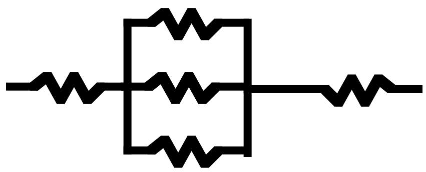
Exam 3
Physics 132

Short Answer Section. Please answer all of the questions.

1. Resistors are arranged as show below. Each resistor is 3 Ohms. What is the equivalent resistance?



$$\frac{1}{R_3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3\Omega} \Rightarrow R_3 = 1\Omega$$
$$R_{eq} = 1\Omega + 3\Omega + 3\Omega = 7\Omega$$

2. State Kirchoff's two laws clearly.

1. The sum of the voltage differences around any closed loop in a circuit must be zero.
2. The net flow of current into a junction must equal the net flow of current out of the junction.

3. State the voltage convention that is used with Kirchoff's voltage law.

1. If the analysis path crosses a resistor in the *same* direction as the current flows through the resistor, then $V = -iR$

2. If the analysis path crosses a resistor in the *opposite* direction as the current flows through the resistor, then $V = iR$

3. If the analysis path crosses an emf from the short (negative) side to the long (positive) side, then $V = +\varepsilon$.

4. If the analysis path crosses an emf from the long (positve) side to the short (negative) side, then $V = -\varepsilon$.

4. An unknown resistor is hooked to a 10mF capacitor. If the time it takes for the voltage on the capacitor to fall to 1/e of its initial charged value is 2 s, how large is the resistor? Sketch the Voltage vs. time graph for the voltage on the capacitor. (Hint, when the voltage has fallen to 1/e of its initial value, $V = V_0\, e^{-1}$).

$$\tau = R\,C$$

$$R = \frac{\tau}{C} = 200\Omega$$

Sketch is exponential decay curve.

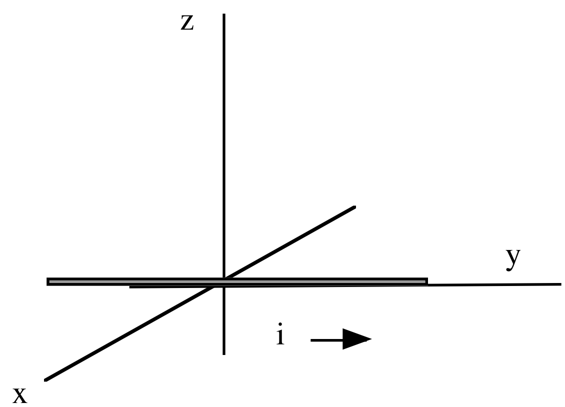
5. A proton has a speed of $v = 2\times10^{6} m/s$ when it enters a region where a uniform B field of 0.1T is present. What is the magnitude of the force that the proton experience? What will the radius of curvature of its path be?

$$F = qvB = 3.2\times10^{-14}$$

$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB} = 0.20875m$$

6. A straight wire of length 1m carries a current of 3 A in the $\hat{j}$ direction as shown below. An external uniform B field of magnitude 2 T points in the $-\hat{i}$ direction. What magnitude **and** direction force does the wire experience?



$$\vec{F} = i\,\vec{l}\times\vec{B} = 3A\cdot 1m\,\hat{j}\times 2T\,(-\hat{i})$$
$$= 6\,\hat{k}$$

7. A square loop with side 0.2m has 10 turns. Five amps flow through each turn. What maximum torque could the loop experience when placed in a 10 T field and at what angle would this occur? What maximum energy could the loop have and what angle would this energy occur?

$$\mu = NiA = 10\cdot 5A\cdot(0.2m)^2 = 2A\,m^2$$

$$\tau = \mu B\sin\theta$$

$$\tau_{\max} = \mu B = 20Nm \quad (@\theta = 90^\circ)$$

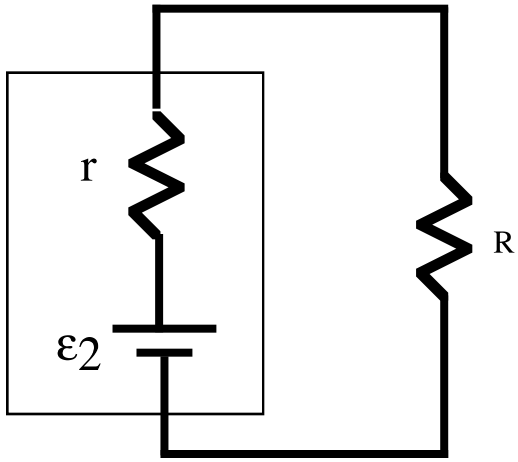
$$U = -\mu B\cos\theta$$

$$U_{\max} = \mu B = 20Nm \quad (@\theta = 180^\circ)$$

8. How is a real battery different from an ideal battery?

Ideal batteries 1) Provide a constant voltage no matter what current is drawn, 2) Live forever

9. What is the current in the circuit shown below? Take r=5 Ohms, R=1000 Ohms, and the emf to be 12 V. What is the power in r? If r=0, what current would flow through R?



$$\varepsilon_2 = i(r + R)$$

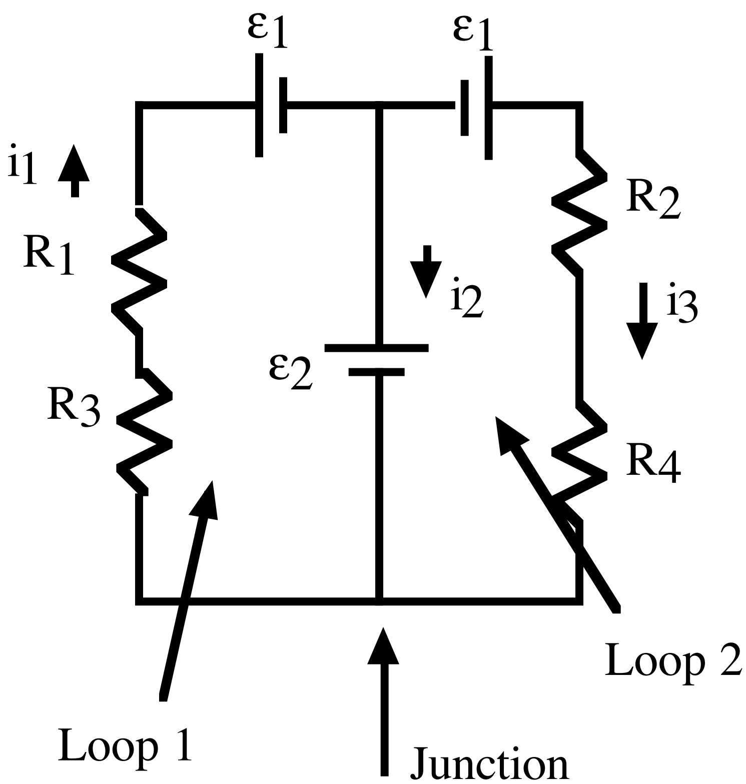
$$i = \frac{\varepsilon_2}{r + R} = 1.19 \times 10^{-2} A$$

$$P = i^2 r = 7.13 \times 10^{-4} W$$

$$i = \frac{\varepsilon_2}{R} = 1.2 \times 10^{-2} A$$

Problems. Please work 2 of three problems

1. Consider the circuit below



R_1 =200 Ohms, R_2 = 250 Ohms, R_3 = 300 Ohms, R_4 = 350 Ohms, ε_1 =20 V and ε_2 = 10 V.

a. Using the loops indicated, write Kirchoff's voltage loop rule for each loop.

$$-i_1 R_3 - i_1 R_1 - \varepsilon_1 - \varepsilon_2 = 0$$

$$+\varepsilon_1 + \varepsilon_2 - i_3 R_2 - i_3 R_4 = 0$$

b. Apply Kirchoff's junction law to the junction indicated.

$$i_1 = i_2 + i_3$$

c. Solve for the currents.

$$-i_1 R_3 - i_1 R_1 - \varepsilon_1 - \varepsilon_2 = 0$$

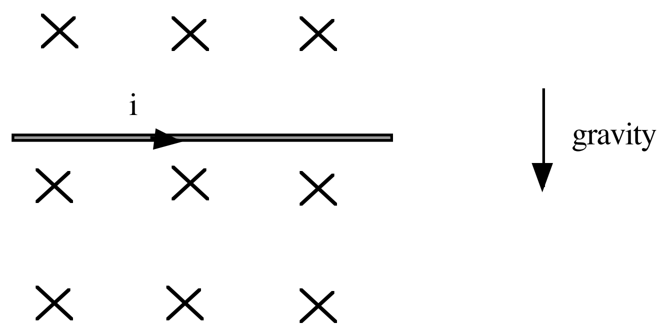
$$i_1 = -\frac{\varepsilon_1 + \varepsilon_2}{R_3 + R_1} = -\frac{20V + 10V}{300\Omega + 200\Omega} = -0.06A$$

$$+\varepsilon_1 + \varepsilon_2 - i_3 R_2 - i_3 R_4 = 0$$

$$i_3 = \frac{\varepsilon_1 + \varepsilon_2}{R_2 + R_4} = \frac{20V + 10V}{350\Omega + 250\Omega} = 0.05A$$

$$i_2 = i_1 - i_3 = -0.06A - 0.05A$$
$$= -0.11A$$

2. A magnetic levitation train uses the force on a current conducting wire to allow a train to float just above a track, as shown below. The rod below has a mass 1 kg and a length of 1m. B=2T



a) What current is necessary to get the rod to just float? (Hint: Balance the magnetic force against the weight, mg).

$$mg = ilB$$

$$i = \frac{mg}{lB} = \frac{1kg \cdot 9.8m/s^2}{1m \cdot 2T} = 4.9A$$

b) Taking the resistance of the rod to be 0.1 Ohms, what battery voltage would be necessary to drive this current with the current you found in part a).

$$V = iR = 4.9A \cdot 0.1\Omega = 0.49V$$

c) Draw the simple loop circuit and compute the power in the rod as resistor.

$$P = i^2 R = (4.9A)^2 \cdot 0.1\Omega = 2.4W$$

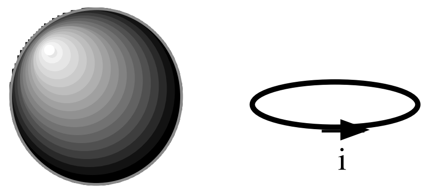
d) You would need to be able to support the battery as well as the rod in a real train. If the mass of the battery, wires, and all the other stuff needed to build this device had a mass of 2 kg, how much more current would you need to support it. What voltage battery would you now need?

$$mg = ilB$$

$$i = \frac{mg}{lB} = \frac{2kg \cdot 9.8m/s^2}{1m \cdot 2T} = 9.8A$$

$$V = iR = 9.8A \cdot 0.1\Omega = 0.98V$$

3. A proton has a magnetic dipole moment $\mu = 1.411 \times 10^{-26} J/T$.



proton

a. If the proton were composed of a single circular current loop with a radius of $r = 0.8 \times 10^{-15} m$, what current would be necessary to produce this dipole moment?

$$\mu = NiA$$
$$i = \frac{\mu}{NA} = \frac{1.411 \times 10^{-26} J/T}{1 \cdot \pi \cdot (0.8 \times 10^{-15})^2} = 7017.74A$$

b. If this current came from taking the charge of the proton as a point and making it go around in the loop described, how long would it take for one orbit? (hint: i=q/t).

$$i = \frac{q}{t}$$
$$t = \frac{q}{i} = \frac{1.6 \times 10^{-19}}{7017.74A} = 2.28 \times 10^{-23} s$$

c. If the proton were placed in a 1 T field, what is the maximum torque that it could experience?

$$\tau = \mu B \sin\theta$$
$$\tau_{\max} = \mu B = 1.411 \times 10^{-26} J/T \cdot 1T = 1.411 \times 10^{-26} Nm$$

d. If the proton were placed in a 1T field, what are the maximum and minimum magnetic potential energies that it could have by virtue of the orientation of its dipole moment? What is the maximum change in energy that it could experience? What would it do for this maximum change to occur?

$$U = -\mu B \cos\theta$$
$$U_{\max} = \mu B = 1.411 \times 10^{-26} \cdot 1T = 1.411 \times 10^{-26} J \quad (\theta = 180°)$$
$$U_{\min} = -\mu B = -1.411 \times 10^{-26} \cdot 1T = -1.411 \times 10^{-26} J \quad (\theta = 0°)$$
$$\Delta U = U_{\max} - U_{\min} = 2.822 \times 10^{-26} J \quad (\mathit{flip\ from}\ 180° \ \mathit{to}\ 0°)$$

Bonus. What speed would the charge have in part b?

$$v = \frac{2\pi r}{t} = \frac{2\pi \cdot 0.8 \times 10^{-15} m}{2.28 \times 10^{-23} s} = 2.2 \times 10^{8} m/s$$

(This is a significant fraction of the speed of light.)

Some useful formulae

Charge on the proton: $1.6\times10^{-19}C$

Charge on the electron $-1.6\times10^{-19}C$

$\varepsilon_0 = 8.85\times10^{-12}C^2/(Nm^2)$

$m_p = 1.67\times10^{-27}kg$

$m_d = 3.33\times10^{-27}kg$

Surface area of a sphere: $A = 4\pi r^2$

Surface area of cylinder: $A = 2\pi a L + 2\pi a^2$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Volume of a cylinder: $V = \pi a^2 L$