Physics 132

Short Answer

1. Explain how the Hall effect can be used to determine the sign of the charge carriers in a metal conductor.

2. A proton with a velocity v=1x10^6 m/s i enters a region of uniform magnetic field given by B=0.1T j. What are the magnitude and direction of the force on the proton?

$$\begin{aligned}\vec{F} &= q\vec{v}\times\vec{B}\\ &= 1.6\times10^{-19}C\cdot 1.0\times10^{6}m/s\,\hat{i}\times 0.1T\ \hat{j}\\ &= 1.6\times10^{-14}N\ \hat{k}\end{aligned}$$

3. The proton in the previous problem is seen to travel in a circular path. What is the radius of that path?

$$\frac{mv^2}{r} = 1.6\times10^{-14}N$$

$$r = \frac{mv^2}{1.6\times10^{-14}N} = \frac{1.67\times10^{-27}kg\cdot(1\times10^{6}m/s)^2}{1.6\times10^{-14}N}$$

4. A current carrying wire makes an angle of 30 degrees with respect to the x axis in the xy plane. A magnetic field is present with the form B=0.1 **i**-0.2 **j**+0.3 **k**. If the wire carries a current of 5 Amps and has a length of 0.5 m, what force does the wire experience.

$$\begin{aligned}\vec{l} &= 0.5\cos30\hat{i} + 0.5\sin30\hat{j} = 0.433\hat{i} + 0.25\hat{j}\\ F &= i\vec{l}\times\vec{B}\\ &= 5\cdot(0.433\hat{i} + 0.25\hat{j})\times(0.1\hat{i} - 0.2\hat{j} + 0.3\hat{k})\end{aligned}$$

5. A square loop with side 0.5 m and 10 turns of wire is oriented so that a normal to its plane makes an angle of 60 degrees with respect to a uniform 1 T field. Compute the magnetic dipole, the energy and the torque on the dipole.

$$\begin{aligned}\mu &= N\,i\,A = 10\cdot i\cdot(0.5m)^2 = 2.5i\\ U &= -\vec{\mu}\cdot\vec{B} = -2.5i\cdot 1T\cdot\cos60\\ \vec{\tau} &= \vec{\mu}\times\vec{B}\\ \tau &= \mu B\sin\theta = 2.5i\cdot 1T\cdot\sin60\end{aligned}$$

6. A 100 ohm resistor is connected to a 10 volt power supply. Compute what the current is if the power supply is an ideal battery. What will happen if the the battery is not ideal and why?

If the battery is ideal, the current would be...

$$i = \frac{\varepsilon}{R} = \frac{10V}{100\Omega} = 0.1A$$

7. State Kirchoff's two laws clearly.

1. The sum of the voltage differences around any closed loop in a circuit must be zero.
2. The net flow of current into a junction must equal the net flow of current out of the junction.

8. State the voltage convention that is used with Kirchoff's voltage law.

1. If the analysis path crosses a resistor in the *same* direction as the current flows through the resistor, then $V = -iR$

2. If the analysis path crosses a resistor in the *opposite* direction as the current flows through the resistor, then $V = iR$

3. If the analysis path crosses an emf from the short (negative) side to the long (positive) side, then $V = +\varepsilon$.

4. If the analysis path crosses an emf from the long (positve) side to the short (negative) side, then $V = -\varepsilon$.

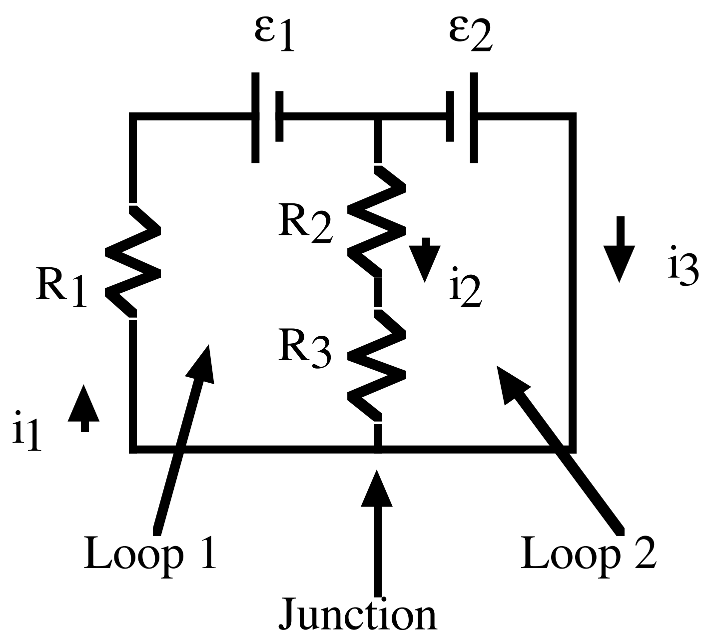
9. A wire carries a current of 1 A in the +x direction and a B field of 0.005 T points in the +y direction. What mass can the wire have and be suspended so that the magnetic force exactly cancels its weight. (Weight acts in the -z direction.)

$$mg = ilB$$

$$m = \frac{ilB}{g}$$

Problems.

1. Consider the circuit below



R1 =100 Ohms, R2 = R3 = 75 Ohms, ε1 =9 V and ε2 = 6 V.

a) Using the loops indicated, write Kirchoff's voltage loop rule for each loop.

$$-i_1R_1 - \varepsilon_1 - i_2R_2 - i_2R_3 = 0$$

Loop 2, analyzed in the clockwise direction

$$+i_2R_3 + i_2R_2 + \varepsilon_2 = 0$$

b) Apply Kirchoff's junction law to the junction indicated.

$$i_1 = i_2 + i_3$$

c) Solve for the currents.

$$+i_2R_3 + i_2R_2 + \varepsilon_2 = 0$$
$$i_2 = -\frac{\varepsilon_2}{R_3 + R_2} = -\frac{6V}{75\Omega + 75\Omega} = -0.04A$$
$$-i_1R_1 - \varepsilon_1 - i_2R_2 - i_2R_3 = 0$$
$$i_1 = \frac{-\varepsilon_1 - i_2R_2 - i_2R_3}{R_1} = \frac{-9V - (-0.04A)\cdot(75\Omega + 75\Omega)}{100\Omega} = -0.03A$$
$$i_1 = i_2 + i_3$$
$$i_3 = i_1 - i_2 = -0.03A - (-0.04A) = 0.01A$$

2. Completely ionized Uranium nuclei are accelerated through a potential difference of 1 MV

a) What energy does the U obtain in the acceleration.

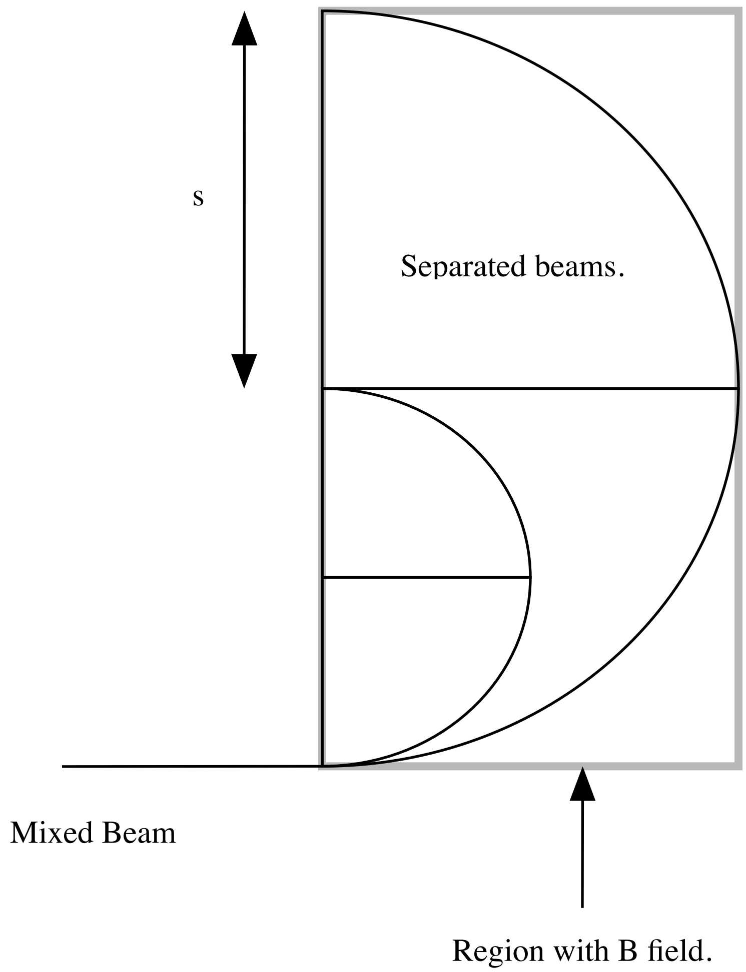
$$\begin{aligned} U &= q\Delta V = 92e\cdot 1MV = 92MeV \\ &= 92\cdot 1\times 10^{6}\cdot 1.6\times 10^{-19}J \\ &= 1.472\times 10^{-11}J \end{aligned}$$

b) There are two isotopes: ^{235}U and ^{238}U. What velocity does each ion have?

$$U = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2U}{m}}$$
$$v_{235} = \sqrt{\frac{2U_{235}}{m_{235}}} = \sqrt{\frac{2\cdot 1.472\times 10^{-11}J}{235\cdot 1.67\times 10^{-27}kg}} = 8.66\times 10^{6}m/s$$
$$v_{238} = \sqrt{\frac{2U_{238}}{m_{238}}} = \sqrt{\frac{2\cdot 1.472\times 10^{-11}J}{238\cdot 1.67\times 10^{-27}kg}} = 8.61\times 10^{6}m/s$$

c) The beam is directed into a region of uniform magnetic field. How can this be used to separate the isotopes. How large does the region need to be so that both beams bend through 180 degrees?

d) What separation do the beams have after bending through 180 degrees?



$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB}$$

$$r_{235} = \frac{m_{235}v_{235}}{qB} = \frac{235 \cdot 1.67 \times 10^{-27}\,kg \cdot 8.66 \times 10^{6}\,m/s}{92 \cdot 1.6 \times 10^{-19}\,C \cdot 1T} = 0.2309m$$

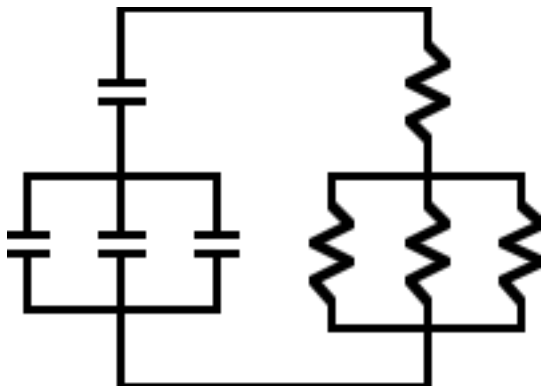
$$r_{238} = \frac{m_{238}v_{238}}{qB} = \frac{238 \cdot 1.67 \times 10^{-27}\,kg \cdot 8.61 \times 10^{6}\,m/s}{92 \cdot 1.6 \times 10^{-19}\,C \cdot 1T} = 0.2324m$$

$$s = 2 \cdot 0.2324m - 2 \cdot 0.2309m = .003m$$

e) What speeds to the ions have after the bend through 180 degrees?

The velocities are unchanged. The force is radially inward and does no work.

3. Consider the arrangement of capacitors and resistors below.



a) Find the equivalent capacitance and resistance and redraw the circuit using them. All C's are 3 milliFarad and all resistors are 1000 Ohms.

Parallel Capacitors

$$C_{eq} = C + C + C = 9mF$$

Parallel Resistors

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{1000\Omega} \Rightarrow R_{eq} = 333.3\Omega$$

Eq. Cap in series with last capacitor

$$\frac{1}{C_{eq'}} = \frac{1}{C} + \frac{1}{C_{eq}}$$

$$= \frac{1}{3mF} + \frac{1}{9mF}$$

$$= \frac{4}{9mF}$$

$$C_{eq'} = \frac{9}{4}mF$$

Eq. Resistor in series with last resistor

$$R_{eq'} = R + R_{eq} = 1000\Omega + 333.3\Omega = 1333.3\Omega$$

b) The equivalent capacitance is charged to 9 Volts. How much charge is stored on it? How much energy is stored in it.

$$q_0 = CV = \frac{9}{4}\times 10^{-3}F\cdot 9V = \frac{81}{4}\times 10^{-3}C$$
$$U = \frac{1}{2}CV^2 = \frac{1}{2}\cdot\frac{9}{4}\times 10^{-3}F\cdot (9V)^2 = \frac{729}{8}\times 10^{-3}J$$

c) The equivalent capacitance is allowed to discharge through the equivalent resistance. What is the 1/e time for this process? What equation describes the charge on the capacitor? What equation describes the current through the resistor?

$$t_{1/e} = RC = 1333.3\Omega\cdot\frac{9}{4}\times 10^{-3}F = 3s$$
$$q = q_0 e^{-t/t_{1/e}}$$
$$i = \frac{dq}{dt} = -\frac{q_0}{t_{1/e}}e^{-t/t_{1/e}}$$

d) Sketch: the charge on the capacitor verses time, the current through the equivalent resistor as a function of time and the power used by the resistor as a function of time.

All three curves have the same general exponential shape. Their slopes are different, but this is only a sketch.