Chapter 30

30.1 In Fig. 30-37, the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$ where the flux is in milliWebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop at t=2.0s? (b) In what direction will the current flow through the resistor.

The magnitude of the induced emf is given by

$$\Phi_B = 6.0t^2 + 7.0t$$
$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = 12.0t + 7.0$$
$$|\varepsilon(2)| = 12.0 \cdot 2 + 7.0 = 31mV$$

The flux is pointing out of the paper and rising. The coil reacts to oppose this rising outward field by generating a field that points into the paper. To do this, the current must flow to the left through the resistor.

30.3 A small loop of area $6.8\,mm^2$ is placed inside a long solenoid that has 854 turns/cm and carries a sinusoidally varying current i of amplitude 1.28 A and angular frequency 212 rad/s. The central axis of the loop and the solenoid coincide. What is the amplitude of the emf induced in the loop

The emf generated can be calculated by finding the rate of change of magnetic flux through the inner loop that results from the changing field due to the solenoid

$$B = \mu_0 n i = \mu_0 n i_0 \sin\omega t$$
$$\varphi_B = BA_{loop} = \mu_0 n i_0 \sin\omega t \cdot A_{loop}$$
$$\frac{d\varphi_B}{dt} == \omega\,\mu_0\, n\, i_0\, A_{loop} \cos\omega t$$

$$\omega\,\mu_0\, n\, i_0\, A_{loop} = 212 \cdot 4\pi \times 10^{-7} \cdot 85400\ turns/m \cdot 1.28A \cdot 6.8 \times 10^{-6} m^2$$
$$= 1.98 \times 10^{-4} V$$

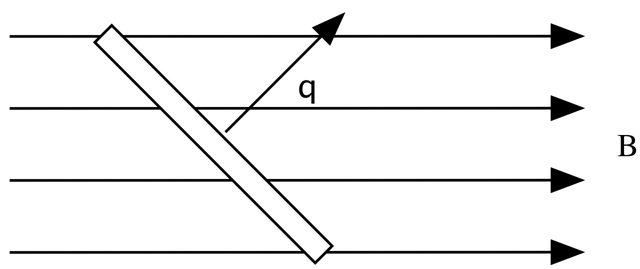
30.7 In Fig. 30-40, a wire forms a closed circular loop, with radius R=2.0 m and resistance $4.0\,\Omega$. The circle is centered on a long straight wire; at time t=0, the current in the long straing wire is 5.0A rightward. Thereafter, the current changes according to $i = 5A - (2A/s^2)t^2$. The straight wire is insulated; so there is no electrical contact between the current induced and the wire of the loop. What is the magnitude of the current induced in the loop at times t>0?

Since the field lines encircle the straight wire, every field line that passes through the circle upward also passes through the circle downward, so the net flux through the circle is always zero, no matter what the current is. The flux never changes, so the emf induced is 0.

30.13 A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field B as indicated. The coil is connected to co rotating cylinders against which metal brushes slide to make contact. (a) Show that the emf matches the given expression. (b) What value of Nab gives an emf such that its maximum value is 150V. Take the angular speed to be 60 rev/s and the field to be 0.5 T.

This is a classic induced emf problem. You are being asked to calculate the expression that describes how a generator works.

We begin by writing out the flux for the loop in the configuration below (shown edge on).



$$\begin{aligned}\Phi_B &= BA\cos\theta \\ &= BNab\cos\theta \\ \theta &= \omega t = 2\pi f t \\ \Phi_B &= BA\cos 2\pi f t\end{aligned}$$

Note that since the loop is rotating, we have substituted in for the angle in terms of the angular velocity and then frequency. We can now find the induced emf

$$\Phi_B = B\ Nab\cos 2\pi f t$$
$$\varepsilon = -\frac{d\Phi_B}{dt} = 2\pi f\ BNab\sin(2\pi f t)$$

The maximum value of the emf occurs when the sine is 1. We can now solve for the loop parameters.

$$\varepsilon_0 = 2\pi f\ BNab$$
$$Nab = \frac{\varepsilon_0}{2\pi f B} = \frac{150V}{2\pi \cdot 60 \cdot 0.5T} = 0.796$$

30.15 In Fig. 30-48, a stiff wire bent into a semicircle of radius $a = 2.0cm$ is rotated at constant angular speed 40 rev/s in a uniform 20 mT magnetic field. What are the (a) frequency and (b) amplitude of the emf in the loop.

This problem is really the same as 30.13, except the area is a half-circle.

$$\begin{aligned} \varphi_B &= BA\cos\theta \\ &= BNab\cos\theta \\ \theta &= \omega t = 2\pi f t \\ \varphi_B &= BA\cos 2\pi f t \end{aligned}$$

$$\varphi_B = B\frac{\pi r^2}{2}\cos 2\pi f t$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = 2\pi f B\frac{\pi r^2}{2}\sin(2\pi f t)$$

$$2\pi f B\frac{\pi r^2}{2} = 3.15\times 10^{-3}V$$

The frequency is 40 Hz.

30.29 If 50.0 cm of copper wire (diameter 1.00mm) is formed into a circular loop and placed perpendicular to a uniform magnetic field that is increasing at the constant rate of $10mT/s$, at what rate i thermal energy generated in the loop.

We find the area of the wire loop first.

$$\begin{aligned} 0.5m &= 2\pi r \\ r &= \frac{0.5m}{2\pi} = 7.96\times 10^{-2}m \\ A_{loop} &= \pi r^2 = 1.99\times 10^{-2}m^2 \end{aligned}$$

Now that we know the area, we can compute the change in flux and induced emf.

$$|\,\varepsilon\,| = \left|\frac{d\varphi_B}{dt}\right| = \left|\frac{d}{dt}(BA_{loop})\right| = A_{loop}\frac{dB}{dt} = 1.99\times 10^{-4}Tm^2$$

The power will depend on the resistance of the wire.

$$\begin{aligned} R &= \rho_{Cu}\frac{L}{A_{wire}} = 1.69\times 10^{-8}\Omega m\cdot\frac{0.5m}{\pi(0.5\times 10^{-3}m)^2} = 1.08\times 10^{-2}\Omega \\ P &= \frac{\varepsilon^2}{R} = 3.67\times 10^{-6}W \end{aligned}$$

30.31 In Fig. 30-56, a metal rod is forced to move with constant velocity $\vec{v}$ along two parallel metal rails connected with a strip o f metal at one end. A magnetic field of magnitude B=0.350T points out of the page. (a) If the rails are separated by L=25.0 cm and the speed of the rod is 55.0 cm/s, what emf is generated? (b)If the rod has a resistance of 18 Ohms and the rails and connectors have negligible resistance, what is the current in the rod? (c) At what rate is energy being transferred to the thermal energy

The emf generated is given by calculating the rate at which the flux is changing. In this problem, the flux changes because the area changes.

$$\varphi = BA = Blvt$$
$$|\varepsilon| = \left|-\frac{d\varphi}{dt}\right| = |-Blv| = 0.350 \cdot 0.25m \cdot 0.55m/s = 0.048V$$

Now that we know the emf, we can compute the current.

$$i = \frac{\varepsilon}{R} = \frac{0.048V}{18\Omega} = 0.00267\ A$$

We can also compute the power.

$$P = i^2R = (0.00267\ A)^2 \cdot 18\Omega = 1.29 \times 10^{-4}W$$

30.34 In Fig. 30-58, a long rectangular conducting loop of width L, resistance R and mass m is hung in a horizontal uniform magnetic field B that is directed into the page and that exists only above line aa. The loop is then dropped during its fall, it accelerates until it reaches a certain terminal speed v_t. Ignoring air drag, find the an expression for v_t.

The terminal velocity is reached when the upward force on the top wire equals the weight of the loop. This force arises because of the current running through this wire. We begin by writing an equation the sets the forces equal

$$mg = iLB$$
$$i = \frac{mg}{LB}$$

Now we need to find the current. We do this by first writing the emf and then finding the current.

$$|\varepsilon| = BLv_t$$
$$i = \frac{\varepsilon}{R} = \frac{BLv_t}{R}$$

The final step is to set the current expressions equal and solve for the velocity

$$\frac{BLv_t}{R} = \frac{mg}{LB}$$
$$v_t = \frac{mgR}{L^2B^2}$$

30.40 The inductance of a closely packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5 mA.

$$L = \frac{N\varphi}{i}$$
$$\varphi = \frac{iL}{N} = \frac{5\times 10^{-3}A\cdot 8\times 10^{-3}H}{400} = 1\times 10^{-7}Tm^2$$

30.44 A 12 H inductor carries a current of 2A. At what rate must the current be changed to produce a 60V emf in the inductor.

$$\varepsilon = L\frac{di}{dt}$$
$$\frac{di}{dt} = \frac{\varepsilon}{L} = \frac{60V}{12H} = 5A/s$$

30.50 The switch in Fig 30-20 is closed on a at time t=0. What is the ratio $\varepsilon_L/\varepsilon$ of the inductor's self-induced emf to the battery's emf (a) just after t=0 and (b) at $t = 2.00\tau_L$? (c) At what multiple of τ_L will $\varepsilon_L/\varepsilon = 0.5$?

The equation for rising current in an RL circuit is

$$i = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$$

From this expression, we can find the voltage across the inductor.

$$\varepsilon_L = L\frac{di}{dt} = L(\frac{\varepsilon}{R}\cdot\frac{1}{\tau_L}\cdot e^{-t/\tau_L})$$
$$\tau_L = \frac{L}{R}$$
$$\varepsilon_L = \varepsilon e^{-t/\tau_L}$$
$$\frac{\varepsilon_L}{\varepsilon} = e^{-t/\tau_L}$$

(a) At t=0

$$\frac{\varepsilon_L}{\varepsilon} = 1$$

(b) At $t = 2.00\tau_L$

$$\frac{\varepsilon_L}{\varepsilon} = e^{-2\tau_L/\tau_L} = e^{-2}$$

(c) The time for $\varepsilon_L/\varepsilon = 0.5$ is

$$\frac{\varepsilon_L}{\varepsilon} = \frac{1}{2} = e^{-t/\tau_L}$$
$$\ln(\frac{1}{2}) = -\frac{t}{\tau_L}$$
$$t = -\tau_L \ln(\frac{1}{2})$$

30.55 A solenoid having an inductance of $6.30\ \mu H$ is connected in series with a $1.2\ k\Omega$ resistor. (a) If a 14.0 V battery is connected across the pair, how long will it take for the current through the resistor to reach 80% of its final value? b) What is the current through the resistor at time $t = 1.0\tau_L$

It's convenient to compute the time constant first.

$$\tau_L = \frac{L}{R} = \frac{6.30 \times 10^{-6} H}{1.2 \times 10^{3} \Omega} = 5.25 \times 10^{-9} s$$

We can write the current in the RL circuit and find the time for part (a)

$$i = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L}) = i_s(1 - e^{-t/\tau_L})$$
$$\frac{i}{i_s} = 0.8 = 1 - e^{-t/\tau_L}$$
$$e^{-t/\tau_L} = 1 - 0.8 = 0.2$$
$$-\frac{t}{\tau_L} = \ln(0.2)$$
$$t = -\tau_L \ln(0.2) = 8.45 \times 10^{-9} s$$

We can find the current at one time constant.

$$i = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L}) = \frac{14V}{1.2 \times 10^{3}\Omega}(1 - e^{-\tau_L/\tau_L}) = 7.37 \times 10^{-3} A$$

30.61 At t=0, a battery is connect to a series arrangement of a resistor and an inductor. If the inductive time constant is 37 ms, at what time is the rate at which the energy is dissipated in the resistor equal to the rate at which energy is stored in the inductors's magnetic field.

$$
\begin{aligned}
&P_R = i^2 R = \left(\frac{\varepsilon}{R}(1 - e^{-t/\tau_L})\right)^2 R \\
&\frac{dU_L}{dt} = \frac{d}{dt}(\frac{1}{2}Li^2) = Li\frac{di}{dt} = L(\frac{\varepsilon}{R}(1 - e^{-t/\tau_L}))(\frac{\varepsilon}{R}\frac{e^{-t/\tau_L}}{\tau_L}) \\
&\frac{dU_L}{dt} = P_R \\
&L(\frac{\varepsilon}{R}(1 - e^{-t/\tau_L}))(\frac{\varepsilon}{R}\frac{e^{-t/\tau_L}}{\tau_L}) = \left(\frac{\varepsilon}{R}(1 - e^{-t/\tau_L})\right)^2 R \\
&\frac{\varepsilon^2}{R}(1 - e^{-t/\tau_L}) \cdot e^{-t/\tau_L} = \frac{\varepsilon^2}{R}(1 - e^{-t/\tau_L})^2 \\
&e^{-t/\tau_L} - e^{-2t/\tau_L} = 1 - 2e^{-t/\tau_L} + e^{-2t/\tau_L} \\
&3e^{-t/\tau_L} - 2e^{-2t/\tau_L} = 1 \\
&2e^{-2t/\tau_L} - 3e^{-t/\tau_L} + 1 = 0 \\
&x = e^{-t/\tau_L} \\
&2x^2 - 3x - 1 = 0 \\
&x = \frac{+3 - \sqrt{3^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} = \frac{1}{2} \\
&e^{-t/\tau_L} = \frac{1}{2} \\
&t = -\tau_L \ln(\frac{1}{2})
\end{aligned}
$$

Note: The book has this as the answer for 30.59. It's actually the answer to 30.61.

30.67 What must be the magnitude of a uniform electric field if it is to have the same energy density as that possessed by a 0.50 T magnetic field?

$$
\begin{aligned}
&\frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2\mu_0}B^2 \\
&E = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}B = 1.5 \times 10^8 V/m
\end{aligned}
$$