

Chapter 30

30.1 In Fig. 30-37, the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0 t^2 + 7.0 t$ where the flux is in milliWebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop at $t=2.0s$? (b) In what direction will the current flow through the resistor.

The magnitude of the induced emf is given by

$$\begin{aligned}\Phi_B &= 6.0 t^2 + 7.0 t \\ |\varepsilon| &= \left| \frac{d\Phi_B}{dt} \right| = 12.0 t + 7.0 \\ |\varepsilon(2)| &= 12.0 \cdot 2 + 7.0 = 31 mV\end{aligned}$$

The flux is pointing out of the paper and rising. The coil reacts to oppose this rising outward field by generating a field that points into the paper. To do this, the current must flow to the left through the resistor.

30.3 A small loop of area 6.8 mm^2 is placed inside a long solenoid that has 854 turns/cm and carries a sinusoidally varying current i of amplitude 1.28 A and angular frequency 212 rad/s. The central axis of the loop and the solenoid coincide. What is the amplitude of the emf induced in the loop

The emf generated can be calculated by finding the rate of change of magnetic flux through the inner loop that results from the changing field due to the solenoid

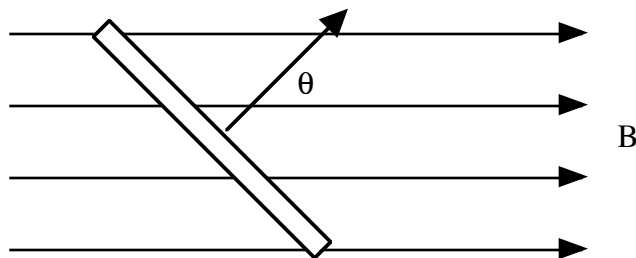
$$\begin{aligned}B &= \mu_0 n i = \mu_0 n i_0 \sin \omega t \\ \varphi_B &= B A_{loop} = \mu_0 n i_0 \sin \omega t \cdot A_{loop} \\ \frac{d\varphi_B}{dt} &= \omega \mu_0 n i_0 A_{loop} \cos \omega t\end{aligned}$$

$$\begin{aligned}\omega \mu_0 n i_0 A_{loop} &= 212 \cdot 4\pi \times 10^{-7} \cdot 85400 \text{ turns / m} \cdot 1.28 A \cdot 6.8 \times 10^{-6} m^2 \\ &= 1.98 \times 10^{-4} V\end{aligned}$$

30.13 A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field B as indicated. The coil is connected to co rotating cylinders against which metal brushes slide to make contact. (a) Show that the emf matches the given expression. (b) What value of Nab gives an emf such that its maximum value is 150V. Take the angular speed to be 60 rev/s and the field to be 0.5 T.

This is a classic induced emf problem. You are being asked to calculate the expression that describes how a generator works.

We begin by writing out the flux for the loop in the configuration below (shown edge on).



$$\begin{aligned}\Phi_B &= BA \cos \theta \\ &= B N a b \cos \theta \\ \theta &= \omega t = 2\pi f t \\ \Phi_B &= B A \cos 2\pi f t\end{aligned}$$

Note that since the loop is rotating, we have substituted in for the angle in terms of the angular velocity and then frequency. We can now find the induced emf

$$\begin{aligned}\Phi_B &= B N a b \cos 2\pi f t \\ \varepsilon &= -\frac{d\Phi_B}{dt} = 2\pi f B N a b \sin(2\pi f t)\end{aligned}$$

The maximum value of the emf occurs when the sine is 1. We can now solve for the loop parameters.

$$\begin{aligned}\varepsilon_0 &= 2\pi f B N a b \\ N a b &= \frac{\varepsilon_0}{2\pi f B} = \frac{150V}{2\pi \cdot 60 \cdot 0.5T} = 0.796\end{aligned}$$

30.15 In Fig. 30-48, a stiff wire bent into a semicircle of radius $a = 2.0\text{cm}$ is rotated at constant angular speed 40 rev/s in a uniform 20 mT magnetic field. What are the (a) frequency and (b) amplitude of the emf in the loop.

This problem is really the same as 30.13, except the area is a half-circle.

$$\begin{aligned}\varphi_B &= B A \cos \theta & \varphi_B &= B \frac{\pi r^2}{2} \cos 2\pi f t \\ &= B N a b \cos \theta \\ \theta &= \omega t = 2\pi f t & \varepsilon &= -\frac{d\varphi_B}{dt} = 2\pi f B \frac{\pi r^2}{2} \sin(2\pi f t) \\ \varphi_B &= B A \cos 2\pi f t\end{aligned}$$

$$2\pi f B \frac{\pi r^2}{2} = 3.15 \times 10^{-3} V$$

The frequency is 40 Hz.

30.29 If 50.0 cm of copper wire (diameter 1.00mm) is formed into a circular loop and placed perpendicular to a uniform magnetic field that is increasing at the constant rate of 10mT/s , at what rate is thermal energy generated in the loop.

We find the area of the wire loop first.

$$0.5\text{m} = 2\pi r$$

$$r = \frac{0.5\text{m}}{2\pi} = 7.96 \times 10^{-2}\text{m}$$

$$A_{\text{loop}} = \pi r^2 = 1.99 \times 10^{-2}\text{m}^2$$

Now that we know the area, we can compute the change in flux and induced emf.

$$|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| = \left| \frac{d}{dt}(BA_{\text{loop}}) \right| = A_{\text{loop}} \frac{dB}{dt} = 1.99 \times 10^{-4}\text{Tm}^2$$

The power will depend on the resistance of the wire.

$$R = \rho_{\text{Cu}} \frac{L}{A_{\text{wire}}} = 1.69 \times 10^{-8}\Omega\text{m} \cdot \frac{0.5\text{m}}{\pi(0.5 \times 10^{-3}\text{m})^2} = 1.08 \times 10^{-2}\Omega$$

$$P = \frac{\mathcal{E}^2}{R} = 3.67 \times 10^{-6}\text{W}$$

30.30 A loop antenna of area $A = 2.00\text{cm}^2$ and resistance $5.21\mu\Omega$ is perpendicular to a uniform magnetic field of magnitude $17\mu\text{T}$. The field magnitude drops to zero in 2.96 ms. How much thermal energy is produced in the loop by the changing field.

$$U = P \cdot t$$

$$P = i^2 R = \left(\frac{\mathcal{E}}{R}\right)^2 R = \frac{\mathcal{E}^2}{R}$$

$$|\mathcal{E}| = \frac{\Delta\Phi}{\Delta t} = \frac{0 - 17 \times 10^{-6}\text{T} \cdot 2.00 \times 10^{-4}\text{m}^2}{2.96 \times 10^{-3}\text{s}} = 1.15 \times 10^{-6}$$

$$P = 2.53 \times 10^{-7}\text{W}$$

$$U = 7.495 \times 10^{-10}\text{J}$$