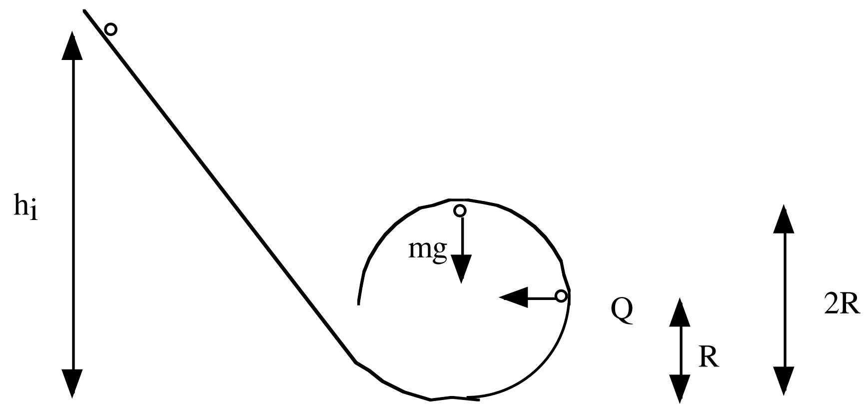
Chapter 11

11.8 In the Figure, a solid brass ball of mass 0.280 g will roll smoothly along a loop-the-loop track when released from rest along the straight section. The circular loop has radius $R = 14.0cm$ and the ball has radius $r \ll R$. (a) What is h if the ball is on the verge of leaving the track when it reaches the top of the loop if the ball is release at height $h = 6.00R$, what are the (b) magnitude and (c) direction of the horizontal force component acting on the ball at point Q.



When the ball is just about to come off the track, the centripetal force is entirely due to gravity. This allows us to solve for the velocity

$$\frac{mv^2}{R} = mg$$
$$v^2 = Rg$$

Now that we know the velocity, we can use energy conservation. The initial state is where we release the ball, the final state is at the top of the loop.

$$E_i = mgh \qquad\qquad E_f = \frac{1}{2}mv^2 + \frac{1}{2}I\,\omega^2 + mg(2R)$$

$$
\begin{aligned}
E_i &= E_f \\
mgh &= mg(2R) + \frac{1}{2}mv^2 + \frac{1}{2}I\,\omega^2 \\
mgh &= mg(2R) + \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}mr^2)\frac{v^2}{r^2} \\
mgh &= mg(2R) + \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}m)v^2 \\
h &= \frac{g(2R) + \frac{1}{2}v^2 + \frac{1}{2}(\frac{2}{5})v^2}{g} \\
&= \frac{g(2R) + \frac{1}{2}Rg + \frac{1}{2}(\frac{2}{5})Rg}{g} \\
&= (2R) + \frac{1}{2}R + \frac{1}{5}R \\
&= \frac{27}{10}R
\end{aligned}
$$

In the second part of this problem, we need the velocity so that we can compute the centripetal force again, since it is the net inward force. In this case, we know the initial height. The force at Q will be to the left.

$$
\begin{aligned}
E_i &= E_f \\
mg(6R) &= mg(R) + \frac{1}{2}mv^2 + \frac{1}{2}I\,\omega^2 \\
mg(6R) &= mg(R) + \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}mr^2)\frac{v^2}{r^2} \\
mg(5R) &= \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}m)v^2 \\
v^2 &= \frac{g(5R)}{\frac{1}{2} + \frac{1}{2}(\frac{2}{5})} \\
&= \frac{10}{7}\cdot g(5R) \\
&= \frac{50}{7}gR
\end{aligned}
$$

$$
\begin{aligned}
F &= \frac{mv^2}{R} = \frac{m\cdot\frac{50}{7}gR}{R} \\
&= \frac{50}{7}m\,g
\end{aligned}
$$

11.9 A solid cylinder of radius 10cm and mass 12kg starts from rest and rolls without slipping a distance of 6m down a house roof that is inclined at 30 degrees (a) What is the angular speed of the cylinder about its center as it leaves the house roof? (b) The roof's edge is 5.0m. How far horizontally from the roof's edge does the cylinder hit the level ground.

This problem is very similar to the previous problem. We begin with energy conservation. Take the ground as zero height.

$$H = 5m + 6m\sin 30 = 8m \qquad h = 5m$$

$$E_i = mgH \qquad E_f = mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$E_f = E_i$$

$$mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$$

$$mgh + \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}mR^2)\frac{v^2}{R^2} = mgH$$

$$\frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}m)v^2 = mg(H-h)$$

$$\frac{1}{2}v^2 + \frac{1}{4}v^2 = g(H-h)$$

$$\frac{3}{4}v^2 = g(H-h)$$

$$v = \sqrt{\frac{4}{3}g(H-h)} = \sqrt{\frac{4}{3}\cdot 9.8m/s^2(8m-5m)}$$

$$= 6.26m/s$$

$$\omega = \frac{v}{r} = \frac{6.26m/s}{0.1m} = 62.6rad/s$$

This problem differs from the previous problem in that the cylinder is not launched horizontally. It has an initial velocity in the y direction as well as in the x direction...

$$x_i = 0 \qquad y_i = h$$

$$x_f = ? \qquad y_f = 0$$

$$v_{ix} = 6.26\cos 30 = 5.42m/s \qquad v_{iy} = -6.26\sin 30 = -3.13m/s$$

$$v_{fx} = v_{ix} \qquad v_{fy} = ?$$

$$a_x = 0 \qquad a_y = -g$$

$$t = ?$$

Again we find the time

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$

$$0 = h + v_{iy}t - \frac{1}{2}g t^2$$

$$t = \frac{-v_{iy} \pm \sqrt{v_{iy}^{\ 2} - 4 \cdot (-\frac{1}{2}g) \cdot h}}{2 \cdot (-\frac{1}{2}g)} = \frac{-(-3.13) \pm \sqrt{(-3.13)^2 - 4 \cdot (-\frac{1}{2} \cdot 9.8) \cdot 5}}{2 \cdot (-\frac{1}{2} \cdot 9.8)}$$

$$= 0.74 s$$

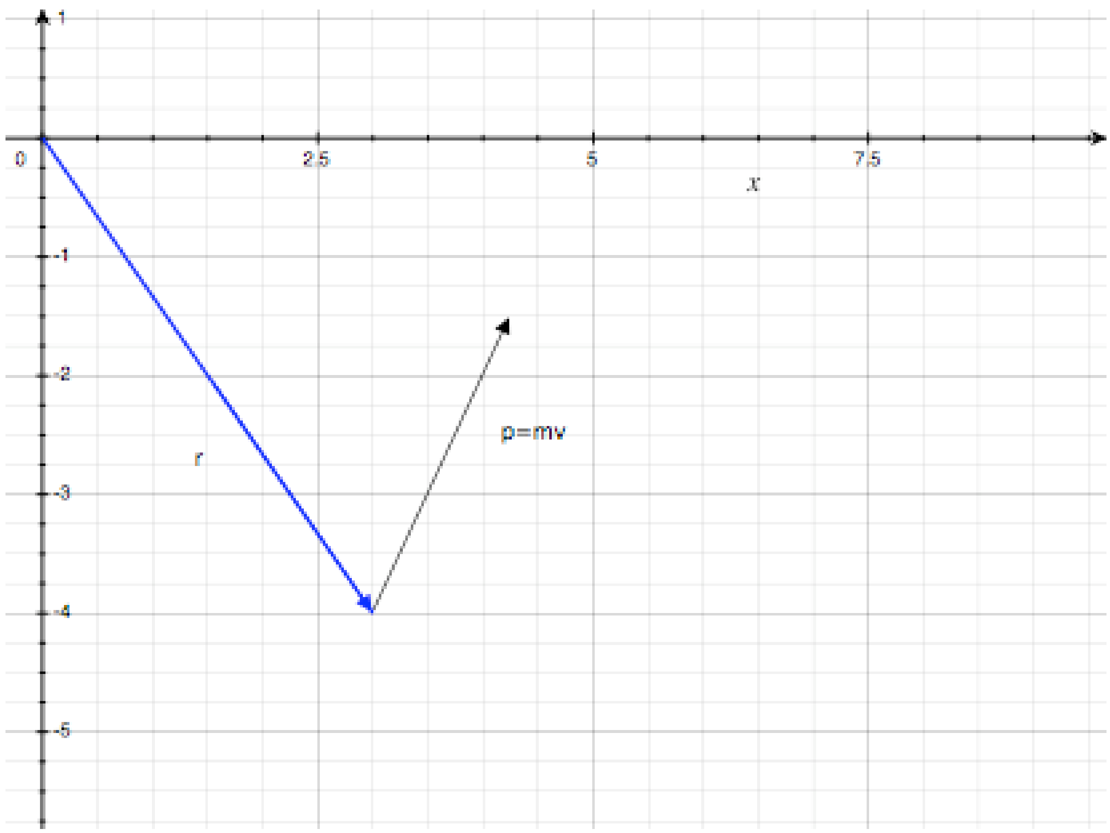
Again we find the horizontal distance

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$
$$= 0 + v_{ix}t + 0$$
$$x_f = 5.42 m/s \cdot 0.74 s$$
$$= 4.01 m$$

This problem is considerably more complicated because the projectile is not launched horizontally.

11.26 A 2.0 kg particle-like object moves in a plane with a velocity components $v_x = 30 m/s$ and $v_y = 60 m/s$ as it passes through the point with (x,y) coordinates of (3.0, -4.0) m. Jus then, in unit-vector notatin, what is the angular momentum and (b) the point (-2.0, -2.0) m?

We can draw what is happening in each case. The r is defined as coming from the axis and ending where the force is applied. In the first case

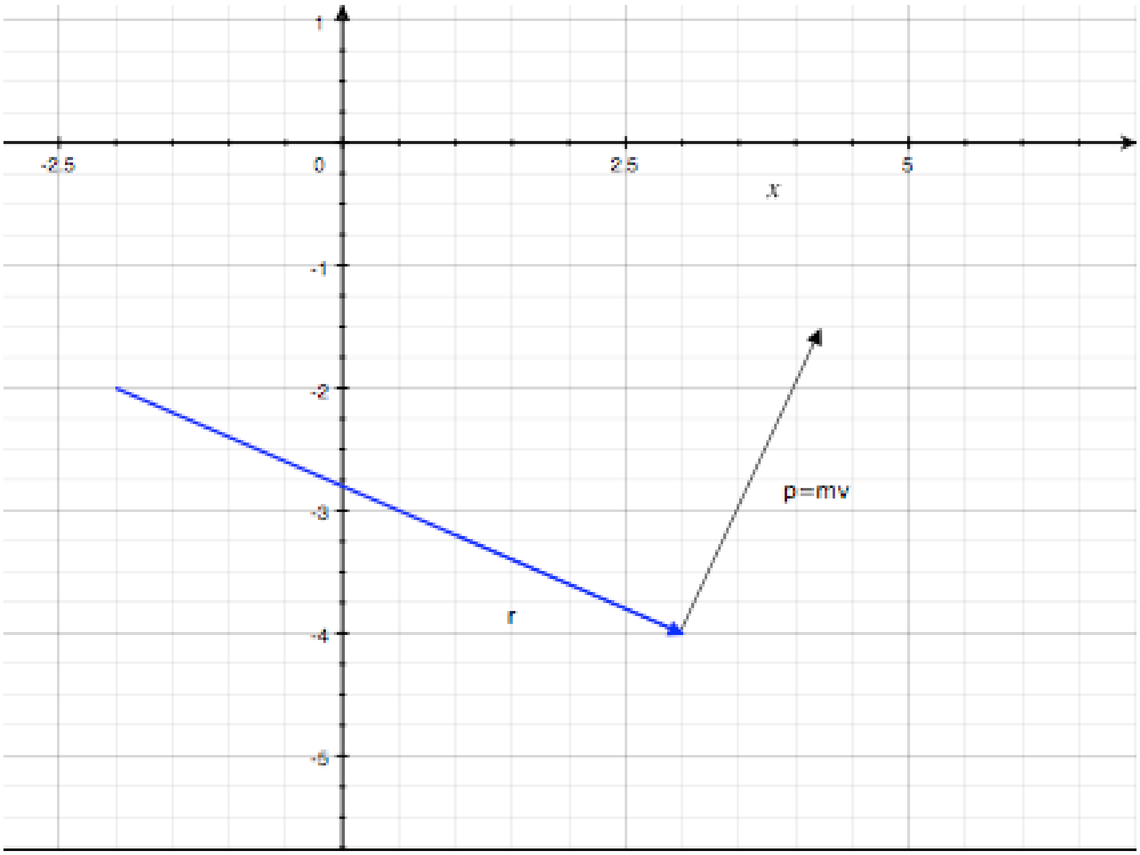


$$\vec{r} = (3.0 - 0.0)\hat{i} + (-4.0 - 0.0)\hat{j} = 3.0m\,\hat{i} - 4.0m\,\hat{j}$$

$$\vec{p} = m\vec{v} = 60kg\,m/s\,\hat{i} + 120kg\,m/s\,\hat{j}$$

$$L = \vec{r} \times \vec{p} = \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 0 \\ 60 & 120 & 0 \end{bmatrix} = (3 \cdot 120 - (-4) \cdot 60)\hat{k} = 600\,kgm^2/s\,\hat{k}$$

In the second case, the axis is at (-2,2), so we need to recalculate r.



$$\vec{r} = (3.0-(-2.0))\hat{i} + (-4.0-(-2.0))\hat{j} = 5.0m\ \hat{i} - 2.0m\ \hat{j}$$

$$\vec{p} = m\vec{v} = 60kg\,m/s\ \hat{i} + 120kg\,m/s\ \hat{j}$$

$$L = \vec{r}\times\vec{p} = \det\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & -2 & 0\\ 60 & 120 & 0\end{bmatrix} = (5\cdot 120-(-2)\cdot 60)\hat{k} = 720kgm^2/s\,\hat{k}$$

11.36 A sanding disk with rotational inertia $I = 1.2\times 10^{-3} kg\,m^2$ is attached to an electric dril whose motor delivers a torque of magnitued 16Nm about the central axis of the disk. About that axis and with the torque applied ro 33ms, what is the magnitued of the angular momentum and angular velocity of the disk.

We begin by finding the angular velocity after 33ms. We begin by finding the angular acceleration.

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{16Nm}{1.2\times 10^{-3} kg\,m^2} = 1.33\times 10^4 rad/s^2$$

Now find the angular velocity

$$\omega_f = \omega_i + \alpha t = 0 + 1.33\times 10^4 \cdot 0.033 s = 440\, rad/s$$

Now we can find the angular momentum.

$$L = I\,\omega_f = 0.528\, kg\, m^2/s$$

11.37

11.43

11.48 The rotational inertial of a collapsing spinning star changes to 1/3 its initial value. What is the ratio of the new rotational kinetic energy to the initial kinetic energy?

$$I_f\,\omega_f = I_i\,\omega_i$$

$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{I_i}{(I_i/3)}\omega_i$$

$$\omega_f = 3\omega_i$$

$$R = \frac{\cancel{\tfrac{1}{2}} I_f\,\omega_f^{\,2}}{\cancel{\tfrac{1}{2}} I_i\,\omega_i^{\,2}} = \frac{\cancel{\tfrac{1}{2}}(I_i/3)(3\omega_i)^2}{\cancel{\tfrac{1}{2}} I_i\,\omega_i^{\,2}} = 3$$