

**Exam 1**  
**Physics 132**

**Short Answer Section. Please answer all of the questions.**

1. What is the force of attraction between a Uranium nucleus (charge +92e) and an inner shell electron that is  $0.1 \times 10^{-10} m$  away?

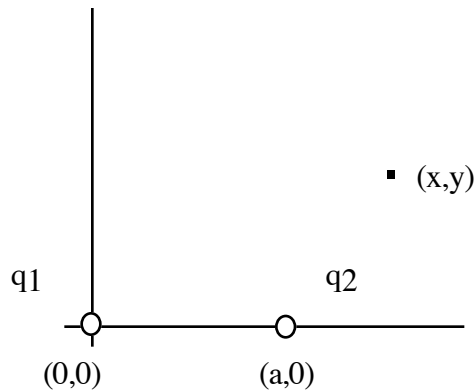
$$F = \frac{q_1 q_2}{4\pi \epsilon_0 r^2} = \frac{1.6 \times 10^{-19} \cdot 92 \cdot 1.6 \times 10^{-19}}{4\pi \epsilon_0 \cdot (0.1 \times 10^{-10} m)^2} = 2.12 \times 10^{-4} N$$

2. A beam of protons has a current of  $1 \mu A$ . How long does it take for 1 mC of charge to accumulate

$$i = \frac{\Delta q}{\Delta t}$$

$$\Delta t = \frac{\Delta q}{i} = \frac{1 \times 10^{-3} C}{1 \times 10^{-6} A} = 1000 s$$

3. What is the electric field at the location  $(x, y)$  shown below? Hint: You will need to write out  $\vec{r}, r, \hat{r}$  for each charge.



$$\begin{aligned} \vec{r}_1 &= (x-0)\hat{i} + (y-0)\hat{j} & \vec{r}_2 &= (x-a)\hat{i} + (y-0)\hat{j} & \vec{E}_1 &= \frac{q_1}{4\pi\epsilon_0 r_1^2} \hat{r}_1 \\ &= x\hat{i} + y\hat{j} & &= (x-a)\hat{i} + y\hat{j} & \vec{E}_2 &= \frac{q_2}{4\pi\epsilon_0 r_2^2} \hat{r}_2 \\ r_1 &= \sqrt{x^2 + y^2} & r_2 &= \sqrt{(x-a)^2 + y^2} & \vec{E} &= \vec{E}_1 + \vec{E}_2 \\ \hat{r}_1 &= \frac{\vec{r}_1}{r} = \frac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}} & \hat{r}_2 &= \frac{\vec{r}_2}{r} = \frac{(x-a)\hat{i} + y\hat{j}}{\sqrt{(x-a)^2 + y^2}} & & \end{aligned}$$

4. What is the electric field inside a conductor and why?

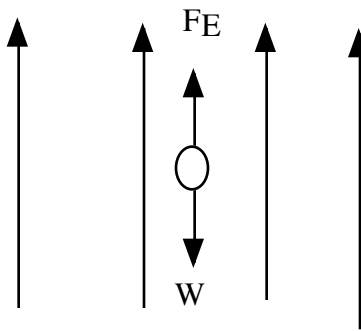
The electric field is zero. Charges will move until the field becomes zero.

5. A positive point charge charge  $+50e$  is placed at the center of a cube. What is the electric flux through the top AND bottom faces.?

$$\phi = \frac{1}{3} \cdot \frac{50e}{\epsilon_0}$$

(Each side of the cube represents 1/6 the total closed surface. The upper AND lower faces therefore represent 1/3 of the total closed surface)

6. A particle with a mass  $m = 1\mu\text{g}$  is placed in a field as shown below. What electric force exactly balances the particle's weight. If the Electric field is  $100 \text{ N/C}$ , what charge is on the particle.  $g = 9.8 \frac{m}{s^2}$

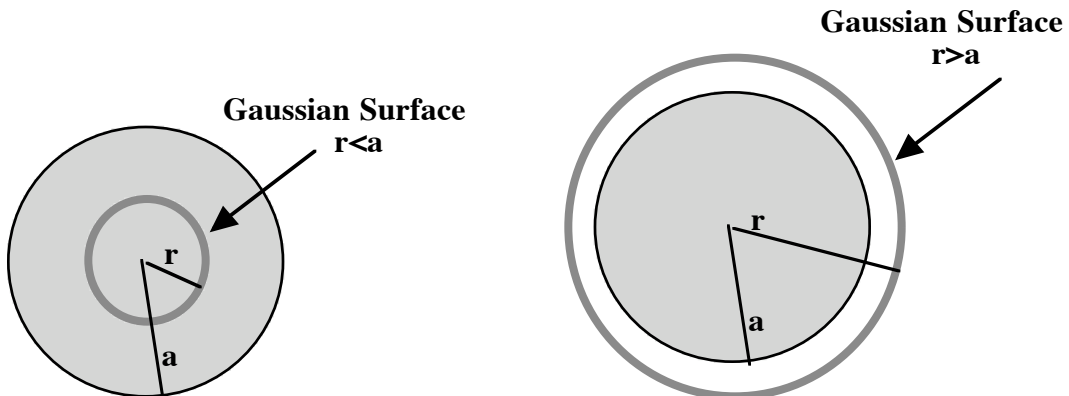


$$F_E = mg = 1 \times 10^{-9} \text{ kg} \cdot 9.8 \text{ m/s}^2 = 9.8 \times 10^{-9} \text{ N}$$

$$qE = F_E$$

$$q = \frac{F_E}{E} = \frac{9.8 \times 10^{-9} \text{ N}}{100 \text{ N/C}} = 9.8 \times 10^{-11} \text{ C}$$

7. A uniformly charged sphere has a charge density  $\rho$  and radius  $a$ . How much charge is enclosed by a Gaussian surface of radius  $r$  where  $r < a$ ? How much charge is enclosed by a Gaussian surface with  $r > a$



$$q_{enc} = \rho \cdot \frac{4}{3} \pi r^3$$

$$q_{enc} = \rho \cdot \frac{4}{3} \pi a^3$$

8. What is the electric flux, in terms of electric field E, for each of the Gaussian surfaces in problem 8. Use this flux to find the field for  $r < a$  and  $r > a$  in problem 7.

$$\varphi = \oint \vec{E} \cdot d\vec{A} = E \cdot 4\pi r^2$$

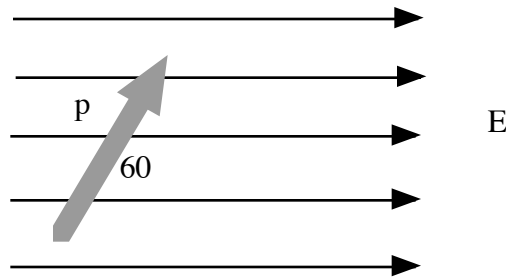
$$\varphi = \frac{q_{enc}}{\epsilon_0}$$

$$E = \frac{q_{enc}}{4\pi \epsilon_0 r^2}$$

$$E_{r < a} = \frac{q_{enc}}{4\pi \epsilon_0 r^2} = \frac{\rho \cdot \frac{4}{3} \pi r^3}{4\pi \epsilon_0 r^2} = \frac{\rho r}{3\epsilon_0}$$

$$E_{r > a} = \frac{q_{enc}}{4\pi \epsilon_0 r^2} = \frac{\rho \cdot \frac{4}{3} \pi a^3}{4\pi \epsilon_0 r^2} = \frac{\rho a^3}{3\epsilon_0 r^2}$$

9. A dipole is composed of a charge  $+e$  and  $-e$  separated by a distance  $1.0 \times 10^{-10}$ . What is the electric dipole moment  $p$  of this dipole? If it is oriented at an angle  $\theta = 60^\circ$  with respect to a uniform electric field  $E = 1000 \text{ N/C}$ , what torque does the dipole experience?

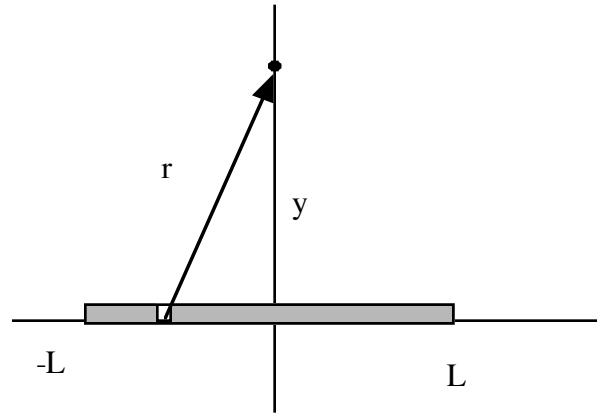


$$p = qd = 1.6 \times 10^{-19} \cdot 1.0 \times 10^{-10} = 1.6 \times 10^{-29} \text{ Cm}$$

$$\tau = pE \sin \theta = 1.6 \times 10^{-29} \text{ Cm} \cdot 1000 \text{ N/C} \cdot \sin 60 = 1.39 \times 10^{-26} \text{ Nm}$$

**Problems: Please work 2 of the 3 problems.**

1. Consider a line of charge along the x axis as shown below. The line has uniform charge density and extends from  $-L$  to  $L$ . We are interested in the field at the point shown, along the y axis a distance  $y$  from the origin.



a) Write  $dq$  for a little length of charge

$$dq = \lambda dx$$

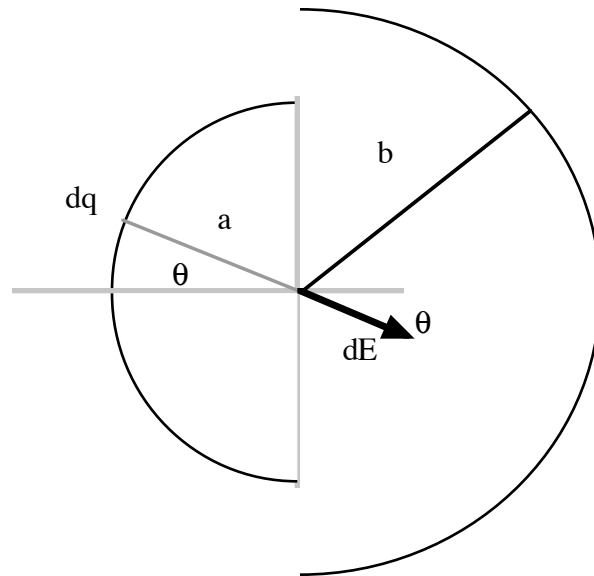
b) Write the  $r$ ,  $\vec{r}$ ,  $\hat{r}$  for the charge  $dq$  that you have defined.

$$\begin{aligned}\vec{r} &= (0 - x)\hat{i} + (y - 0)\hat{j} \\ &= -x\hat{i} + y\hat{j} \\ r &= \sqrt{x^2 + y^2} \\ \hat{r} &= \frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}\end{aligned}$$

c) Compute the field at the point shown. You may need the integral  $\int \frac{dx}{(y^2 + x^2)^{3/2}} = \frac{x}{y^2(y^2 + x^2)^{1/2}}$

$$\begin{aligned}d\vec{E} &= \frac{dq}{4\pi\epsilon_0 r^2} \hat{r} = \frac{\lambda dx}{4\pi\epsilon_0 (x^2 + y^2)} \cdot \frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}} = \\ E_y &= \int_{-L/2}^{L/2} \frac{\lambda y dx}{4\pi\epsilon_0 (x^2 + y^2)^{3/2}} = \frac{\lambda y}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{dx}{(x^2 + y^2)^{3/2}} \\ &= \frac{\lambda y}{4\pi\epsilon_0} \cdot \frac{x}{y^2(y^2 + x^2)^{1/2}} \Bigg|_{-L/2}^{L/2} \\ &= \frac{\lambda}{4\pi\epsilon_0} \cdot \frac{L}{y(y^2 + (L/2)^2)^{1/2}} \\ E_x &= 0\end{aligned}$$

2. Consider the two half circles of charge below or charge below. One half circle has radius  $a$  and charge per unit length  $\lambda_a$  and the second has radius  $b$  charge per unit length  $\lambda_b$ . A little charge  $dq$  produces a  $dE$  as shown from the left half circle.



**Consider just the left half circle with radius  $a$  first. Assume that both charge densities are positive.**

a) What direction will the field point after integrating?

...to the right.

b) Write  $dq$  for a little arc length of charge? Write the arc length in terms of the radius  $a$  and an angle  $d\theta$ ?

$$dq = \lambda_a dl = \lambda_a a d\theta$$

c) Write the magnitude of  $dE$  that is produced by  $dq$ ?

$$E = \frac{dq}{4\pi \epsilon_0 r^2} = \frac{\lambda_a a d\theta}{4\pi \epsilon_0 a^2} = \frac{\lambda_a d\theta}{4\pi \epsilon_0 a}$$

d) Write the component of  $dE$  that will survive in terms of  $\cos\theta$ ?

$$dE_x = dE \cos\theta = \frac{\lambda_a d\theta}{4\pi \epsilon_0 a} \cos\theta$$

e) Integrate  $\theta$  from  $-\pi/2$  to  $+\pi/2$  to find the field.

$$E_x = \int_{-\pi/2}^{\pi/2} \frac{\lambda_a d\theta}{4\pi \epsilon_0 a} \cos\theta = \frac{\lambda_a}{2\pi \epsilon_0 a}$$

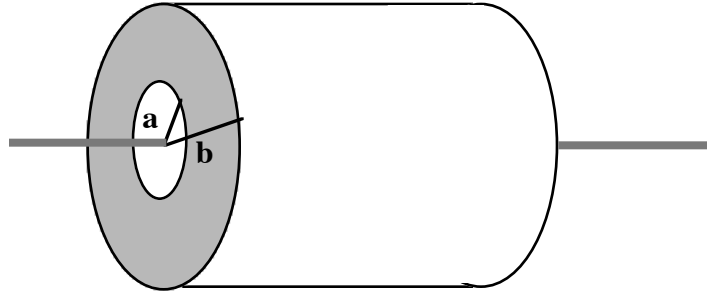
f) Now knowing the field due to the left half circle, what is the field due to both half circles. Hint: You can just write the field due to the right half circle down with appropriate substitution and then add the fields together as appropriate.

$$E_x = \frac{\lambda_a}{2\pi\epsilon_0 a} - \frac{\lambda_b}{2\pi\epsilon_0 b}$$

**Bonus)** What is the relationship between  $\lambda_a$  and  $\lambda_b$  such that the field is zero?

$$0 = \frac{\lambda_a}{2\pi\epsilon_0 a} - \frac{\lambda_b}{2\pi\epsilon_0 b}$$
$$\frac{\lambda_a}{a} = \frac{\lambda_b}{b}$$

3. Consider the **conducting** infinite cylindrical shell shown below . It has inner radius  $a$  and outer radius  $b$ . The shell is electrically neutral, but a line of charge with charge per unity length  $\lambda$  lies along the axis.



**Choose a Gaussian surface with  $r < a$  and length  $l$ .**

a) Write an expression for the charge enclosed by this surface.

$$q_{enc} = \lambda l$$

b) Use Gauss' Law to find the field for radii less than  $a$ .

$$\oint \vec{E} \cdot d\vec{A} = \frac{\lambda l}{\epsilon_0}$$

$$E \cdot 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$

**Choose a Gaussian surface for  $a < r < b$ . This surface is in the conductor.**

c) What is the field for  $a < r < b$ ?

$$E = 0$$

d) What is the charge enclosed in your Gaussian surface? What is the charge on the inner surface of the cylinder and what is the surface charge density on that surface (Hint:  $\lambda_a = \frac{q_{inner}}{2\pi a l}$ )

$$\oint \vec{E} \cdot d\vec{A} = 0 = \frac{q_{enc}}{\epsilon_0}$$

$$0 = q_{center} + q_{inner}$$

$$q_{inner} = -q_{center}$$

$$\sigma_{inner} = \frac{q_{inner}}{2\pi a l} = \frac{-q_{center}}{2\pi a l}$$

e). If the cylinder is electrically neutral, what charge resides on the outer surface. What is the surface charge density on the outer surface at radius  $b$ ?

$$q_{cylinder} = 0 = q_{inner} + q_{outer}$$

$$0 = q_{outer} + q_{inner}$$

$$= -q_{inner} = q_{center}$$

$$\sigma_{outer} = \frac{q_{outer}}{2\pi b l} = \frac{q_{center}}{2\pi b l}$$

**Some useful formulae**

Charge on the proton:  $+1.6 \times 10^{-19} \text{ C}$   
Charge on the electron:  $-1.6 \times 10^{-19} \text{ C}$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N m}^2}$$

Surface area of a sphere:  $A = 4\pi r^2$

Surface area of cylinder:  $A = 2\pi aL + 2\pi a^2$

Volume of a sphere:  $V = \frac{4}{3}\pi r^3$

Volume of a cylinder:  $V = \pi a^2 L$