Chapter 29

29.5 In Fig. 29-36, two circular arcs have radii $a = 13.5\,cm$ and $b = 10.7\,cm$, subtend angle $\theta = 74°$, carry current $i = 0.411A$, and share the same center of curvature P. What are (a) the magnitude and (b direction of the net field at P.

This is exactly the same kind of problem as 29.6. See the full derivation below. Only the radii and angles are different. We begin near the end of the previous problem, changing the limits of integration

$$\vec{B} = \frac{\mu_0 i}{4\pi\, a} \int_{-\theta/2}^{\theta/2} d\theta\, \hat{k} = \frac{\mu_0 i}{4\pi\, a} \cdot \pi\, \hat{k}$$

$$= \frac{\mu_0 i}{4\pi\, a} \cdot \theta\, \hat{k}$$

$$\theta = \frac{74°}{180°} \cdot \pi = 0.4111\pi$$

$$\vec{B} = \frac{\mu_0 i}{4\pi\, a} 0.4111\pi \cdot \hat{k}$$

$$= \frac{0.4111\mu_0 i}{4\,a} \hat{k}$$

To get the full field, we can add the contributions from each arc, and again taking into the paper as positive.

$$\vec{B} = \left(\frac{0.4111\mu_0 i}{4\,b} - \frac{0.4111\mu_0 i}{4\,a}\right)\hat{k}$$

$$= -1.028 \times 10^{-7} T$$

Field points out of paper.

29.6 In Fig. 29-38, two semicircular arcs have radii $R_2 = 7.8\,cm$ and $R_1 = 3.15\,cm$ and carry current $i = 0.281A$, and share the same center of curvature C. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at C?

To do this problem, we begin by writing the Biot-Savart Law and describe what the field at (0,0,0) will look like.

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{i\, d\vec{l} \times \hat{r}}{r^2}$$

Carefully write $\vec{r}, r, \hat{r}$, and $d\vec{l}$ for the tiny current element given. Hint. You can find $d\vec{l}$ by taking the derivative of $\vec{r}$, Our observation point is taken to be the origin (0,0,0). Note that we are finding the field due to a generic arc with radius a that is centered on the origin.

$$\vec{l} = x\hat{i} + y\hat{j} = a\cos\theta\hat{i} + a\sin\theta\,\hat{j}$$

$$\frac{d\vec{l}}{d\theta} = -a\sin\theta\hat{i} + a\cos\theta\,\hat{j}$$

$$d\vec{l} = (-a\sin\theta\hat{i} + a\cos\theta\,\hat{j})d\theta$$

$$\vec{r} = (0 - a\cos\theta)\hat{i} + (0 - a\sin\theta)\,\hat{j} + (0-0)\hat{k}$$
$$= -a\cos\theta\hat{i} - a\sin\theta\,\hat{j}$$

$$r = \sqrt{(-a\cos\theta)^2 + (a\sin\theta)^2} = \sqrt{a^2} = a$$

$$\hat{r} = \frac{-a\cos\theta\hat{i} - a\sin\theta\,\hat{j}}{a} = -\cos\theta\hat{i} - \sin\theta\,\hat{j}$$

Set up to compute the field at (0,0,0). Be sure to show that the field that results will only point in the z direction.

$$\vec{B} = \frac{\mu_0}{4\pi}\int_0^{\pi}\frac{i\,d\vec{l}\times\hat{r}}{r^2} =$$
$$= \frac{\mu_0 i}{4\pi}\int_0^{\pi}\frac{(-a\sin\theta\,\hat{i} + a\cos\theta\,\hat{j})\times(-\cos\theta\hat{i} - \sin\theta\,\hat{j})}{a^2}d\theta$$
$$= \frac{\mu_0 i}{4\pi a^2}\int_0^{\pi}(-a\sin\theta\,\hat{i} + a\cos\theta\,\hat{j})\times(-\cos\theta\hat{i} - \sin\theta\,\hat{j})d\theta$$
$$= \frac{\mu_0 i}{4\pi a}\int_0^{\pi}(\sin^2\theta + \cos^2\theta)\hat{k}\,d\theta$$

$$\vec{B} = \frac{\mu_0 i}{4\pi a}\int_0^{2\pi}(\sin^2\theta + \cos^2\theta)\hat{k}\,d\theta$$
$$= \frac{\mu_0 i}{4\pi a}\int_0^{\pi}d\theta\hat{k} = \frac{\mu_0 i}{4\pi a}\cdot\pi\hat{k}$$
$$= \frac{\mu_0 i}{4a}\hat{k}$$

To get the total field, we sum contributions from each half circle. If we count into the paper as positive,

$$\vec{B} = \left(\frac{\mu_0 i}{4R_1} - \frac{\mu_0 i}{4R_2}\right)\hat{k}$$
$$= 1.67\times10^{-6}T$$

29.8 In Fig. 29-39, a wire forms a semicircle of radius $R = 9.26cm$ and two radial straight segments each of length $L = 13.1cm$. The wire carries current $i = 34.8mA$. What are (a) magnitude and (direction of the net magnetic field

The radial wires do not contribute to the field. The field due to the arc is given by

$$B = \frac{\mu_0 i\varphi}{4\pi R}$$

$$\varphi = \pi$$

$$B = \frac{\mu_0 i}{4R} = 1.18\times 10^{-7}T$$

29.12 In Fig. 29-43, point P is at perpendicular distance $R = 2.00cm$.from a very long straight wire carrying a current. The magnetic field $\vec{B}$ set up at point P is due the contributions from all the identical current length elements $i\,d\vec{s}$ along the wire. What is the distance to the element making the (a) the greatest contribution to the field $\vec{B}$ and (b) 10% of the grates contribution?

Since the unit vector always has length 1, all we need to worry about is the r^2 in the denominator.

$$d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{i\,d\vec{s}\times\hat{r}}{r^2}$$

$$i\,d\vec{s} = i\,dx\,\hat{i}$$

$$r^2 = s^2 + P^2$$

The piece of current that produces the biggest contribution is the one with the smallest r. This occurs at s=0. To find the 10% point, we can solve the following equation for s

$$\frac{1}{s_{10}^2 + P^2} = 0.10\cdot\frac{1}{P^2}$$

$$\frac{P^2}{s_{10}^2 + P^2} = 0.10$$

$$\frac{s_{10}^2 + P^2}{P^2} = 10$$

$$s_{10}^2 + P^2 = 10P^2$$

$$s_{10}^2 = 9P^2$$

$$s_{10} = 3P = 6.00cm$$

29.28 In Fig 29-56, part of a long insulate wire carrying current $i = 5.78mA$ is bent into a circular section of radius $R = 1.89cm$. In unit-vector notation, what is the magnetic field at the center of curvature Cif the circular section (a) lies in the plain of the page as shown and J(b) is perpendicular to the plane of the page after being rotated 90 degrees as indicated

The resultant field can be thought of as just the vector sum of the field due to an infinite wire and a circular loop. In case (a), both fields point out of the page, in the k direction, while in (b), the field due to the loop points in the i direction.

$$B_{straightwire} = \frac{\mu_0 i}{2\pi R} = 6.12\times 10^{-8} T$$

$$B_{loop} = \frac{\mu_0 i}{2R} = 1.92\times 10^{-7} T$$

$$\vec{B} = 6.12\times 10^{-8}\,\hat{k} + 1.92\times 10^{-7}\,\hat{k}$$

$$= 2.534\times 10^{-7}\,\hat{k}$$

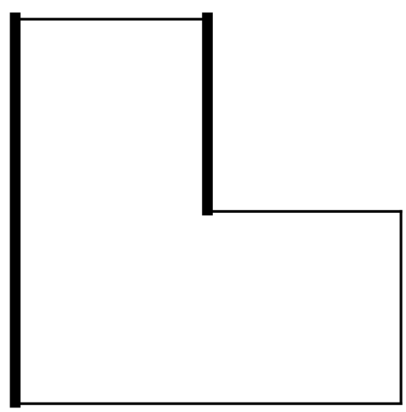
$$B_{straightwire} = \frac{\mu_0 i}{2\pi R} = 6.12\times 10^{-8} T$$

$$B_{loop} = \frac{\mu_0 i}{2R} = 1.92\times 10^{-7} T$$

$$\vec{B} = 1.92\times 10^{-7}\,\hat{i} + 6.12\times 10^{-8}\,\hat{k}$$

29.33 In Fig. 29-61, length a is 4.7 cm and current i is 13A. What are the magnitude and direction of the magnetic field at point P. Note that these are not long sections of wire.

By symmetry, we actually only have two independent calculations to do: one for the long side and one for the short side. (shown darkened below).



We begin with the long side. We consider the vertical side with length 2a, running from 0 to 2a, taking the point P at (2a,2a).

$$\vec{B} = \frac{\mu_0}{4\pi}\int_{-\infty}^{+\infty} \frac{i\,d\vec{l}\times\hat{r}}{r^2}$$

$$d\vec{l} = dy\,\hat{j}$$

$$\vec{r} = (2a-0)\,\hat{i} + (2a-y)\,\hat{j} + (0-0)\hat{k}$$

$$r = \sqrt{(2a-0)^2 + (2a-y)^2 + (0-0)^2}$$

$$\hat{r} = \frac{(2a)\,\hat{i} + (2a-y)\,\hat{j}}{\sqrt{(2a)^2 + (2a-y)^2}}$$

We can compute the field (out of paper).

$$\vec{B} = \frac{\mu_0}{4\pi}\int_{2a}^{0} \frac{i(dy\,\hat{j})\times((2a)\,\hat{i} + (2a-y)\,\hat{j})}{\left((2a)^2 + (2a-y)^2\right)^{3/2}}$$

$$= \frac{\mu_0}{4\pi}\int_{2a}^{0} \frac{i(dy\,\hat{j})\times((2a)\,\hat{i} + (2a-y)\,\hat{j})}{\left((2a)^2 + (2a-y)^2\right)^{3/2}}$$

$$= -\frac{\mu_0 i 2a}{4\pi}\int_{0}^{2a} \frac{dy\,\hat{k}}{\left((2a)^2 + (2a-y)^2\right)^{3/2}}$$

$$= \frac{\mu_0 ia}{2\pi}\cdot\frac{1}{4\sqrt{2}a^2}\hat{k}$$

$$= \frac{\mu_0 i}{8\sqrt{2}\pi}\cdot\frac{1}{a}\hat{k}$$

Now we can calculate the field due to one of the short sides that is a distance a from P. We'll think of the short length as lying along the y axis again. In this case, the point P is at (a,a)

$$\vec{B} = \frac{\mu_0}{4\pi}\int_{-\infty}^{+\infty} \frac{i\,d\vec{l}\times\hat{r}}{r^2}$$

$$d\vec{l} = dy\,\hat{j}$$

$$\vec{r} = (a-0)\,\hat{i} + (a-y)\,\hat{j} + (0-0)\,\hat{k}$$

$$r = \sqrt{(a-0)^2 + (a-y)^2 + (0-0)^2}$$

$$\hat{r} = \frac{(a)\,\hat{i} + (a-y)\,\hat{j}}{\sqrt{(a)^2 + (a-y)^2}}$$

We can compute the field into the paper.

$$\vec{B} = \frac{\mu_0}{4\pi}\int_0^a \frac{i(dy\,\hat{j})\times(a\,\hat{i}+(a-y)\,\hat{j})}{\left((a)^2+(a-y)^2\right)^{3/2}}$$

$$= \frac{\mu_0}{4\pi}\int_0^a \frac{i(dy\,\hat{j})\times(a\,\hat{i}+(a-y)\,\hat{j})}{\left((a)^2+(2a-y)^2\right)^{3/2}}$$

$$= -\frac{\mu_0 i a}{4\pi}\int_0^a \frac{dy\,\hat{k}}{\left((a)^2+(a-y)^2\right)^{3/2}}$$

$$= -\frac{\mu_0 ia}{4\pi}\cdot\frac{1}{\sqrt{2}a^2}$$

$$= -\frac{\mu_0 i}{4\sqrt{2}\pi}\cdot\frac{1}{a}$$

The net field is

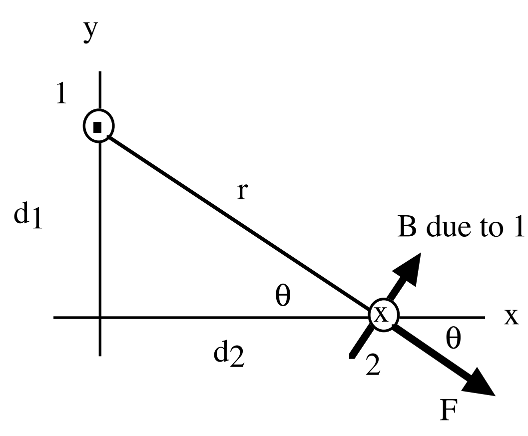
$$B = 2\frac{\mu_0 i}{8\sqrt{2}\pi}\cdot\frac{1}{a}\hat{k} - 2\frac{\mu_0 i}{4\sqrt{2}\pi}\cdot\frac{1}{a}\hat{k}$$

$$= -\frac{\mu_0 i}{4\sqrt{2}\pi}\cdot\frac{1}{a}\hat{k}$$

$$= -1.96\times10^{-5}T\hat{k}$$

into of paper.

29.35 Figure 29-63 shows wire 1 in cross section; The wire is long and straight, carries a current of 4mA out of the page and is at distance $d_1 = 2.40cm$ from a surface. Wire 2, which is parallel to wire 1 and also long, is at horizontal distance $d_2 = 5.00cm$ from wire 1 and carries a current of 6.80 mA into the page. What is the x component of the magnetic force per unit length on wire 2 due to wire 1?



Wires with opposite current REPEL. We can see this by computing the magnitude of B that wire 2 sees due to Wire 1, and then computing the force on wire 2 due to that field. We then compute the x-component.

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{2\pi\sqrt{d_1^2 + d_2^2}} = 1.44 \times 10^{-8} T$$
$$F = ilB$$
$$F_l = \frac{F}{l} = iB = 9.81 \times 10^{-11} N/m$$
$$F_{lx} = F_l \cos\theta = 8.84 \times 10^{-11} N/m$$

29.43 Each of the eight conductors in Fig 29-68 carries 2A of current into or out of the page. Two paths are indicated for the line integral. What is the value of the integral for (a) path 1 and (b) path 2.

The value of the integral is equal to $\mu_0\, i_{enc}$. For path (a) $\mu_0\, i_{enc} = \mu_0(2A + 2A - 2A) = \mu_0 \cdot 2A$ and for path (b) $\mu_0\, i_{enc} = \mu_0(2A + 2A - 2A - 2A) = 0$ Note that the sign depends on how you define the coordinate system.

29.45 Figure 29-70 shows a cross section across a diameter of a long cylindrical conductor of radius $a = 2.00cm$ and carrying a uniform current 170 A. What is the magnitude of the current's magnetic field at radial distance (a) 0, (b) 1.00 cm (c) 2.00 (wire surface) and (d) 4 cm.

We begin by computing the current density:

$$J = \frac{i}{\pi a^2} = \frac{170A}{\pi(0.02m)^2} = 1.353 \times 10^5 A/m^2$$

We can now use Ampere's Law

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 i$$

For r<a

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 i$$
$$B \cdot (2\pi r) = \mu_0 J \pi r^2$$
$$B = \frac{\mu_0 J r}{2}$$

For r>=a

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 i$$
$$B \cdot (2\pi r) = \mu_0 J \pi a^2$$
$$B = \frac{\mu_0 J a^2}{2r}$$

You can compute at various radii.

29.49 A 200 turn solenoid having a length of 25 cm and a diameter of 10 cm carries a current of 0.29 A. Calculate he magnitude of the magnetic filed B inside the solenoid.

$$B = \mu_0 n i = 4\pi \times 10^{-7} \cdot \frac{200}{0.25m} \cdot 0.29A = 2.92 \times 10^{-4} T$$

29.51 A toroid having a square cross section, 5.0 cm on a side and an inner radius of 15.0 cm has 500 turns and carries a current of 0.800 A. What is the magnetic field at (a) the inner radius and (b) the outer radius.

This is also an Ampere's law problem that we worked out in class.

$$B = \frac{N\mu_0 i}{2\pi r}$$

$$B_{inner} = \frac{500\mu_0 \cdot 0.8A}{2\pi \cdot 0.15} = 5.33 \times 10^{-4}$$

$$B_{outr} = \frac{500\mu_0 \cdot 0.8A}{2\pi \cdot 0.20} = 4.00 \times 10^{-4}$$