Chapter 10

10.9 A disk, initially rotating at 120 rad/s is slowed down with a constant angular acceleration of magnitude of $4.0\, rad/s^2$ (a) How much time does the disk take to stop? (b) Through what angle does the disk rotate during that time?

$$\theta_i = 0$$
$$\theta_f = ?$$
$$\omega_i = 120\ rad/s$$
$$\omega_f = 0$$
$$\alpha = -4\ rad/s^2$$
$$t = ?$$

$$\omega_f = \omega_i + \alpha t$$
$$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0\, rad/s - 120\, rad/s}{-4.0\, rad/s^2}$$
$$= 30s$$
$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$
$$\theta_f = 0 + 120 rad/s \cdot 30s + \frac{1}{2} \cdot -4 rad/s^2 \cdot (30s)^2$$
$$= 1800\, rad$$

10.12 Starting from rest, a disk rotates about its central axis with constant angular acceleration. In 5.0 s, it rotates 25 rad. During that time, what are the magnitudes of (a) the angular acceleration and (b) the average angular velocity? (c) What is the instantaneous angular velocity of the disk at the end of the 5.0 s? (d) With the angular acceleration unchanged, through what additional angle will the disk turn during the next 5s.

This problem is the angular analog of many of the constant linear acceleration problems that we did.
Parts (a) and (b) ... the angular acceleration and average angular velocity.

$$\theta_i = 0$$
$$\theta_f = 25 rad$$
$$\omega_i = 0\ rad/s$$
$$\omega_f = ?$$
$$\alpha = ?$$
$$t = 5s$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$
$$\theta_f = 0 + 0 + \frac{1}{2}\alpha t^2$$
$$\alpha = \frac{2\theta_f}{t^2} = \frac{2 \cdot 25 rad}{(5s)^2} = 2 rad/s^2$$
$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{25 rad - 0 rad}{5s - 0s} = 5 rad/s$$

We can compute the instantaneous angular velocity...part (c)

$$\omega_f = \omega_i + \alpha t$$
$$= 0 rad/s + 2 rad/s^2 \cdot 5s$$
$$= 10 rad/s$$

Through what additional angle will the disk turn through. We'll consider our starting angle at t=5s as zero so that we can easily compute how far the disk turns in the next 5s. We need to remember that the disk has an initial angular velocity for the second 5s.

$$\theta_i = 0$$
$$\theta_f = ?$$
$$\omega_i = 10\ rad/s \qquad \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$
$$\omega_f = ?$$
$$\alpha = 2 rad/s^2 \qquad \theta_f = 0 + 10 rad/s \cdot 5s + \frac{1}{2}\cdot 2 rad/s^2 \cdot (5s)^2$$
$$t = 5s \qquad = 75 rad$$

10.18 A vinyl record is played by rotating the record so that an approximately circular groove in the vinyl slides under a stylus. Bumps in the groove run into the stylus causing it to oscillate. The equipment convert those oscillation to electrical signals and then to sound. Suppose that a record turns at the rate of 33 1/3 rev min, the groove being played is at a radius of 10. cm and the bumps in the groove are uniformly separated by the 1.75 mm. At what rate (hits/second do the bumps hit the stylus.

The easiest way to do this is to consider one complete turn of the record. If we know how many bumps are in one complete turn and how long a turn takes, we will be able to compute hits/sec.

$$circumference = (space/bump) \times (number\ of\ bumps\ in\ one\ turn)$$
$$(number\ of\ bumps\ in\ one\ turn) = \frac{circumference}{space/bump} = \frac{2\pi \cdot 10cm}{0.175cm} = 359$$

$$\frac{33\frac{1}{3} rev}{60s} = \frac{1 rev}{T}$$
$$T = \frac{60s}{33\frac{1}{3}} = 1.8s$$

$$\frac{bumps}{\text{sec}} = \frac{359}{1.8s} = 199.4 \approx 200$$

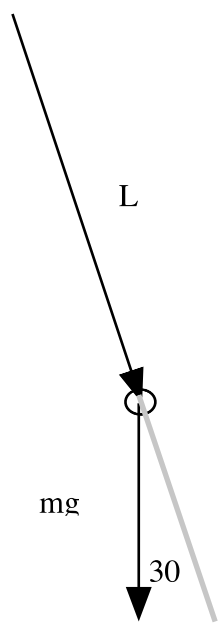
10.33 Calculate the rotational inertia of a wheel that has a kinetic energy of 24,400J when rotating at 602 rev/min.

$$\omega = \frac{602 rev}{\min} \cdot \frac{2\pi\, rad}{1 rev} \cdot \frac{1\min}{60 s} = 63.04 rad / s$$

$$KE_R = \frac{1}{2} I\omega^2$$

$$I = \frac{2KE_R}{\omega^2} = \frac{2 \cdot 24{,}400 J}{(63.04 rad / s)^2} = 12.28 kgm^2$$

10.45 A small ball of mass 0.75 kg is attached to one end of a 1.25 m long massless rod and the other end of the rod is hung from a pivot. When the resulting pendulum is 30 degrees from the vertical, what is the magnitude of the gravitational torque calculated about the pivot



$$|\tau| = r F \sin\theta = L \cdot mg \cdot \sin 30° = 4.59 Nm$$

10.50 If a 32.0 Nm torque on a wheel causes angular acceleration of $\alpha = 25 rad / s^2$, what is the wheel's rotational inertia?

$$\tau = I\alpha$$

$$I = \frac{\tau}{\alpha} = \frac{32 Nm}{25 rad / s^2} = 1.28 kgm^2$$

10.59 A 32.0 kg wheel, essentially a thin hoop with radius 1.2 m is rotating at 280 rev/min. It must be brought to a stop in 15s. (a) How much work must be done to stop it? (b) What is the required power?

To proceed, we need to compute the angular velocity in rad/s and we need to know the moment of inertia.

$$\omega_i = \frac{280rev}{\min} \cdot \frac{2\pi\ rad}{1rev} \cdot \frac{1\min}{60s} = 29.32rad/s$$

$$\omega_f = 0rad/s$$

$$I = mr^2 = 32kg \cdot (1.2m)^2 = 46.08kgm^2$$

$$W = \Delta KE = KE_f - KE_i = \frac{1}{2}I\omega_f^{\ 2} - \frac{1}{2}I\omega_i^2 = 0 - \frac{1}{2} \cdot 46.08kgm^2 \cdot (29.32rad/s)^2$$

$$= -19{,}806.6J$$

$$P_{avg} = \frac{|W|}{t} = \frac{19{,}806.6J}{15s} = 1320.4W$$

10.66 A uniform spherical shell of mass M and radius R rotates about a vertical axis on frictionless bearings (Fig 11-45). A massless cord passes around the equator of the shell over a pulley of rotational inertia I and radius r and is attached to a small object of mass m. There is no friction on the pulley's axle; the cord does not slip on the pulley. What is the speed of the object after it falls a distance h form rest. use energy considerations.

This is an energy conservation problem

$$E_i = mgh \qquad\qquad E_f = \frac{1}{2}mv^2 + \frac{1}{2}I_{sph}\,\omega_{sph}^{\ \ 2} + \frac{1}{2}I_p\,\omega_p^{\ 2}$$

Now set the final energy equal to the initial energy...

$$E_f = E_i$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I_{sph}\,\omega_{sph}^{\ \ 2} + \frac{1}{2}I_p\,\omega_p^{\ 2} = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{3}MR^2)\frac{v}{R^2}^2 + \frac{1}{2}I_p\frac{v}{r^2}^2 = mgh$$

$$v^2(m + \frac{2}{3}M + \frac{I_p}{r^2}) = 2mgh$$

$$v = \sqrt{\frac{2mgh}{(m + \frac{2}{3}M + \frac{I_p}{r^2})}}$$

10.69 Similar to problem done in class.