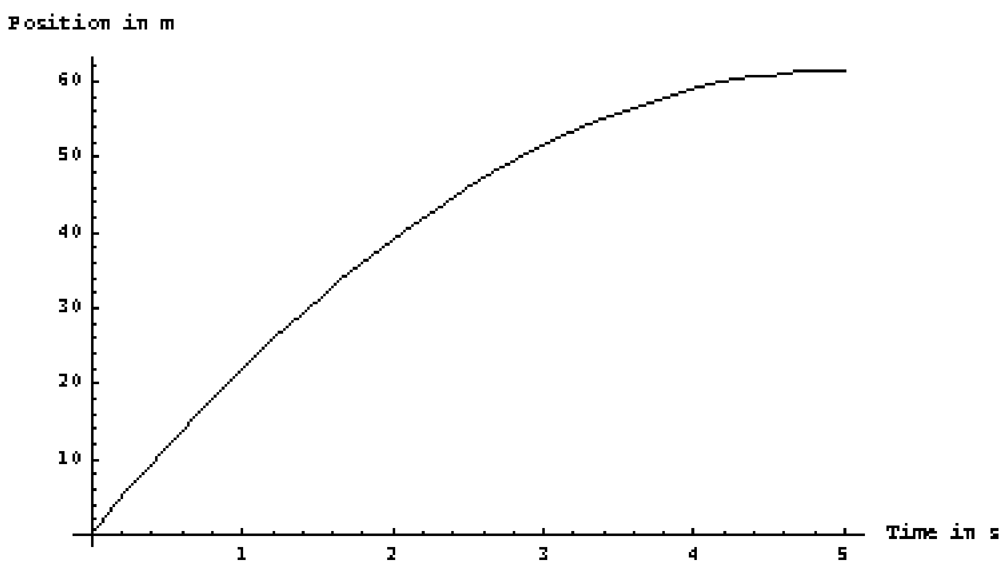
This is a classic constant acceleration problem. We begin by writing out what we know and then solving for time.

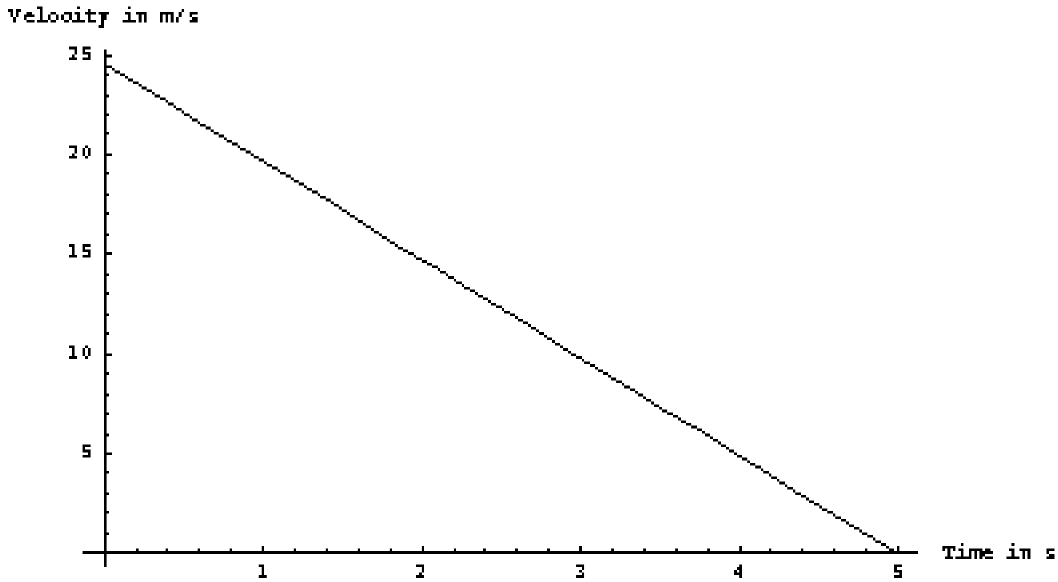
$$x_i = 0$$
$$x_f = ?$$
$$v_i = 24.6 m/s$$
$$v_f = 0$$
$$a = -4.92 m/s^2$$
$$t = ?$$

$$v_f = v_i + at$$
$$0 = v_i + at$$
$$t = -\frac{v_i}{a} = -\frac{24.6\, m/s}{-4.92\, m/s^2} = 5s$$

b. Now we can find the stopping point

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$
$$x_f = 0 + (24.6 m/s) \cdot 5s + \frac{1}{2} \cdot (-4.92 m/s^2) \cdot (5s)^2$$
$$= 61.5\, m$$





2.26 A world's land speed record was set by Colonel John P. Stapp when in March 1954 he rode a rocket-propelled sled that moved along a track at 1020 km/h. He and the sled were brought to a stop in 1.4 s. In terms of g, what acceleration did he experience while stopping?

$$v_i = \frac{1020km}{hr} \cdot \frac{1000m}{1km} \cdot \frac{1hr}{3600s} = 283.3m/s$$

$$v_f = 0$$

$$t = 1.4s$$

$$v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t} = \frac{0 - 283.3m/s}{1.4s} = -202.381m/s^2$$

$$\#g's = \frac{202.381m/s^2}{9.8m/s^2} = 20.65$$

2.29 A certain elevator cab has a total. run of 190 m and a maximum speed of 305 m/min, and it accelerates from rest and then back to rest at $1.22m/s^2$. (a) How far does the cab move while accelerating to full speed from rest? (b) How long does it take to make the nonstop 190m run, starting and ending at rest.

$$y_f = ?$$

$$y_i = 0$$

$$v_f = \frac{305m}{\min} \cdot \frac{1\min}{60s} = 5.083m/s$$

$$v_i = 0m/s$$

$$a = 1.22m/s^2$$

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$

$$v_f^2 = 0 + 2a(y_f - 0)$$

$$y_f = \frac{v_f^2}{2a} = \frac{(5.083m/s)^2}{2 \cdot 1.22m/s^2}$$

$$= 10.59m$$

We see that the acceleration process takes 10.59m. The deceleration will take the same distance. This leaves $d = 190 - 10.59m - 10.59m = 168.82$ to be traveled at top speed. First we'll find the time for the acceleration and deceleration, and then find the time for the section at top speed.

$$v_f = 5.083 m/s$$
$$v_i = 0 m/s$$
$$v_f = v_i + a t$$
$$t = \frac{v_f - v_i}{a} = \frac{5.083 m/s - 0}{1.22 m/s^2} = 4.17s$$

$$t = \frac{d}{v} = \frac{168.82m}{5.083m/s} = 33.2s$$

Now that we know the time for the acceleration, deceleration and constant speed.

$$t = 33.2s + 4.17s + 4.17s = 41.54s$$

2.31 Figure 2-20 depicts the motion of a particle moving along an x axis with a constant acceleration
What are the (a) magnitude and (b) direction of the particle's acceleration?

We know that for constant acceleration, the position is given by

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

We can see from the graph that the initial position (at t=0) is $x_i = -2m$. We have the positions at two additional times. We can use these positions to find the initial velocity and acceleration. We use these positions and times to solve for the initial velocity and acceleration. We take the position equation for 1s, solve for the initial velocity and then plug that expression in to find the acceleration.

$$x(1) = 0m$$
$$x(2) = 6m$$

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$
$$0 = -2m + v_i \cdot 1s + \frac{1}{2} a (1s)^2$$
$$6 = -2m + v_i \cdot 2s + \frac{1}{2} a (2s)^2$$

$$0 = -2 + v_i + \frac{1}{2} a$$
$$v_i = 2 - \frac{1}{2} a$$
$$0 = -8 + 2 \cdot (2 - \frac{1}{2} a) + 2a$$
$$= -4 + a$$
$$a = 4 m/s^2$$
$$v_i = 0 m/s$$

2.37 When a high-speed passenger train traveling at 161 km/h rounds a bend, the engineer is shocked to see that a locomotive has improperly entered onto the track from the siding is a distance D=676 m ahead (Fig 2-24). The locomotive is moving at 29 km/h. the engineer of the high speed train immediately applies the brakes (a) What must be the magnitude of the resulting constant deceleration if a collision is to be just avoided? (b) Assume the engineer is at x=0 when, at t=0, he first spots the locomotive. Sketch curves for the e locomotive and the high speed train for the case in which a collision is just avoided and is not quite avoided.

We begin by writing what we know about each train

High Speed Train

$x_{ih} = 0$

$x_{fh} = ?$

$v_{ih} = \frac{161km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = 44.72m/s$

$v_{fh} = ?$

$a_h = ?$

Low Speed Train

$x_{il} = 676m$

$x_{fl} = ?$

$v_{il} = \frac{29km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = 8.06m/s$

$v_f = v_i$

$a_l = 0$

The correct condition is that the velocity of the fast train matches the velocity of the slow train. If the trains have not collided by the time this happens, they never will. After this occurs, the fast train will fall further and further behind. We need to set the two positions equal to each other to find out when the collision will occur.

$$x_{ih} + v_{ih}t + \frac{1}{2}a_h t^2 = x_{il} + v_{il}t + \frac{1}{2}a_l t^2$$

$$0 + v_{ih}t + \frac{1}{2}a_h t^2 = x_{il} + v_{il}t + 0$$

$$v_{il} = v_{ih} + a_h t$$

$$t = \frac{v_{il} - v_{ih}}{a_h}$$

$$v_{ih} \cdot \left(\frac{v_{il} - v_{ih}}{a_h}\right) + \frac{1}{2}a_h\left(\frac{v_{il} - v_{ih}}{a_h}\right)^2 = x_{il} + v_{il} \cdot \left(\frac{v_{il} - v_{ih}}{a_h}\right)$$

$$\left(\frac{v_{ih}(v_{il} - v_{ih})}{a_h}\right) + \frac{1}{2}\left(\frac{(v_{il} - v_{ih})^2}{a_h}\right) = x_{il} + \left(\frac{v_{il}(v_{il} - v_{ih})}{a_h}\right)$$

$$v_{ih}(v_{il} - v_{ih}) + \frac{1}{2}(v_{il} - v_{ih})^2 = a_h x_{il} + v_{il}(v_{il} - v_{ih})$$

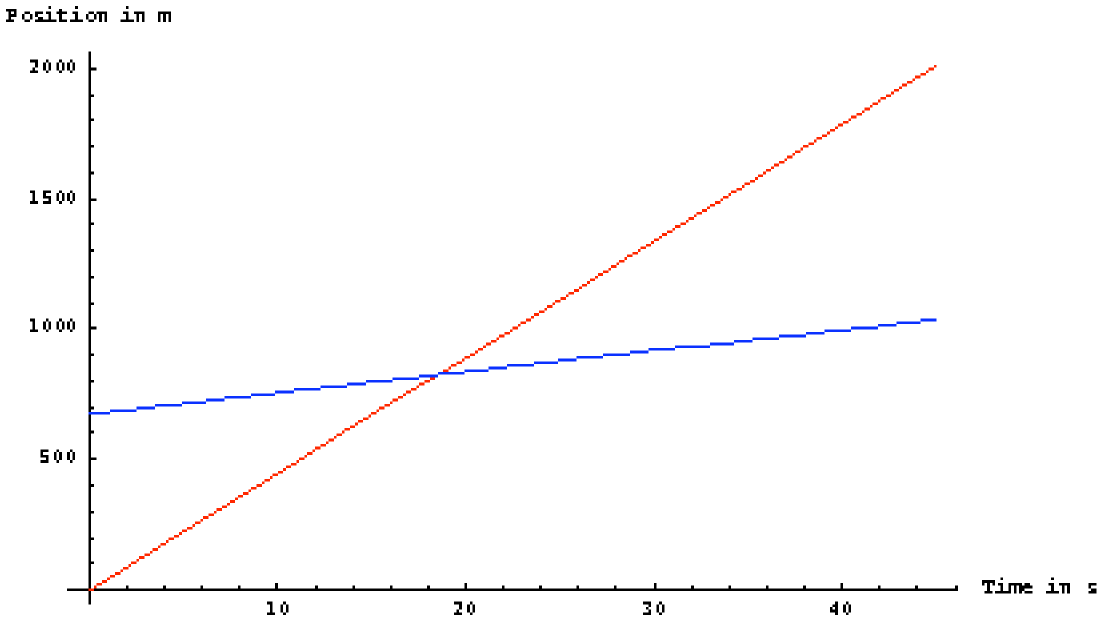
$$v_{ih}(v_{il} - v_{ih}) - v_{il}(v_{il} - v_{ih}) + \frac{1}{2}(v_{il} - v_{ih})^2 = a_h x_{il}$$

$$(v_{ih} - v_{il})(v_{il} - v_{ih}) + \frac{1}{2}(v_{il} - v_{ih})^2 = a_h x_{il}$$

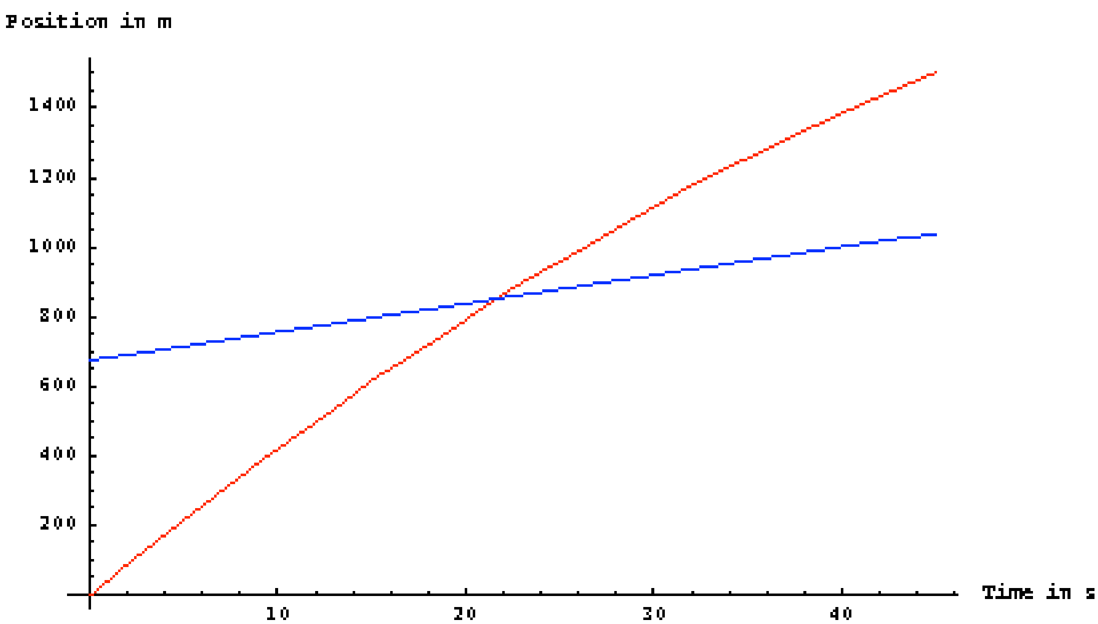
$$a_h = \frac{(v_{ih} - v_{il})(v_{il} - v_{ih}) + \frac{1}{2}(v_{il} - v_{ih})^2}{x_{il}}$$

$$a_h = -0.994$$

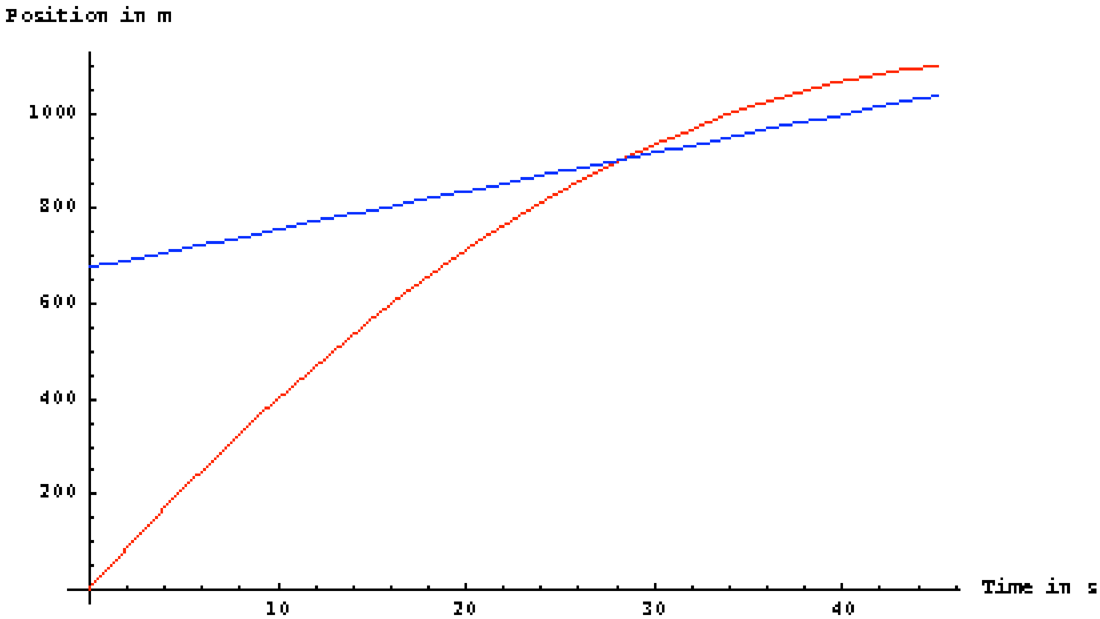
$$a = 0\, m/s^2$$



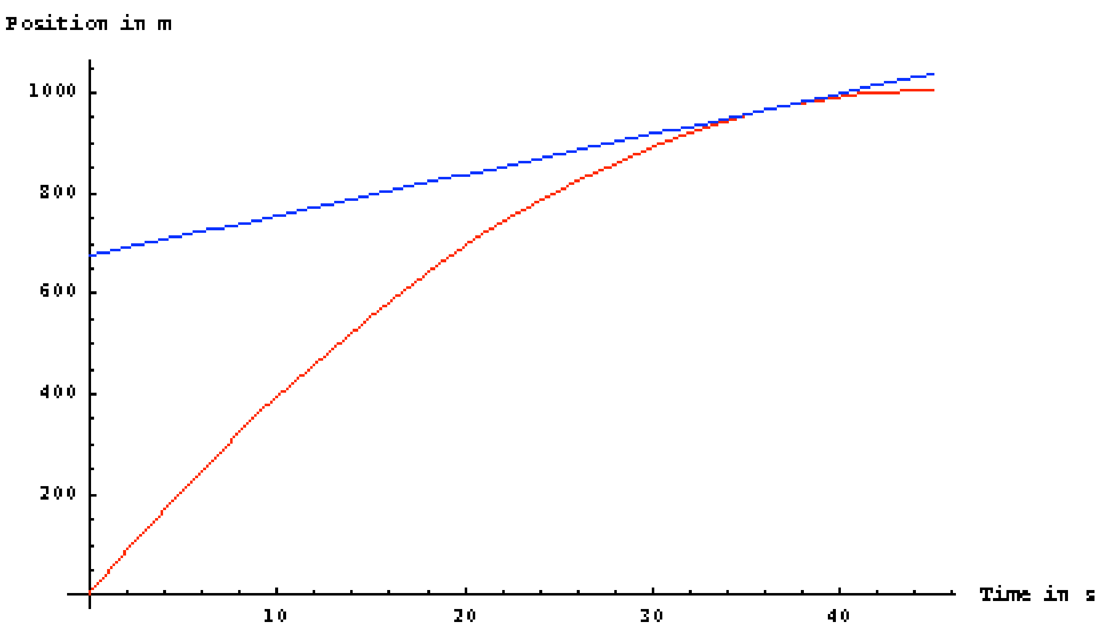
$a = -0.5\,m/s^2$



$a = -0.9\;m/s^2$



$$a = -0.994\ m/s^2$$



Notice that in the last picture, the slopes of the curves match just where they meet.

2.42 A hoodlum throws a stone vertically downward with an initial speed of 12m/s from the roof of a building, 30 m above the ground. How long does it take the stone to reach the ground? (b)What is the speed of the stone at impact.

$$y_i = 30m$$
$$y_f = 0m$$
$$v_i = -12m/s$$
$$v_f = ?$$
$$t = ?$$
$$a = -g$$

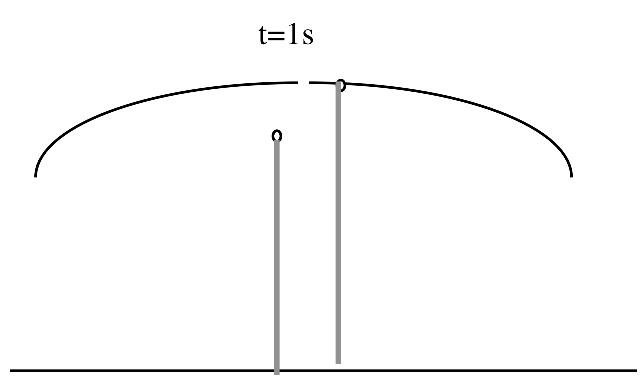
(a). We compute the time. We use the solution to a quadratic equation and choose the positive sign to get the positive time.

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$
$$0 = 30 - 12t - \frac{1}{2}\cdot 9.8\cdot t^2$$
$$t = \frac{12 \pm \sqrt{12^2 - 4(\frac{-9.8}{2})(30)}}{2\cdot(\frac{-9.8}{2})}$$
$$t = 1.54s$$

(b) Now that we know the time, we can find the velocity. The direction is downward and the speed would be the absolute value of the velocity for this one dimensional problem.

$$\begin{aligned} v_f &= v_i + at \\ &= -12m/s - 9.8m/s^2 \cdot 1.54s \\ &= -27.1m/s \end{aligned}$$

2.52 A stone is dropped from a river from a bridge 43.9 m above the water. Another stone is thrown vertically down 1.00 s after the first is dropped. The stones strike the water at the same time (a) What is the initial speed of the second stone? (b) Plot the velocity vs. time on a graph for each stone, taking zero time at the instant the first stone is released.



We begin by writing out the equations of motion for each stone. Notice that we write the time slightly differently for the thrown stone, since it's initial velocity occurs at $t = 1s$. We recognize then that for the thrown rock, $t = t_f - t_i \ so \ t \Rightarrow t-1$

Dropped rock

$$y_i = 43.9m$$
$$y_f = 0m$$
$$v_i = 0$$
$$v_f = ?$$
$$a = -g$$

Thrown rock

$$t = 1s$$
$$y_i = 43.9m$$
$$y_f = 0m$$
$$v_i = ?$$
$$v_f = ?$$
$$a = -g$$

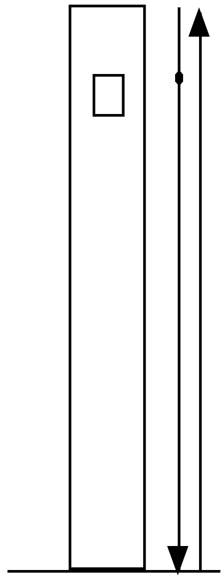
First, we calculate the time for the dropped rock to fall.

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$
$$0 = y_i - \frac{1}{2}gt^2$$
$$t = \sqrt{\frac{2y_i}{g}} = 3s$$

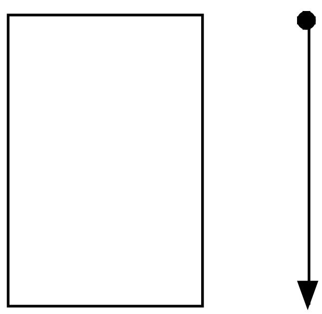
Now that we know this time, we can find the initial velocity of the thrown rock

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$
$$0 = y_i + v_i(t-1) - \frac{1}{2}g(t-1)^2$$
$$v_i = \frac{\frac{1}{2}g(t-1)^2 - y_i}{(t-1)} = -12.15m/s$$

2.57 A steel ball is dropped from a building 's roof and passes a window, taking 0.125 s to fall from the top to the bottom of the window, a distance of 1.2m It then falls to a sidewalk and bounces back past the window, moving from bottom to top in 0.125s. Assume that the upward flight is an exact reverse of the fall. The time the ball spends below the bottom of the window is 2.0 s. How tall is the building.



We need to do this problem in parts. First consider the ball's trip across the window on the way down. We can find the velocity the ball had at the top of the window.



$$y_f - y_i = -1.2m$$
$$v_i = ?$$
$$v_f = ?$$
$$a = -g$$
$$t = 0.125s$$

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$

$$y_f - y_i = v_i t - \frac{1}{2}gt^2$$

$$v_i = \frac{(y_f - y_i) + \frac{1}{2}g t^2}{t} = -8.99m/s$$

Now that we know how fast the ball was going at the top of the window, we can calculate how far it had to fall to reach that speed. This will give us the distance from the top of the building to the top of the window.

$$y_f - y_i = ?$$
$$v_i = 0$$
$$v_f = -8.99$$
$$a = -g$$
$$t = ?$$

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$
$$v_f^2 = 0 - 2g(y_f - y_i)$$
$$(y_f - y_i) = -\frac{v_f^2}{2g} = -4.12m$$

The top of the window is 4.12m from the top of the building.

Now we can consider the fall from the top of the window to the ground. We know the velocity at the top of the window and the time it takes to reach the ground. We can use this information to find the absolute height of the top of the window from the ground.

$$y_f = 0$$
$$y_i = ?$$
$$v_i = -8.99$$
$$v_f = ?$$
$$a = -g$$
$$t = 1.125s$$

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$
$$0 = y_i + v_i t - \frac{1}{2}gt^2$$
$$y_i = \frac{1}{2}gt^2 - v_i t = 16.32m$$

Now we know the distance from the ground to the top of the window and from the top of the window to the top of the building. The height of the building is

$$h = 16.32m + 4.12m = 20.44m$$