Exam 4

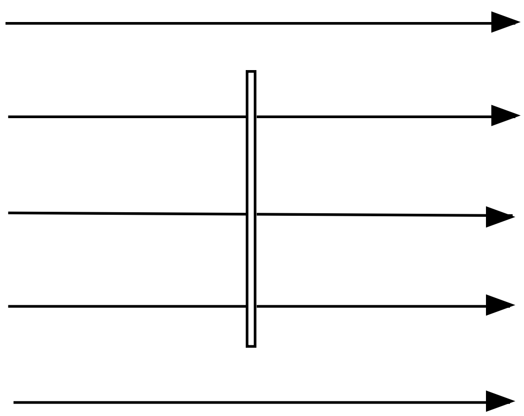
Please answer all of the Short Answer questions below.

1. An airplane flies through a magnetic field of $0.5\times10^{-3}T$ that is perpendicular to its wings at a constant speed. If the wingspan is 30m, and an EMF of 1V develops across the wings, what is its speed? Hint: Treat this as a bar moving through a field.

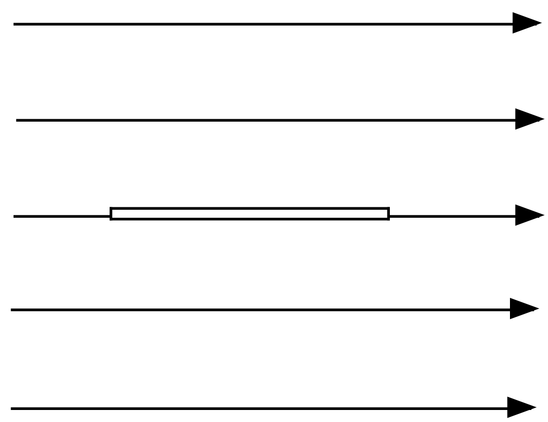
$$\varepsilon = Blv$$

$$v = \frac{\varepsilon}{Bl} = \frac{1V}{0.5\times10^{-3}T\cdot30m} = 66.67m/s$$

2. A uniform magnetic field of 4T passes through a square loop of wire with 100 turns. The loop has a side of 0.5m . Initially, it is oriented at 0 degrees with respect to the field(left picture). What is the magnetic flux through the loop? The loop is then turned to an orientation 90 degrees (right picture). What is the flux when the loop is in this orientation? Over what time period Δt does this turn need to occur for the loop to develop an emf of 1 Volt.



Initial State



Final State

$$\varphi = BA\cos\theta$$

$$\varphi_i = BA\cos0^\circ = (4T)\cdot(100\cdot(0.5m)^2)\cos0^\circ = 100Tm^2$$

$$\varphi_f = BA\cos90^\circ = 0Tm^2$$

$$|\varepsilon| = \left|-\frac{\Delta\varphi}{\Delta t}\right|$$

$$\Delta t = \left|-\frac{\Delta\varphi}{\varepsilon}\right| = \left|\frac{100Tm^2}{1V}\right| = 100s$$

3. A cylindrical solenoid is 2m long and has 100,000 turns ($N = 100{,}000$). Use Ampere's Law to calculate the field inside the solenoid if the current is 10A? Using the field, compute the energy density of the magnetic field.

$$n = \frac{100,000 turns}{2m} = 50,000\ turns/m$$

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 i_{enc}$$

$$Bl = \mu_0 N_{end}\, i = \mu_0\, nl\, i$$

$$B = \mu_0\, n\, i = 0.628T$$

$$u_B = \frac{1}{2\mu_0} B^2 = 1.57 \times 10^5\, J/m^3$$

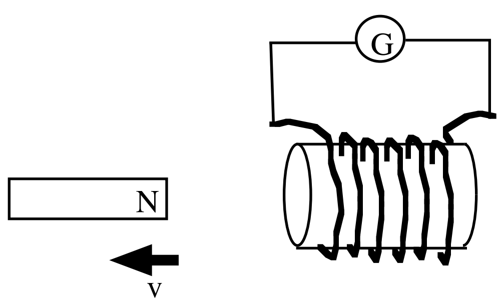
4. Use *Ampere's Law* to find the field outside a long beam of current. Assume that the beam has current density J and radius a. *You are calculating for r>a.*

$$\oint \vec{B} \cdot d\vec{r} = \mu_0\, i$$

$$B \cdot (2\pi r) = \mu_0\, J\, \pi\, a^2$$

$$B = \frac{\mu_0\, J\, a^2}{2r}$$

5. The **north** end of a magnet is **moved out** a coil connected to a galvanometer as shown below. Assuming that the needle of the galvanometer deflects toward the side that current enters it, which way will the needle point due to the induced current and why. Note: We did this in lab.



The magnetic flux due to the North pole points to the right, and as the North pole is moved away, the flux passing through the solenoid decreases. The solenoid will respond by trying to replace the lost flux to the right. To do this, a current will be induced that flows down the front of the solenoid. The current will from from right to left through the galvanometer, so the needle will deflect right.

6. Write Maxwell's equations and briefly explain each equation.

$$(a)\quad \oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$$

$$(b)\quad \oint \vec{E}\cdot d\vec{r} = -\frac{d\varphi_B}{dt}$$

$$(c)\quad \oint \vec{B}\cdot d\vec{A} = 0$$

$$(d)\quad \oint \vec{B}\cdot d\vec{r} = \mu_0 i_{encl} + \mu_0\varepsilon_0\frac{d\varphi_E}{dt}$$

(a) Gauss' Law for E: The total flux of E through an enclosed surface is proportional to the charge enclosed

(b) A changing magnetic flux induces an electric field.

(c) Gauss' Law for B: There is no free magnetic charge.

(d) A magnetic field is produced by either a current or a changing electric flux.

7. If the inductor in an RLC circuit is 500mH, the capacitance is 0.1 microFarads and the resistance is 1000 Ohms, what would X_L, X_C and Z be for this circuit at $\omega = 4{,}000s^{-1}$. Is this circuit at resonance?

$$X_L = \omega L = 4000s^{-1}\cdot 0.5H = 2000\Omega$$

$$X_C = \frac{1}{\omega C} = 2500\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 1118\Omega$$

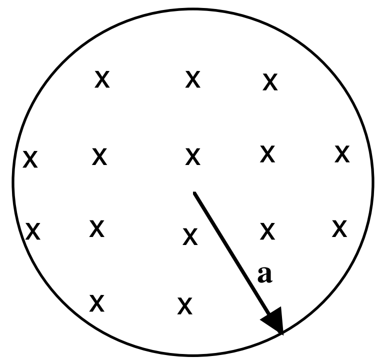
...not at resonance

8. You wish to tune in the Cubs on WGN, the Cubs' radio station. WGN's frequency f is 720 kHz. If the inductor in your circuit is 200 mH, what capacitor do you need? Note: Your radio is an RLC circuit.

$$2\pi f = \sqrt{\frac{1}{LC}}$$

$$C = \frac{1}{(2\pi f)^2 L} = 1.105\times 10^{-6}F$$

9. A uniform electric field has a magnitude given by $E = 10\,t^2 + 5$ and points into the page as shown in the picture. What is the changing *electric* flux $\frac{d\varphi_E}{dt}$ due to the changing electric field $\frac{dE}{dt}$ through a circle of radius a=1m? What is the displacement current and what magnetic field is produced at the edge of the circle.



Viewed from above.
E is pointing downward.

$$\varphi_E = E \cdot A = (10t^2 + 5) \cdot \pi a^2$$

$$\frac{d\varphi_E}{dt} = 20t \cdot \pi a^2$$

$$i_d = \varepsilon_0 \frac{d\varphi_E}{dt} = 20t \cdot \varepsilon_0 \cdot \pi a^2$$

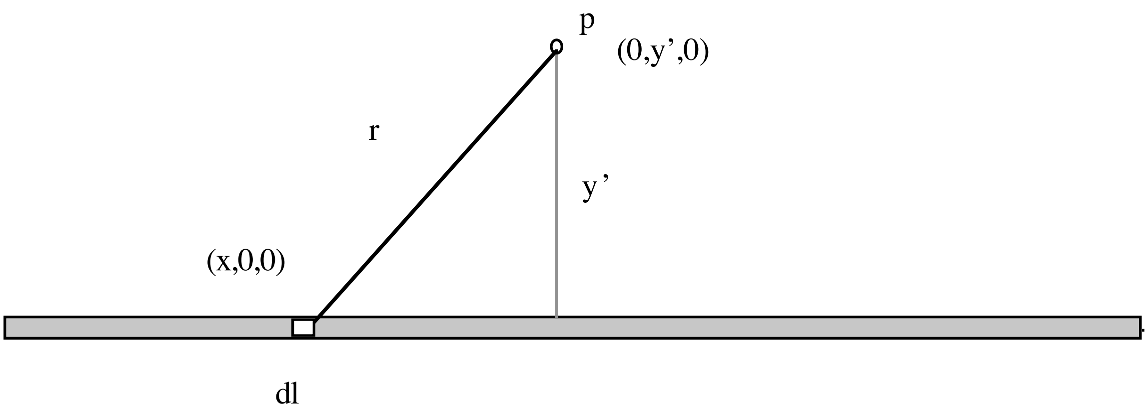
$$\oint \vec{B} \cdot d\vec{r} = \mu_0 i_d$$

$$B \cdot 2\pi r = \mu_0 i_d$$

$$B = \frac{\mu_0 i_d}{2\pi r}$$

Long Problems: Please work 2 of 3 long problems

1. A finite wire is shown below. It runs along the x axis from -L/2 to +L/2. We are interested in the B field at point P. Note: $\hat{i}\times\hat{j}=\hat{k}$. The current i runs in the +x direction.



a. Write a little length of current $d\vec{l}$ is in terms of dx. Be sure to indicate the direction.

$$d\vec{l} = dx\,\hat{i}$$

b. Write $\vec{r}, r, \hat{r}$.

$$\vec{r} = (0-x)\,\hat{i} + (y'-0)\,\hat{j}$$
$$r = \sqrt{x^2 + y'^2}$$
$$\hat{r} = \frac{-x\hat{i} + y'\hat{j}}{\sqrt{x^2 + y'^2}}$$

c. Write the Biot-Savart Law and use the results from (a) and (b) to set up the integration.

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{i\,d\vec{l}\times\hat{r}}{r^2}$$
$$= \frac{\mu_0 i}{4\pi}\int_{-L/2}^{+L/2} \frac{(dx\,\hat{i})\times\left(\frac{-x\hat{i}+y'\hat{j}}{\sqrt{x^2+y'^2}}\right)}{x^2+y'^2}$$
$$= \frac{\mu_0 i}{4\pi}\int_{-L/2}^{+L/2} \frac{(dx\,\hat{i})\times(-x\hat{i}+y'\hat{j})}{(x^2+y'^2)^{3/2}}$$
$$= \frac{\mu_0 i}{4\pi}\int_{-L/2}^{+L/2} \frac{y'\,dx\,\hat{k}}{(x^2+y'^2)^{3/2}}$$

d. Evaluate the integral to calculate the field. Note: you may need the integral. Hint: You may need:

$$\int_{-L/2}^{+L/2} \frac{dx}{(x^2+y'^2)^{3/2}} = \frac{x}{y'^2\sqrt{y'^2+x^2}}\Big]_{-L/2}^{+L/2} = \frac{L}{y'^2\sqrt{y'^2+(\frac{L}{2})^2}}$$

$$\vec{B} = \frac{\mu_0 i}{4\pi}\int_{-L/2}^{+L/2} \frac{y'\,dx\,\hat{k}}{(x^2+y'^2)^{3/2}}$$

$$B_z = \frac{\mu_0 i y'}{4\pi}\int_{-L/2}^{+L/2} \frac{dx}{(x^2+y'^2)^{3/2}} = \frac{\mu_0 i y'}{4\pi}\cdot\frac{L}{y'^2\sqrt{y'^2+(\frac{L}{2})^2}}$$

$$= \frac{\mu_0 i}{4\pi y'}\cdot\frac{L}{\sqrt{y'^2+(\frac{L}{2})^2}}$$

Bonus: Show that if you let the limits of integration go to infinity, you get the infinite wire result

$$B_z = \frac{\mu_0 i}{4\pi y'}\cdot\frac{L}{\sqrt{y'^2+(\frac{L}{2})^2}} = \frac{\mu_0 i}{4\pi y'}\cdot\frac{2L}{\sqrt{4y'^2+L^2}} =$$

$$B_z = \lim_{L\to\infty}\frac{\mu_0 i}{4\pi y'}\cdot\frac{2L}{\sqrt{4y'^2+L^2}}$$

$$B_z = \frac{\mu_0 i}{2\pi y'}$$

2. Consider an RLC circuit with R=200 Ohms, L = 600 mH and C=15 microF.

a. Write the differential equation that describes this circuit by using Kirchoff's voltage loop rule. What is the natural oscillating. frequency ω_0 ?

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = 333.33s^{-1}$$

b. Now consider driving this circuit. What are X_L,X_C, and Z for this circuit?

Take the driving frequency to be $\omega = 300s^{-1}$

$$X_L = \omega L = 300s^{-1} \cdot 0.6H = 180\Omega$$

$$X_C = \frac{1}{\omega C} = 222.2\Omega$$

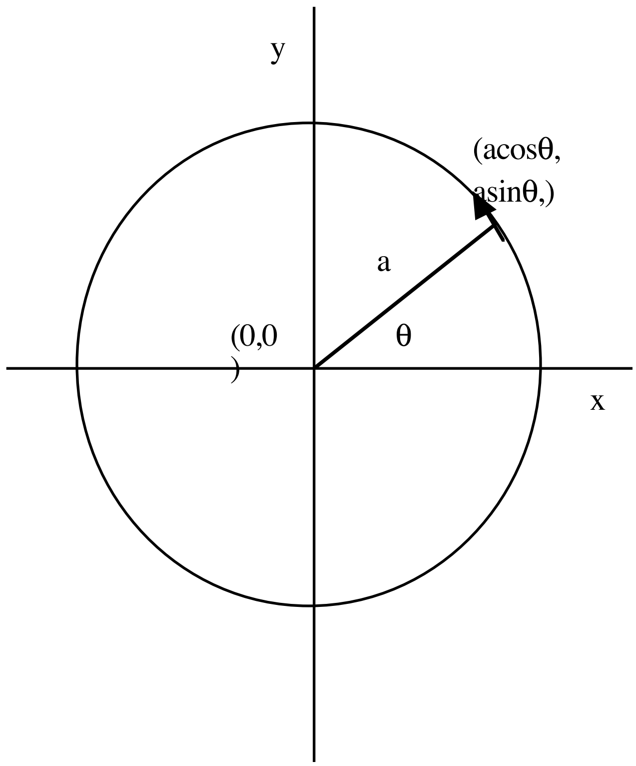
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 205.41\Omega$$

c. If the RMS Voltage is 120, what RMS current will flow in this circuit? What maximum current will flow.

$$i_{rms} = \frac{V_{rms}}{Z} = \frac{120V}{205.41\Omega} = 0.587A$$

$$i_0 = \sqrt{2}\, i_{rms} = 0.83A$$

3. Current flows in a circular loop as shown below



a. Sketch the direction of the field inside and outside the circle using our usual x or dot notation. What direction does the field at the center point in unit vector notation?

Field points out of the paper inside the circle and into the paper outside the circle.

Now derive an expression for the field at the center of the loop using the Biot-Savart Law. Take the little length of current to be given by:

$$d\vec{l} = (-\,a\sin\theta\,\hat{i} + a\cos\theta\,\hat{j})\,d\theta$$

b. Write the Biot-Savart Law.

$$\vec{B} = \frac{\mu_0 i}{4\pi}\int \frac{d\vec{l} \times \hat{r}}{r^2}$$

c. Write expressions for $\vec{r}$, r, and $\hat{r}$ using the coordinates of the center and the position little length of current as given by the picture.

$$\vec{r} = (0 - a\cos\theta)\hat{i} + (0 - a\sin\theta)\hat{j}$$
$$r = a$$
$$\hat{r} = \frac{\vec{r}}{r} = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$$

d. Substitute into the the Biot-Savart Law and integrate around the circle to get the field.
Note: $\hat{i}\times\hat{j} = \hat{k},\ \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = 0$ Also: $\sin^2\theta + \cos^2\theta = 1$

$$\vec{B} = \frac{\mu_0 i}{4\pi}\int \frac{(-a\sin\theta\,\hat{i} + a\cos\theta\,\hat{j})\,d\theta \times (-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})}{a^2}$$
$$= \frac{\mu_0 i}{4\pi a}\int (-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})\,d\theta \times (-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$
$$= \frac{\mu_0 i}{4\pi a}\int (\sin^2\theta\,\hat{k} + \cos^2\theta\,\hat{k})\,d\theta = \frac{\mu_0 i}{4\pi a}\int d\theta\,\hat{k}$$
$$= \frac{\mu_0 i}{2a}\hat{k}$$

Hints

$$L = \frac{N\varphi}{i}$$
$$U = \frac{1}{2}L\,i^2$$
$$u_B = \frac{1}{2\mu_0}B^2$$

$$\varphi_B = \vec{B}\cdot\vec{A}$$
$$\varphi_E = \vec{E}\cdot\vec{A}$$

Some useful constants

Charge on the proton: 1.6×10^{-19} C

Charge on the electron	-1.6×10^{-19}C
Perm. of Free space:	$\varepsilon_0 = 8.85\text{x}10^{-12}\ C^2/(N\ m^2)$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}\ Tm/A$
Surface area of a sphere:	$A = 4\pi r^2$
Surface area of cylinder:	$A = 2\pi a L + 2\pi a^2$
Volume of a sphere:	$V = \frac{4}{3}\pi r^3$
Volume of a cylinder:	$V = \pi a^2 L$
Area of a circle:	$A = \pi r^2$
Circumference of a circle:	$A = 2\pi r$