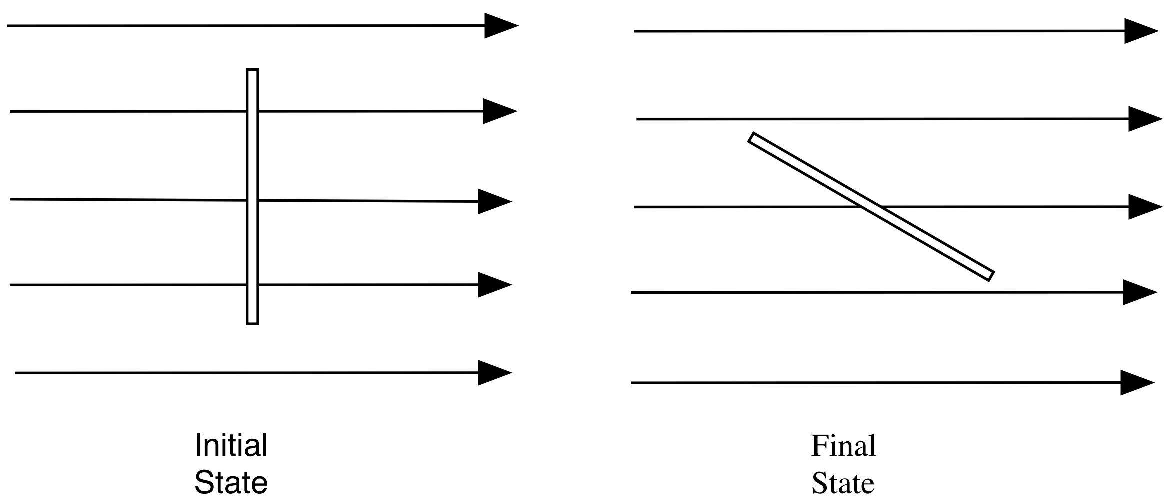
Exam 4
Physics 132

Short Answer Section. Please work all of the short answer problems.

1. An airplane flies through a magnetic field of $0.5\times10^{-3}T$ at a constant speed of 270m/s. If the wingspan is 40m, what EMF develops across the wings. Hint: Treat this as a bar moving through a field.

$$\varepsilon = Blv = 0.5\times10^{-3}T\cdot 40m\cdot 270m/s = 5.4V$$

2. A uniform magnetic field of 10T passes through a square loop of wire with 100 turns. The loop has side 0.1 m and is oriented at 0 degrees with respect to the field. What is the flux through the wire? The loop is turned to 60 degrees. What is the flux in the final state? If the turn occurs in 0.1 seconds, what emf is developed?



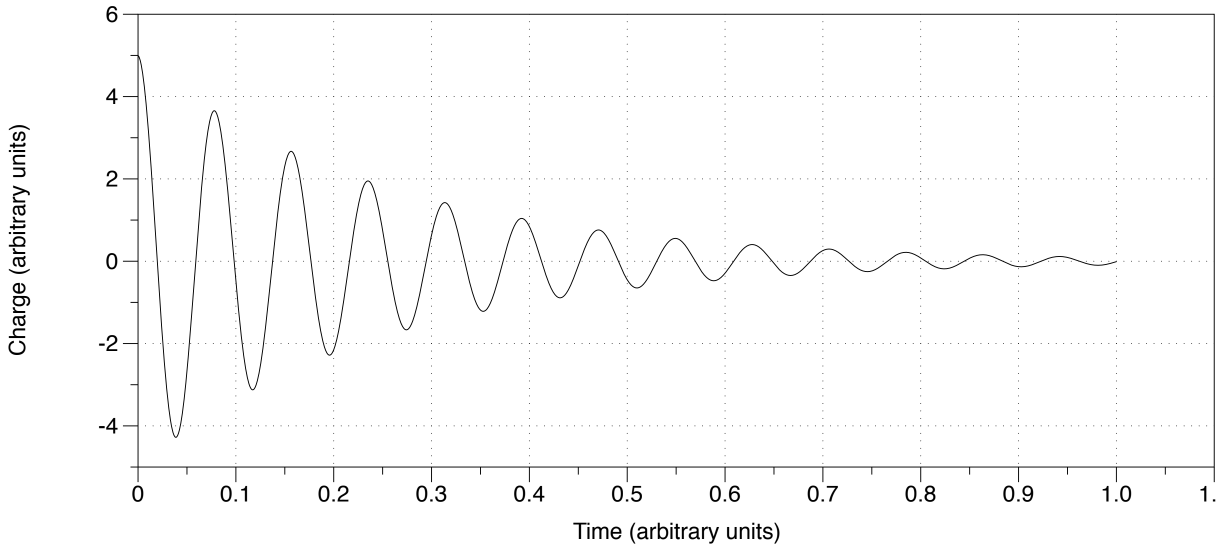
$$\varphi_{Bi} = NAB\cos\theta_i = 100\cdot(0.1m)^2\cdot 10T\cdot\cos 0 = 10Tm^2$$
$$\varphi_{Bf} = NAB\cos\theta_i = 100\cdot(0.1m)^2\cdot 10T\cdot\cos 60 = 5Tm^2$$
$$|\varepsilon| = \left|\frac{\Delta\varphi}{\Delta t}\right| = \left|\frac{\varphi_{Bf}-\varphi_{Bi}}{\Delta t}\right| = 50V$$

3. A 100mH inductor has a current given by $i = 5t^2$. At what time t is the voltage across the inductor 2 V.

$$V = L\frac{di}{dt} = L\cdot 10t$$
$$t = \frac{V}{10L} = \frac{2}{10\cdot 100\times10^{-3}} = 2s$$

4. An RLC Circuit has a very small resistance. Sketch the charge on the capacitor as a function of time. Is this the underdamped, critically damped, or overdamped?

Underdamped



5. Write Maxwell's equations and briefly explain each equation.

$$(a) \quad \oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$$

$$(b) \quad \oint \vec{E} \cdot d\vec{r} = -\frac{d\varphi_B}{dt}$$

$$(c) \quad \oint \vec{B} \cdot d\vec{A} = 0$$

$$(d) \quad \oint \vec{B} \cdot d\vec{r} = \mu_0 i_{encl} + \mu_0 \varepsilon_0 \frac{d\varphi_E}{dt}$$

(a) Gauss' Law for E: The total flux of E through an enclosed surface is proportional to the charge enclosed

(b) A changing magnetic flux induces an electric field.

(c) Gauss' Law for B: There is no free magnetic charge.

(d) A magnetic field is produced by either a current or a changing electric flux.

6. If the inductor in an LC circuit is 100mH and the capacitance is 0.001 microFarads, what is the natural oscillating frequency ω_0 of this circuit. To what frequency f does this correspond?

$$\omega_0 = \sqrt{\frac{1}{LC}} = 1\times 10^5\, s^{-1}$$
$$f_0 = \frac{\omega_0}{2\pi} = 1.59\times 10^4\, Hz$$

7. A cylindrical solenoid is 1m m long and has 10,000 turns total. What current would you need to put through the solenoid to generate a field of 0.5T? (Hint: Calculate B using Ampere's Law). What is the energy density?

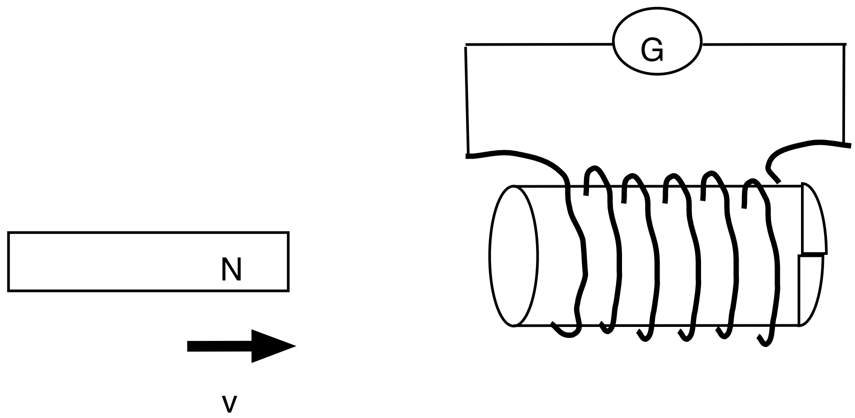
$$\oint \vec{B}\cdot d\vec{l} = \mu_0\, i_{enc}$$
$$Bl = \mu_0\, i\, n\, l$$
$$B = \mu_0 i n$$
$$i = \frac{B}{\mu_0 n} = \frac{0.5T}{4\pi\times 10^{-7}(10000/1m)} = 39.8A$$
$$u = \frac{1}{2\mu_0}B^2 = \frac{1}{2\cdot 4\pi\times 10^{-7}}\cdot(0.5T)^2 = 9.95\times 10^4\, J/m^3$$

8. Use Ampere's Law to find the field due to a long straight wire that has current i.

$$\oint \vec{B}\cdot d\vec{r} = \mu_0\, i$$

$$\oint \vec{B}\cdot d\vec{r} = \mu_0\, i$$
$$B\cdot(2\pi r) = \mu_0\, i$$
$$B = \frac{\mu_0\, i}{2\pi r}$$

BONUS The north end of a magnet is moved into a coil connected to a galvanometer as shown below. Assuming that the needle of the galvanometer deflects toward the side that current enters it, which way will the needle point due to the induced current and why. Note: We did this in lab.



The magnetic flux rises to the right. The coil will respond to oppose this change in flux, so it will develop a current to produce a field to the left. A current will flow up the front face, causing the needle to deflect left.

Problems: Please work 2 of 3 long problems

1. Consider an RLC circuit with R=200 Ohms, L = 500 mH and C=2 microF.

a. Write the differential equation that describes this circuit by using Kirchoff's voltage loop rule. What is the natural oscillating. frequency ω_0 ?

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=0$$
$$\omega_0=\sqrt{\frac{1}{LC}}=\sqrt{\frac{1}{500\times10^{-3}H\cdot2\times10^{-6}F}}=1000s^{-1}$$

b. Now consider driving this circuit. What are X_L,X_C, and Z for this circuit?
Take the driving frequency to be $\omega=900\,s^{-1}$

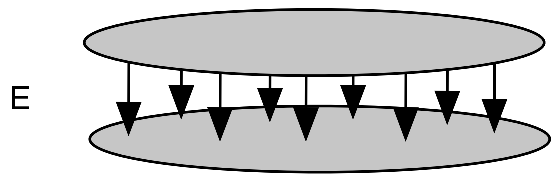
$$X_L=\omega L=900\cdot500\times10^{-3}=450\Omega$$
$$X_C=\frac{1}{\omega C}=555.6\Omega$$
$$Z=\sqrt{(X_L-X_C)^2+R^2}=226.2\Omega$$

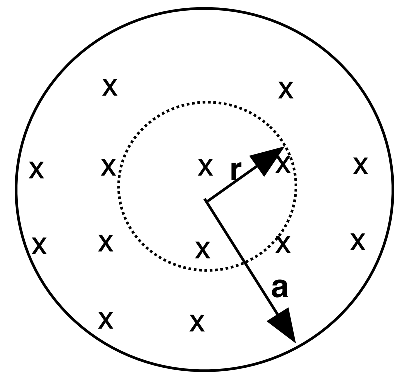
c. If the RMS Voltage is $V_{rms}=120V$, what RMS current will flow in this circuit? What maximum current i_0 will flow.

$$i_{rms}=\frac{V_{rms}}{Z}=\frac{120V}{226.2\Omega}=0.531A$$
$$i_0=\sqrt{2}i_{rms}=0.75A$$

2. A circular parallel plate capacitor with radius a and separation d in an RC circuit charges via the equation

$$q = q_0 \sin \omega t$$





Viewed from above.
E is pointing
downward.

a. What are the voltage and electric field as a function of time?

$$V = \frac{q}{C} = \frac{q_0}{C}\sin \omega t$$
$$E = \frac{V}{d} = \frac{q_0}{Cd}\sin \omega t$$

b. What is the ***electric*** flux through an area between the plates with radius r

$$E = \frac{V}{d} = \frac{q_0}{Cd}\sin \omega t$$
$$\varphi_E = EA = \left(\frac{q_0}{Cd}\sin \omega t\right)\cdot \pi r^2$$

c. What is the displacement i_d current for the area describe by a circle of radius r?

$$\varphi_E = EA = \left(\frac{q_0}{Cd}\sin\omega t\right)\cdot \pi r^2$$

$$i_d = \varepsilon_0 \frac{d\varphi_E}{dt} = \varepsilon_0 \frac{q_0 \pi r^2}{Cd}\omega\cos\omega t$$

d. What is the induced B at at r?

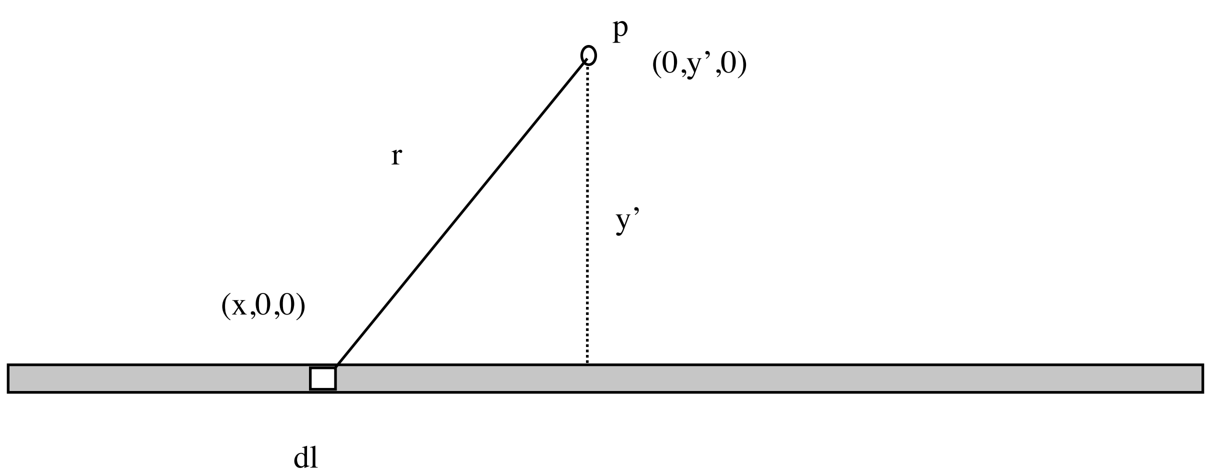
$$\oint \vec{B}\cdot d\vec{l} = \mu_0\, i_d$$

$$B(2\pi r) = \mu_0\, i_d$$

$$B = \frac{\mu_0\, i_d}{(2\pi r)}$$

$$B = \frac{\mu_0 \varepsilon_0 \dfrac{q_0 \pi r^2}{Cd}\omega\cos\omega t}{(2\pi r)} = \mu_0 \varepsilon_0 \frac{q_0 r}{2Cd}\omega\cos\omega t$$

3. A finite wire is shown below. It runs along the x axis from -L/2 to +L/2. We are interested in the B field at point P. Note: $\hat{i}\times\hat{j} = \hat{k}$. The current i runs in the +x direction.



a. Write a little length of current $d\vec{l}$ is in terms of dx. Be sure to indicate the direction.

$$d\vec{l} = dx\,\hat{i}$$

b. Write $\vec{r}, r, \hat{r}$.

$$\vec{r} = (0 - x)\,\hat{i} + (y' - 0)\,\hat{j} = -x\,\hat{i} + y'\,\hat{j}$$
$$r = \sqrt{x^2 + y'^2}$$
$$\hat{r} = \frac{\vec{r}}{r} = \frac{-x\,\hat{i} + y'\,\hat{j}}{\sqrt{x^2 + y'^2}}$$

c. Write the Biot-Savart Law and use the results from (a) and (b) to set up the integration.

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{i d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 i}{4\pi}\int \frac{(dx\,\hat{i}) \times \dfrac{-x\,\hat{i} + y'\,\hat{j}}{\sqrt{x^2 + y'^2}}}{x^2 + y'^2}$$
$$= \frac{\mu_0 i}{4\pi}\int \frac{(dx\,\hat{i}) \times (-x\,\hat{i} + y'\,\hat{j})}{(x^2 + y'^2)^{3/2}}$$

d. Evaluate the integral to calculate the field. Note: you may need the integral. Hint: You may need:

$$\int_{-L/2}^{+L/2} \frac{dx}{(x^2 + y'^2)^{3/2}} = \frac{x}{y'^2\sqrt{y'^2 + x^2}}\Big]_{-L/2}^{+L/2} = \frac{L}{y'^2\sqrt{y'^2 + (\frac{L}{2})^2}}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{i d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 i}{4\pi}\int \frac{(dx\,\hat{i}) \times \dfrac{-x\,\hat{i} + y'\,\hat{j}}{\sqrt{x^2 + y'^2}}}{x^2 + y'^2}$$
$$= \frac{\mu_0 i}{4\pi}\int \frac{(dx\,\hat{i}) \times (-x\,\hat{i} + y'\,\hat{j})}{(x^2 + y'^2)^{3/2}}$$
$$B_z = \frac{\mu_0 i y'}{4\pi}\int \frac{dx}{(x^2 + y'^2)^{3/2}}$$
$$= \frac{\mu_0 i y'}{4\pi} \cdot \frac{x}{y'^2\sqrt{y'^2 + x^2}}\Big]_{-L/2}^{+L/2} = \frac{\mu_0 i}{4\pi} \cdot \frac{L}{y'\sqrt{y'^2 + (\frac{L}{2})^2}}$$

Bonus: Show that if you let the limits of integration go to infinity, you get the infinite wire result

Note: $\mu_0 = 4\pi \times 10^{-7}$ in MKS units.

$m_p = 1.67 \times 10^{-27}\,kg$

$q_p = 1.60 \times 10^{-19}\,C$