## Chapter 11

11.7 A solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance of 6 m down a house roof that is inclined at 30 degrees (a) What is the angular speed of the cylinder about its center as it leaves the house roof? (b) The roof's edge is 5.0 m . How far horizontally from the roof's edge does the cylinder hit the level ground.

This problem is very similar to the previous problem. We begin with energy conservation. Take the ground as zero height.

$$
\begin{aligned}
& H=5 m+6 m \sin 30=8 m \quad h=5 m \\
& E_{i}=m g H \quad E_{f}=m g h+\frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2} \\
& m g h+\frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2}=m g H \\
& m g h+\frac{1}{2} m v^{2}+\frac{1}{2}\left(\frac{1}{2} m R^{2}\right) \frac{v^{2}}{R^{2}}=m g H \\
& \frac{1}{2} m v^{2}+\frac{1}{2}\left(\frac{1}{2} m\right) v^{2}=m g(H-h) \\
& \frac{1}{2} v^{2}+\frac{1}{4} v^{2}=g(H-h) \\
& \frac{3}{4} v^{2}=g(H-h) \\
& v=\sqrt{\frac{4}{3} g(H-h)}=\sqrt{\frac{4}{3} \cdot 9.8 m / s^{2}(8 m-5 m)} \\
&=6.26 m / s \\
& \omega=\frac{v}{r}=\frac{6.26 m / s}{0.1 m}=62.6 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

This problem differs from the previous problem in that the cylinder is not launched horizontally. It has an initial velocity in the $y$ direction as well as in the $x$ direction...

$$
\begin{array}{ll} 
& y_{i}=h \\
x_{i}=0 & y_{f}=0 \\
x_{f}=? & v_{i y}=-6.26 \sin 30=-3.13 \mathrm{~m} / \mathrm{s} \\
v_{i x}=6.26 \cos 30=5.42 \mathrm{~m} / \mathrm{s} & v_{f y}=? \\
v_{f x}=v_{i x} & a_{y}=-g \\
a_{x}=0 & t=?
\end{array}
$$

Again we find the time

$$
\begin{aligned}
& y_{f}=y_{i}+v_{i y} t+\frac{1}{2} a_{y} t^{2} \\
& 0=h+v_{i y} t-\frac{1}{2} g t^{2} \\
& t=\frac{-v_{i y} \pm \sqrt{v_{i y}{ }^{2}-4 \cdot\left(-\frac{1}{2} g\right) \cdot h}}{2 \cdot\left(-\frac{1}{2} g\right)}=\frac{-(-3.13) \pm \sqrt{(-3.13)^{2}-4 \cdot\left(-\frac{1}{2} \cdot 9.8\right) \cdot 5}}{2 \cdot\left(-\frac{1}{2} \cdot 9.8\right)} \\
& =0.74 s
\end{aligned}
$$

Again we find the horizontal distance

$$
\begin{aligned}
x_{f} & =x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2} \\
& =0+v_{i x} t+0 \\
x_{f} & =5.42 \mathrm{~m} / \mathrm{s} \cdot 0.74 \mathrm{~s} \\
& =4.01 \mathrm{~m}
\end{aligned}
$$

This problem is considerably more complicated because the projectile is not launched horizontally.
11.12 In the Figure, a solid brass ball of mass 0.280 g will roll smoothly along a loop-the-loop track when released from rest along the straight section. The circular loop has radius $R=14.0 \mathrm{~cm}$ and the ball has radius $r \ll R$. (a) What is h if the ball is on the verge of leaving the track when it reaches the top of the loop if the ball is release at height $h=6.00 R$, what are the (b) magnitude and (c) direction of the horizontal force component acting on the ball at point Q .


When the ball is just about to come off the track, the centripetal force is entirely due to gravity. This allows us to solve for the velocity

$$
\begin{aligned}
\frac{m v^{2}}{R} & =m g \\
v^{2} & =R g
\end{aligned}
$$

Now that we know the velocity, we can use energy conservation. The initial state is where we release the ball, the final state is at the top of the loop.

$$
\begin{aligned}
E_{i}=m g h & \quad E_{f}=\frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2}+m g(2 R) \\
E_{i} & =E_{f} \\
m g h & =m g(2 R)+\frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2} \\
m g h & =m g(2 R)+\frac{1}{2} m v^{2}+\frac{1}{2}\left(\frac{2}{5} m r^{2}\right) \frac{v^{2}}{r^{2}} \\
m g h & =m g(2 R)+\frac{1}{2} m v^{2}+\frac{1}{2}\left(\frac{2}{5} m\right) v^{2} \\
h & =\frac{g(2 R)+\frac{1}{2} v^{2}+\frac{1}{2}\left(\frac{2}{5}\right) v^{2}}{g} \\
& =\frac{g(2 R)+\frac{1}{2} R g+\frac{1}{2}\left(\frac{2}{5}\right) R g}{g} \\
& =(2 R)+\frac{1}{2} R+\frac{1}{5} R \\
& =\frac{27}{10} R
\end{aligned}
$$

In the second part of this problem, we need the velocity so that we can compute the centripetal force again, since it is the net inward force. In this case, we know the initial height. The force at Q will be to the left.

$$
\begin{aligned}
E_{i} & =E_{f} \\
m g(6 R) & =m g(R)+\frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2} \\
m g(6 R) & =m g(R)+\frac{1}{2} m v^{2}+\frac{1}{2}\left(\frac{2}{5} m r^{2}\right) \frac{v^{2}}{r^{2}} \\
m g(5 R) & =\frac{1}{2} m v^{2}+\frac{1}{2}\left(\frac{2}{5} m\right) v^{2} \\
v^{2} & =\frac{g(5 R)}{\frac{1}{2}+\frac{1}{2}\left(\frac{2}{5}\right)} \\
& =\frac{10}{7} \cdot g(5 R) \\
& =\frac{50}{7} g R \\
F & =\frac{m v^{2}}{R}=\frac{m \cdot \frac{50}{7} g R}{R} \\
& =\frac{50}{7} m g
\end{aligned}
$$

11.28 A 2.0 kg particle-like object moves in a plane with a velocity components $v_{x}=30 \mathrm{~m} / \mathrm{s}$ and $v_{y}=60 \mathrm{~m} / \mathrm{s}$ as it passes through the point with $(\mathrm{x}, \mathrm{y})$ coordinates of $(3.0,-4.0) \mathrm{m}$. Jus then, in unit-vector notation, what is the angular momentum and (b) the point $(-2.0,-2.0) \mathrm{m}$ ?

We can draw what is happening in each case. The r is defined as coming from the axis and ending where the force is applied. In the first case


$$
\begin{aligned}
\vec{r} & =(3.0-0.0) \hat{i}+(-4.0-0.0) \hat{j}=3.0 \mathrm{~m} \hat{i}-4.0 \mathrm{~m} \hat{j} \\
\vec{p} & =m \vec{v}=60 \mathrm{~kg} \mathrm{~m} / \mathrm{s} \hat{i}+120 \mathrm{kgm} / \mathrm{s} \hat{j} \\
L & =\vec{r} \times \vec{p}=\operatorname{det}\left[\begin{array}{ccc}
\hat{i} & \hat{j} & \hat{k} \\
3 & -4 & 0 \\
60 & 120 & 0
\end{array}\right]=(3 \cdot 120-(-4) \cdot 60) \hat{k}=600 \mathrm{kgm}^{2} / \mathrm{s} \hat{k}
\end{aligned}
$$

In the second case, the axis is at $(-2,2)$, so we need to recalculate $r$.


$$
\begin{aligned}
\vec{r} & =(3.0-(-2.0)) \hat{i}+(-4.0-(-2.0)) \hat{j}=5.0 m \hat{i}-2.0 \mathrm{~m} \hat{j} \\
\vec{p} & =m \vec{v}=60 \mathrm{~kg} \mathrm{~m} / \mathrm{s} \hat{i}+120 \mathrm{kgm} / \mathrm{s} \hat{j} \\
L & =\vec{r} \times \vec{p}=\operatorname{det}\left[\begin{array}{ccc}
\hat{i} & \hat{j} & \hat{k} \\
5 & -2 & 0 \\
60 & 120 & 0
\end{array}\right]=(5 \cdot 120-(-2) \cdot 60) \hat{k}=720 \mathrm{kgm}^{2} / \mathrm{s} \hat{k}
\end{aligned}
$$

11.38 A sanding disk with rotational inertia $I=1.2 \times 10^{-3} \mathrm{~kg} \mathrm{~m}^{2}$ is attached to an electric drill whose motor delivers a torque of magnitude 16 Nm about the central axis of the disk. About that axis and with the torque applied for 33 ms , what is the magnitude of the angular momentum and angular velocity of the disk.

We begin by finding the angular velocity after 33 ms . We begin by finding the angular acceleration.

$$
\begin{aligned}
& \tau=I \alpha \\
& \alpha=\frac{\tau}{I}=\frac{16 \mathrm{Nm}}{1.2 \times 10^{-3} \mathrm{~kg} \mathrm{~m}^{2}}=1.33 \times 10^{4} \mathrm{rad} / \mathrm{s}^{2}
\end{aligned}
$$

Now find the angular velocity

$$
\omega_{f}=\omega_{i}+\alpha t=0+1.33 \times 10^{4} \cdot 0.033 \mathrm{~s}=440 \mathrm{rad} / \mathrm{s}
$$

Now we can find the angular momentum.

$$
L=I \omega_{f}=0.528 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}
$$

11.39 The angular momentum of a flywheel having a rotational inertia of $0.140 \mathrm{~kg} \mathrm{~m}^{2}$ about its central axis decreases from 3.0 to $0.800 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}$ in 1.50 s . (a) What is the magnitude of the average torque acting on the flywheel about its central axis during this period? (b) Assuming a constant angular acceleration, through what angle does the flywheel turn? (c) How much work is done on the wheel? (d) what is the average power of the flywheel?

$$
\begin{aligned}
& \tau=\left|\frac{\Delta L}{\Delta t}\right|=\left|\frac{L_{f}-L_{i}}{\Delta t}\right|=\left|\frac{0.800 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}-3.00 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}}{1.5 \mathrm{~s}}\right|=1.47 \mathrm{Nm} \\
& \tau=I \alpha \\
& \alpha=\frac{\tau}{I}=\frac{1.47 \mathrm{Nm}}{0.140 \mathrm{~kg} \mathrm{~m}^{2}}=-10.48 \mathrm{rad} / \mathrm{s}^{2} \\
& \omega_{i}=\frac{L_{i}}{I}=\frac{3.00 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}}{0.140 \mathrm{~kg} \mathrm{~m}^{2}}=21.48 \mathrm{rad} / \mathrm{s} \\
& \theta_{f}=\theta_{i}+\omega_{i} t+\frac{1}{2} \alpha t^{2}=0+21.48 \mathrm{rad} / \mathrm{s} \cdot 1.5 \mathrm{~s}+\frac{1}{2}\left(-10.48 \mathrm{rad} / \mathrm{s}^{2}\right)(1.5 \mathrm{~s})^{2} \\
&=20.43 \mathrm{rad}
\end{aligned}
$$

11.45 A man stands on a platform that is rotating (without friction) win an angular speed of 1.2 $\mathrm{rev} / \mathrm{s}$; his arms are outstretch and he hold a brick in each hand. The rotational inertia of the system consisting of the man, bricks and platform about the central vertical axis of the platform is $6.0 \mathrm{~kg} \mathrm{~m}^{2}$. If by moving the bricks, the man decreases the rotational inertia of the system to $2.0 \mathrm{~kg} \mathrm{~m}^{2}$, what are (a) the resulting angular speed of the platform and (b) the ratio of the new kinetic energy of the system to the original kinetic energy? (c) What source provided the added kinetic energy?

This is an angular momentum conservation problem. Since all of the forces and torques are internal, the angular momentum cannot change. First we'll use conservation of angular momentum to find the final angular speed (a).

$$
\begin{aligned}
L_{f} & =L_{i} \\
I_{f} \omega_{f} & =I_{i} \omega_{i} \\
\omega_{i} & =\frac{1.2 \mathrm{rev}}{\mathrm{~s}} \cdot \frac{2 \pi \mathrm{radians}}{\mathrm{rev}}=7.54 \mathrm{radians} / \mathrm{s} \\
\omega_{f} & =\frac{I_{i}}{I_{f}} \omega_{i}=\frac{6 \mathrm{~kg} \mathrm{~m}^{2}}{2 \mathrm{~kg} \mathrm{~m}^{2}} \cdot 7.54 \mathrm{radians} / \mathrm{s}=22.62 \mathrm{radians} / \mathrm{s}
\end{aligned}
$$

(b) Ratio of Kinetic Energies

$$
\left.\left.R=\frac{K E_{f}}{K E_{i}}=\frac{\frac{1}{2} I_{f} \omega_{f}^{2}}{\frac{1}{2} I_{i} \omega_{i}^{2}}=\frac{\frac{1}{2} \cdot(2 \mathrm{~kg} \mathrm{~m}}{}{ }^{2}\right)(22.62 \mathrm{radians} / \mathrm{s})^{2}\right)=3
$$

(c) Moving the bricks required work. The man's arms did work.
11.46 The rotational inertial of a collapsing spinning star changes to $1 / 3$ its initial value. What is the ratio of the new rotational kinetic energy to the initial kinetic energy?

$$
\begin{aligned}
& I_{f} \omega_{f}=I_{i} \omega_{i} \\
& \omega_{f}=\frac{I_{i}}{I_{f}} \omega_{i}=\frac{I_{i}}{\left(I_{i} / 3\right)} \omega_{i} \\
& \omega_{f}=3 \omega_{i}
\end{aligned}
$$

