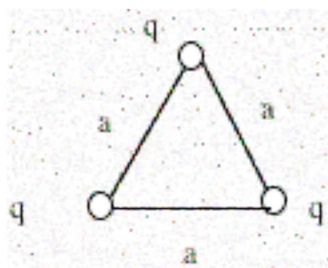
Exam 2
Physics 132

Short Answer Section. Please answer all of the questions.

1. Compute the electric potential on the surface of an hydrogen atom. The charge producing the potential is a single proton at the center of the atom with charge $1.6\times10^{-19}C$ The radius of the atom is $r = 0.529\times10^{-10}m$.

$$V = \frac{q}{4\pi\varepsilon_0 r} = 27.2V$$

2. Three charges are arranged as below. What energy was required to assemble these charges?

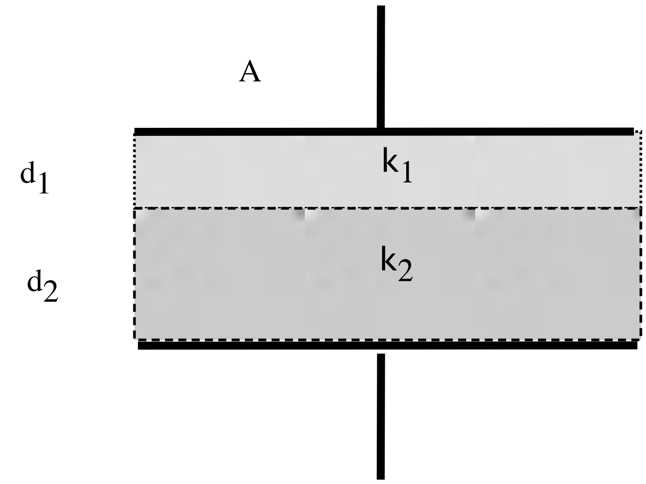


$$U = \frac{q^2}{4\pi\varepsilon_0 a} + \frac{q^2}{4\pi\varepsilon_0 a} + \frac{q^2}{4\pi\varepsilon_0 a} = 3\frac{q^2}{4\pi\varepsilon_0 a}$$

3. A potential difference of 10 kV is used to accelerate the electrons used in a typical old style television (not LCD or plasma). Assuming that each electron falls through 10 kV, what energy will each electron have?

$$U = q\Delta V = 1e\cdot 10kV = 10keV$$

4. Write an expression for the capacitance of the capacitor shown below. The surface area of the plates is A and the dielectric constants and distances are as shown.



$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{\frac{\kappa_1\varepsilon_0 A}{d_1}}+\frac{1}{\frac{\kappa_2\varepsilon_0 A}{d_2}}=\frac{d_1/\kappa_1}{\varepsilon_0 A}+\frac{d_2/\kappa_2}{\varepsilon_0 A}=\frac{d_1/\kappa_1+d_2/\kappa_2}{\varepsilon_0 A}$$

$$C=\frac{\varepsilon_0 A}{d_1/\kappa_1+d_2/\kappa_2}$$

5. A solid copper cylinder has a length of 5000m. What cross-sectional area would this cylinder need to be if it is to have a resistance of 2 Ohms? What current would flow if this cylinder has a potential difference of 100 V. Take the resistivity of copper to be $1.69\times10^{-8}\,\Omega m$

$$R=\rho\frac{L}{A}$$

$$A=\rho\frac{L}{R}=4.225\times10^{-5}\,m^2$$

$$V=iR$$

$$i=\frac{V}{R}=50A$$

6. Five identical 50 mF capacitors are connected in series. What is the equivalent capacitance for this case? The same capacitors are now connected in parallel. What is the equivalent capacitance?

$$C_{parallel}=50mF+50mF+50mF+50mF+50mF=250mF$$

$$\frac{1}{C_{series}}=\frac{1}{50mF}+\frac{1}{50mF}+\frac{1}{50mF}+\frac{1}{50mF}+\frac{1}{50mF}=\frac{5}{50mF}$$

$$C_{series}=10mF$$

7. If a battery of 100V were placed on the *series* system in problem 6. What energy would be stored on the *series* equivalent capacitor ?

$$q=CV=10mF\cdot100V=1000mC=1C$$

8. The density of charge carriers in a wire is $n = 1 \times 10^{10}\, electrons / m^3$. If the drift velocity is $1 m / s$, what is the current density in the wire? If the radius of the wire is 1 mm, what is the current in the wire? The charge on the electron is $-1.6 \times 10^{-19} C$

$$J = nev = 1.6 \times 10^{-9} A / m^2$$
$$i = J \cdot Area = 5.03 \times 10^{-25} A$$

9 The potential on the surface of a solid *conducting* sphere of radius a is 5000 Volts (with zero at infinity). What is the potential at the center of the sphere? Why is this so? Hint: Consider what the field is in a *conducting* sphere and how the potential is related to how the electric field *changes*.

The potential at the center is 5000V. Since the electric field in a conductor is zero, the electric potential remains constant.

10. The electric potential is given below. What are the x and y components of the electric field.?

$$V = \frac{x^2}{2} + 2xy + \frac{y^2}{2}$$

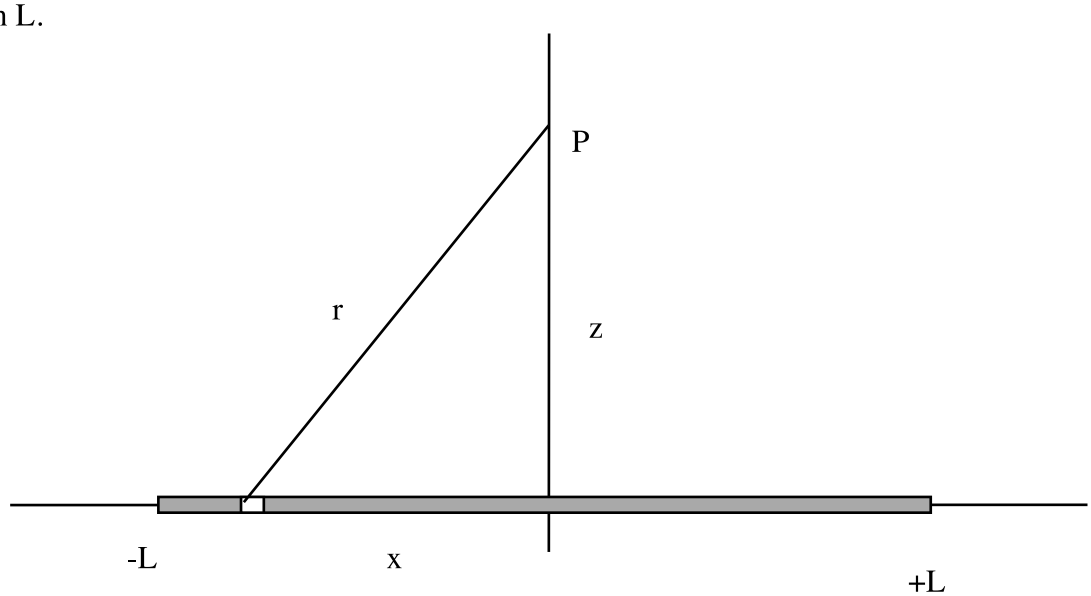
$$E_x = -\frac{\partial V}{\partial x} = -x - 2y$$
$$E_y = -\frac{\partial V}{\partial y} = -2x - y$$

11. An electric heater uses 900 W of power. If the resistance of the heater is 9 Ohms, what current is used by the heater. If the heater is on for 24 hours, how many Joules of energy were used.

$$P = i^2 R$$
$$i = \sqrt{\frac{P}{R}} = 10A$$
$$U = P \cdot t = 900W \cdot (24hrs \cdot \frac{3600s}{1ht}) = 7.78 \times 10^7 J$$

Problems: Please work 2 of the 3 problems. Please indicate which problems you would like to have graded.

1. Consider the charged rod below. Assume that it is uniformly charged with charge per unit length L.



a) Write an expression for the electric potential due to a small charge dq at the point P?

$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

b) Write an expression for the dq and the r in terms of x and the distance z.

$$dq = \lambda dx$$
$$r = \sqrt{x^2 + z^2}$$

c) What is the potential at the point indicated? You may need the integral

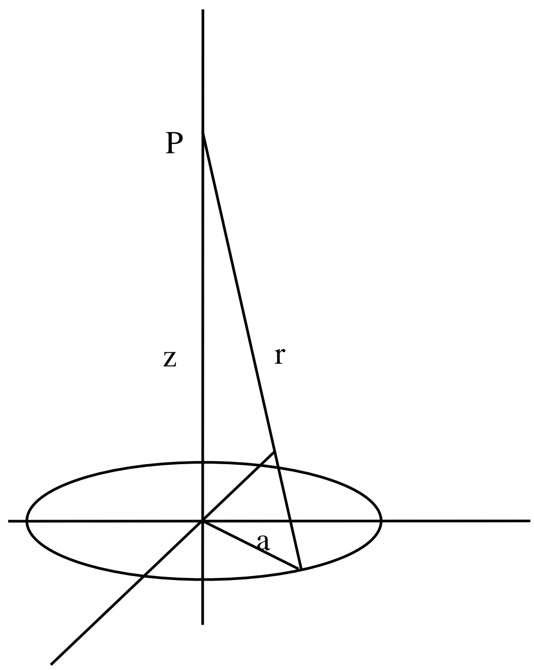
$$\int_{-L}^{L} \frac{dx}{\sqrt{x^2 + z^2}} = \ln[\mathrm{L} + \sqrt{\mathrm{L}^2 + \mathrm{z}^2}] - \ln[-\mathrm{L} + \sqrt{\mathrm{L}^2 + \mathrm{z}^2}]$$

$$
\begin{aligned}
V &= \int_{-L}^{L} \frac{dq}{4\pi\varepsilon_0 r} \\
&= \int_{-L}^{L} \frac{\lambda dx}{4\pi\varepsilon_0 \sqrt{x^2 + z^2}} \\
&= \frac{\lambda}{4\pi\varepsilon_0} \int_{-L}^{L} \frac{dx}{\sqrt{x^2 + z^2}} \\
&= \frac{\lambda}{4\pi\varepsilon_0} \ln[\mathrm{L} + \sqrt{\mathrm{L}^2 + \mathrm{z}^2}] - \ln[-\mathrm{L} + \sqrt{\mathrm{L}^2 + \mathrm{z}^2}]
\end{aligned}
$$

d) Explain how you could use your answer from c) to compute the z component of the electric field, but do not do this calculation.

Take the derivative with respect to z. $E_z = -\frac{\partial V}{\partial z}$

2. Consider the charged circular wire with radius a as shown below. Assume that it is uniformly charged with charge per unit length λ. Note: This is not a disk--its a circular wire.



a) Write an expression for the dq and the r in terms of the radius a, and a small angle $d\theta$ for the circle. (Note: the angle is in the xy plane and goes around the circle).

$$dq = \lambda a d\theta$$
$$r = \sqrt{a^2 + z^2}$$

b) Write the expression for the electric potential at the center due to a small charge dq at the point P?

$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

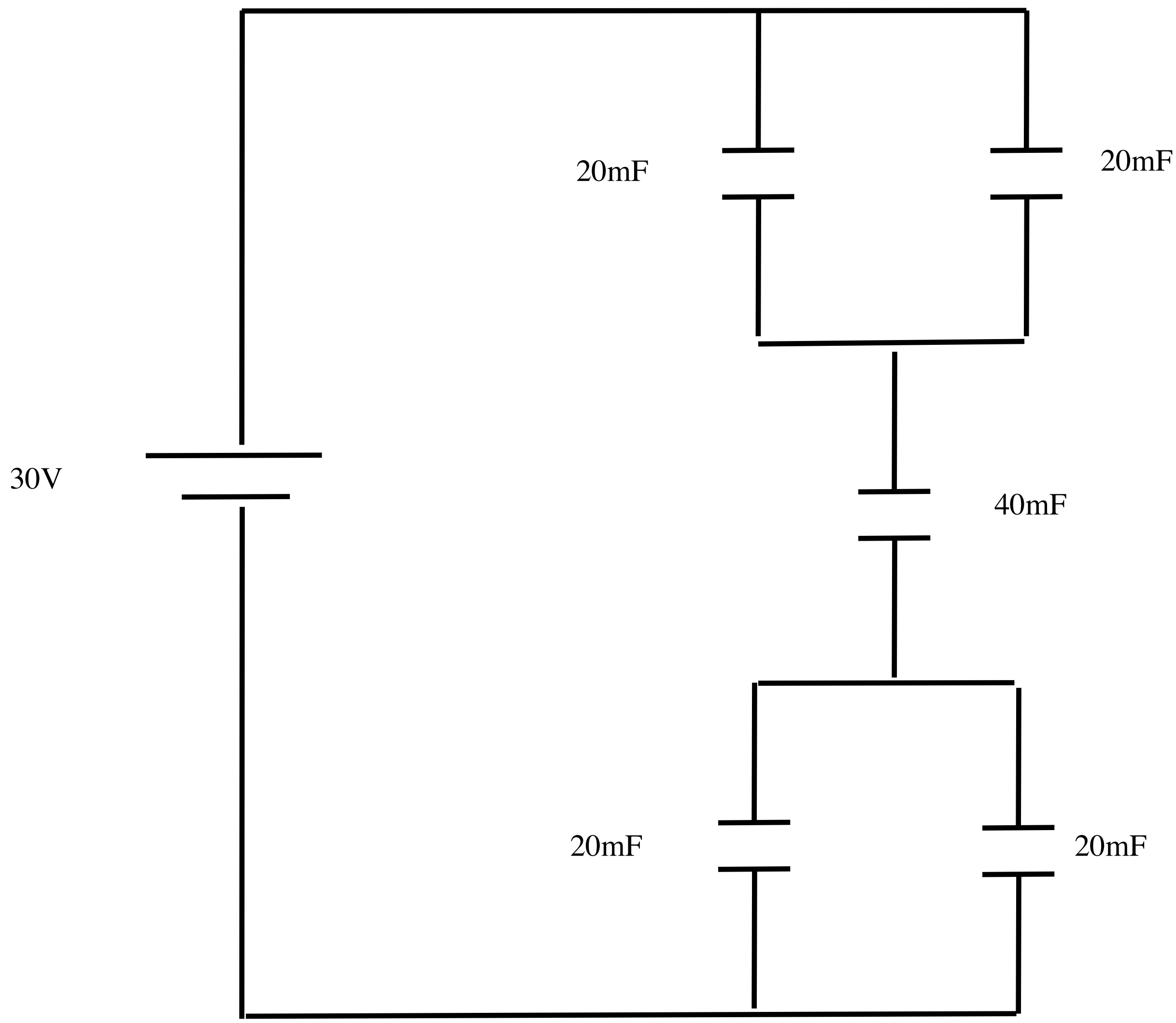
c) What is the potential at point P due to the circle of charge?

$$
\begin{aligned}
V &= \int_0^{2\pi} \frac{dq}{4\pi\varepsilon_0 r} \\
&= \int_0^{2\pi} \frac{\lambda a d\theta}{4\pi\varepsilon_0\sqrt{a^2+z^2}} \\
&= \frac{\lambda a}{4\pi\varepsilon_0\sqrt{a^2+z^2}}\int_0^{2\pi} d\theta \\
&= \frac{\lambda a}{2\varepsilon_0\sqrt{a^2+z^2}}
\end{aligned}
$$

Bonus: Use the potential to find the z-component of the electric field at point P.

$$
\begin{aligned}
E_z &= -\frac{\partial}{\partial z}\left(\frac{\lambda a}{2\varepsilon_0\sqrt{a^2+z^2}}\right) \\
&= \frac{\lambda a z}{2\varepsilon_0\left(a^2+z^2\right)^{3/2}}
\end{aligned}
$$

3. Consider the circuit shown below.



a) What is the equivalent capacitance for this array of capacitors?

$$Top: C_{eqTop} = 20mF + 20mF = 40mF$$

$$Bottom: C_{eqBottom} = 20mF + 20mF = 40mF$$

$$\frac{1}{C_{eq}} = \frac{1}{40mF} + \frac{1}{40mF} + \frac{1}{40mF} = \frac{3}{40mF}$$

$$C_{eq} = \frac{40}{3} mF$$

b) How much charge is stored on the equivalent capacitor? What energy does it store?

$$q = CV$$
$$q_{eq} = C_{eq}V = \frac{40}{3}mF \cdot 30V = 400mC = 0.4C$$
$$U = \frac{1}{2}C_{eq}V^2 = 6000mJ = 6J$$

c) What is the charge on each capacitor?

The charge on series capacitors is the same as on the equivalent capacitor that replaces them. This means that

$$q_{eqTop} = q_{center} = q_{eqBottom} = 400mC$$

Now that we know the charge on the equivalent capacitors, we can calculate the voltage on them and use those voltages to find the charge on each capacitor

$$V_{eqTop} = \frac{q_{eqTop}}{C_{eqTop}} = \frac{400mC}{40mF} = 10V$$
$$V_{leftTop} = V_{rightTop} = V_{eqTop}$$
$$q_{leftTop} = C_{leftTop}V_{leftTop} = 20mF \cdot 10V = 200mC$$
$$q_{rightTop} = C_{rightTop}V_{rightTop} = 20mF \cdot 10V = 200mC$$
$$V_{eqBottom} = \frac{q_{eqBottom}}{C_{eqBottom}} = \frac{400mC}{40mF} = 10V$$
$$V_{leftBottom} = V_{rightBottom} = V_{eqBottom}$$
$$q_{leftBottom} = C_{leftBottom}V_{leftBottom} = 20mF \cdot 10V = 200mC$$
$$q_{rightBottom} = C_{rightBottom}V_{rightBottom} = 20mF \cdot 10V = 200mC$$

d) What is the energy stored in each capacitor? Does it add up to the result in c)?

$$U = \frac{1}{2}C_{leftTop}V_{leftTop}^2 + \frac{1}{2}C_{rightTop}V_{rightTop}^2 + \frac{1}{2}C_{leftBottom}V_{leftBottom}^2 + \frac{1}{2}C_{rightBottom}V_{rightBottom}^2 + \frac{1}{2}C_{center}V_{center}^2$$
$$= \frac{1}{2} \cdot 20mF \cdot (10V)^2 + \frac{1}{2} \cdot 20mF \cdot (10V)^2 + \frac{1}{2} \cdot 20mF \cdot (10V)^2 + \frac{1}{2} \cdot 20mF \cdot (10V)^2 + \frac{1}{2} \cdot 40mF \cdot (10V)^2$$
$$= 6000mJ$$
$$= 6J$$

It matches! Energy is conserved.

Some useful formulae

Charge on the proton: $+1.6\times10^{-19}C$

Charge on the electron $-1.6\times10^{-19}C$

$\varepsilon_0 = 8.85\times10^{-12}\ \dfrac{\text{C}^2}{\text{N m}^2}$

Surface area of a sphere: $A = 4\pi r^2$

Surface area of cylinder: $A = 2\pi a L + 2\pi a^2$

Volume of a sphere: $V = \dfrac{4}{3}\pi r^3$

Volume of a cylinder: $V = \pi a^2 L$