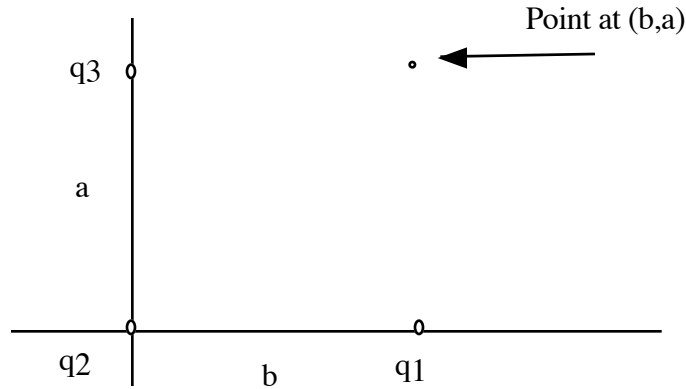


Exam 2
Physics 132

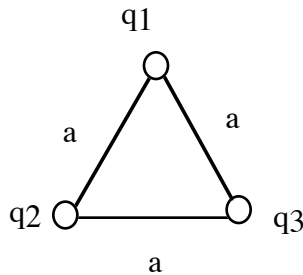
Short Answer Section. Please answer all of the questions.

1. Calculate the potential at the point (b,a) (*corrected*) as shown in the drawing below.



$$V = \frac{q_1}{4\pi\epsilon_0 a} + \frac{q_2}{4\pi\epsilon_0 \sqrt{a^2 + b^2}} + \frac{q_3}{4\pi\epsilon_0 b}$$

2. How much energy is stored in the charge arrangement shown below.



$$U = \frac{q_1 q_2}{4\pi\epsilon_0 a} + \frac{q_1 q_3}{4\pi\epsilon_0 a} + \frac{q_2 q_3}{4\pi\epsilon_0 a}$$

3. A potential difference of 100 kV is used to accelerate the electrons used in a typical television (not LCD or plasma). Assuming that each electron falls through 100 kV, what energy will each electron have?

$$E = q \Delta U = 1e \cdot 100kV = 100keV$$

4. What dielectric constant κ would you need to construct a capacitor that has a capacitance of 0.5F? Assume that your capacitor will have an area 1 m^2 and a separation of 0.1 mm, with a dielectric that fills the gap.

$$C = \frac{\kappa \epsilon_0 A}{d}$$

$$\kappa = \frac{C d}{\epsilon_0 A} = 5.64 \times 10^6$$

5. A copper wire has a radius of 0.002m. What length of this copper wire would have a resistance of 10 Ohms? Take the resistivity of copper to be $\rho = 1.72 \times 10^{-8} \Omega m$.

$$R = \rho \frac{L}{A}$$

$$L = \frac{RA}{\rho} = 7306m$$

(here A is the cross-sectional area of the wire).

6. Five identical 100 mF capacitors are connected in series. What is the equivalent capacitance for this case? If a battery of 100V were placed on each system, what energy would be stored? Now five resistors are connected in the same way with the same battery. What is the equivalent resistance. What current and power are present in the resistor case.

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{5}{100mF}$$

$$C_{eq} = 20mF$$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \cdot 20mF \cdot (100V)^2 = 100,000mJ = 100J$$

$$R_{eq} = R + R + R + R + R = 500\Omega$$

$$V = i R_{eq}$$

$$i = \frac{V}{R_{eq}} = \frac{100V}{500\Omega} = 0.2A$$

$$P = i^2 R = 20W$$

7. A beam of particles protons has a current of 10A. Assuming the beam has a cross sectional area of $1cm^2$ what is the current density in the beam? Now assuming the protons are moving at a speed of $2.5 \times 10^8 m/s$, what is the number density n of the beam?

$$J = \frac{i}{A} = \frac{10A}{1cm^2 \cdot \left(\frac{1m}{100cm}\right)^2} = 100,000 \frac{A}{m^2}$$

$$J = \rho v = n e v$$

$$n = \frac{J}{e v} = \frac{100,000 A / m^2}{1.6 \times 10^{-19} C \cdot 2.5 \times 10^8} = 2.5 \times 10^{15} \text{ particles} / m^3$$

8. The potential on the surface of a solid *conducting* sphere of radius a is 100 Volts (with zero at infinity). What is the potential at the center of the sphere? Why is this so? Hint: Consider what the field is in a *conducting* sphere and how the potential is related to how the electric field *changes*.

The potential can't change because the E field is zero. The E field is proportional to the change in the potential. If the E is zero, the potential must be constant in the region where the E is zero. If it has a value of 100 V on the surface, then it must be unchanged throughout where the E is zero, including the center. The potential is 100V everywhere in the sphere.

9. The electric potential is given below. What are the x and y components of the electric field.?

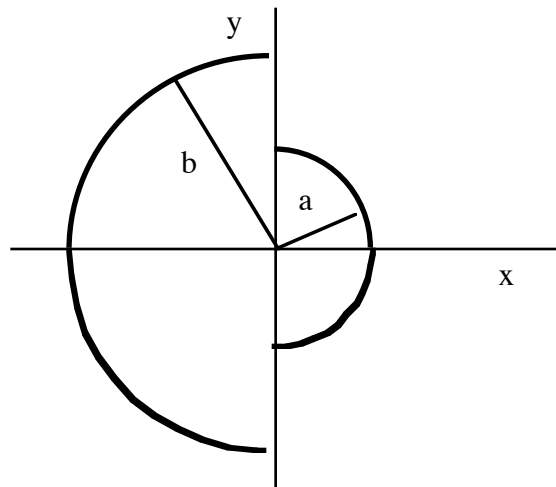
$$V = \frac{x^2}{2} + 2xy + \frac{y^2}{2}$$

$$E_x = -\frac{\partial V}{\partial x} = -(x + 2y)$$

$$E_y = -\frac{\partial V}{\partial y} = -(2x + y)$$

Problems: Please work 2 of the 3 problems. Please indicate which problems you would like to have graded.

1. Consider two charged half circular rods shown below. Assume that the rod with radius a is uniformly charged with charge per unit length λ_a . The rod with radius b is also uniformly charged with charge per unit length λ_b .



a) Write an expression for the dq in terms of the radius a and a small angle $d\theta$ for the smaller circle.

$$dq = \lambda_a a d\theta$$

b) Write the expression for the electric potential at the center due to a small charge dq for the inner circle?

$$dV = \frac{dq}{4\pi\epsilon_0 a}$$

c) What is the potential at the center due to just the smaller arc of charge?

$$V = \int \frac{dq}{4\pi\epsilon_0 a} = \int_0^\pi \frac{\lambda_a a d\theta}{4\pi\epsilon_0 a} = \frac{\lambda_a}{4\pi\epsilon_0} \cdot (\pi - 0) = \frac{\lambda_a}{4\epsilon_0}$$

d) What is the potential at the center due to both arcs of charge?

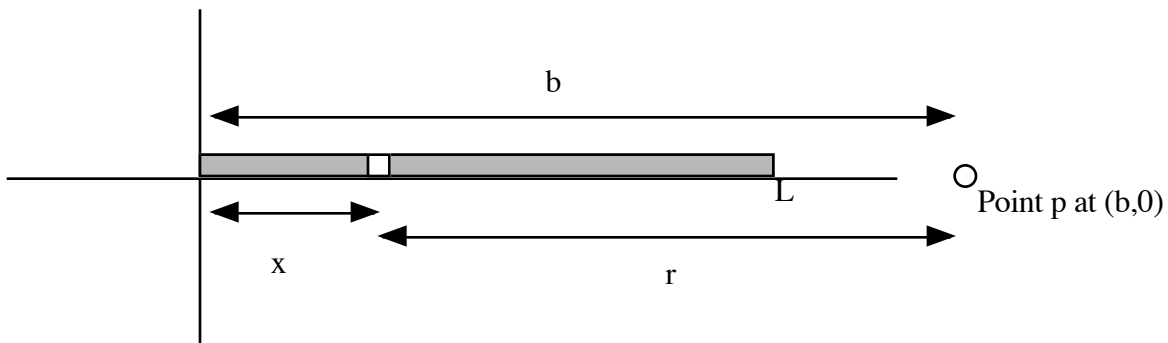
$$V = \frac{\lambda_a}{4\epsilon_0} + \frac{\lambda_b}{4\epsilon_0}$$

bonus) What relationship between λ_a and λ_b would give zero total potential at the center?

$$0 = \frac{\lambda_a}{4\epsilon_0} + \frac{\lambda_b}{4\epsilon_0}$$

$$\lambda_a = -\lambda_b$$

2. Consider the charged rod below. Assume that it is uniformly charged with charge per unit length λ .



a) Write an expression for dq and for r . Your expression for r should be in terms of x and b .

$$dq = \lambda dx$$

$$r = b - x$$

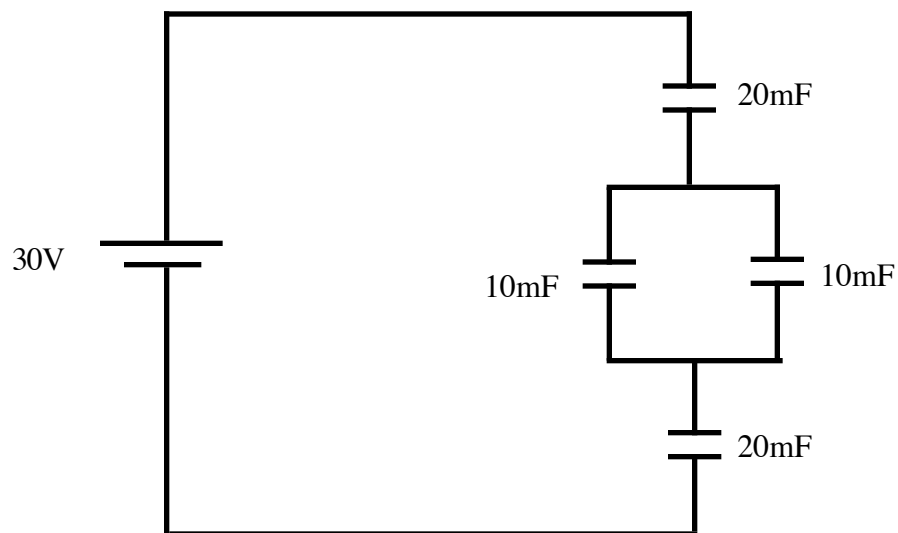
b) Write an expression for the electric potential due to a small charge dq at the point p ?

$$dV = \frac{dq}{4\pi\epsilon_0 r} = \frac{\lambda dx}{4\pi\epsilon_0 (b-x)}$$

c) What is the potential at the point indicated?

$$\begin{aligned} V &= \int_0^L \frac{\lambda dx}{4\pi\epsilon_0 (b-x)} = \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{(b-x)} = -\frac{\lambda}{4\pi\epsilon_0} \ln(b-x) \Big|_0^L \\ &= \frac{\lambda}{4\pi\epsilon_0} [\ln(b) - \ln(b-L)] \end{aligned}$$

3. Consider the circuit shown below.



a) What is the equivalent capacitance.

$$\begin{aligned} C_{\parallel} &= 10mF + 10mF = 20mF \\ \frac{1}{C_{eq}} &= \frac{1}{20mF} + \frac{1}{20mF} + \frac{1}{20mF} = \frac{3}{20mF} \\ C_{eq} &= \frac{20}{3}mF \end{aligned}$$

b) How much charge is stored on the equivalent capacitor? What energy does it store?

$$q = CV = \frac{20}{3}mF \cdot 30V = 200mC$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \cdot \frac{20}{3}mF \cdot (30V)^2 = 3000mJ = 3J$$

c) What is the charge on each capacitor?

Since the equivalent capacitor came from the series combination

$$q_{10} = 200mC$$

Now we need to find the charge on each of the two parallel capacitors. We can find the voltage across the pair since we know the charge.

$$V_{\parallel} = \frac{q}{C_{\parallel}} = \frac{200mC}{20mF} = 10V$$

Now that we know the voltage on the parallel combination, we can find the charge on each of the parallel capacitors.

$$q = CV = 10mF \cdot 10V = 100mC$$

d) What is the energy stored in each capacitor? Does it add up to the result in c)

$$\begin{aligned} U &= \frac{1}{2} \cdot 20mF \cdot 10V^2 + \frac{1}{2} \cdot 20mF \cdot 10V^2 + \frac{1}{2} \cdot 10mF \cdot 10V^2 + \frac{1}{2} \cdot 10mF \cdot 10V^2 \\ &= 3000mJ = 3.0J \end{aligned}$$

It matches!

Some useful formulae

Charge on the proton: $+1.6 \times 10^{-19} C$

Charge on the electron: $-1.6 \times 10^{-19} C$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N m^2}$$

Surface area of a sphere: $A = 4\pi r^2$

Surface area of cylinder: $A = 2\pi aL + 2\pi a^2$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Volume of a cylinder: $V = \pi a^2 L$