Chapter 27

27.2 A certain car battery with 12.0 V emf has an initial carge of 120 A h. Assuming that the potential across the terminals stays constant until the battery is completely discharged, for how many hours can it deliver energy at a the rate of 100W.

The total energy stored in the battery is

$$U = 120 A\,h \cdot \frac{3600 s}{1 h} = 4.32 \times 10^5 J$$

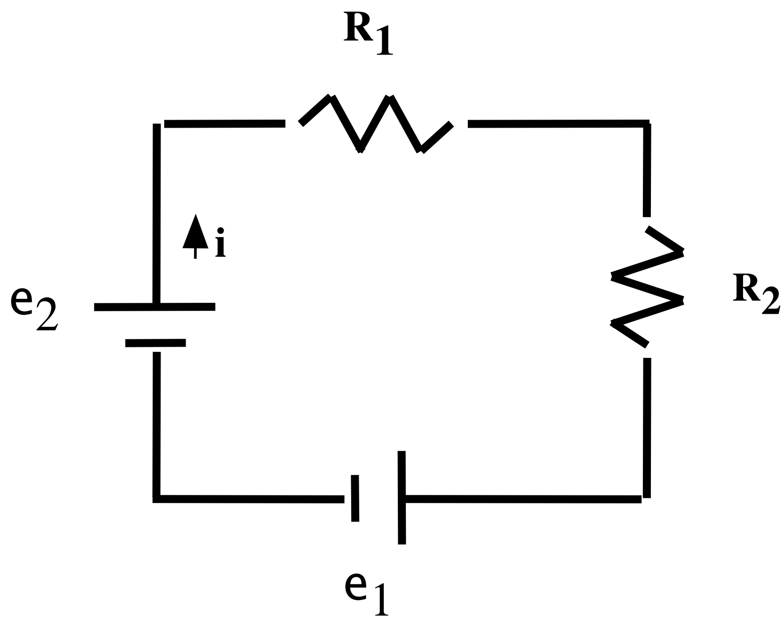
Now we compute how long we can deliver a power of 100 W with this energy available.

$$\begin{aligned} U &= P \cdot t \\ t &= \frac{U}{P} = \frac{4.32 \times 10^5 J}{100 W} \\ t &= 4.32 \times 10^3 s \\ &= 1.2 h \end{aligned}$$

27.3 A 5.0 A current is set up in a circuit for 6.0 min by a rechargeable battery with a 6.0 V emf. By how much is the chemical energy of the battery reduced.

$$\begin{aligned} U &= P \cdot t \\ P &= iV = 5A \cdot 6.0V = 30\ W \\ U &= 30W \cdot 360\ s = 10{,}800J \end{aligned}$$

27.7 In Fig. 27-25, the ideal batteries have emfs $\varepsilon_1 = 12V$ and $\varepsilon_1 = 6V$ and the resistors have resistances $R_1 = 4.0\Omega$ and $R_2 = 8.0\Omega$. What are (a) the current, the dissipation rate in (b) resistor 1 and (c)resistor 2, and the energy transfer rate in (d) battery 1 and (e) battery 2? Is energy being supplied or absorbed by (f) battery 1 and (g) battery 2?



We can solve for the current by first assuming a current direction. We assume a current i as shown above. We begin our analysis path in the lower left hand corner and use the sign convention. Kirchoff's voltage loop rule yields

$$+\varepsilon_2 - iR_1 - iR_2 - \varepsilon_1 = 0$$
$$i = \frac{\varepsilon_2 - \varepsilon_1}{R_1 + R_2} = \frac{6-12}{4+8} = -0.5A$$

The minus sign indicates that the current actually flows in the opposite direction than we assumed.
The current in resistors 1 and 2 is 0.5 A and it flows in the counterclockwise direction. The power in each resistor is:

$$P_1 = i^2 R_1 = (-0.5A)^2 \cdot 4\Omega = 1W$$
$$P_2 = i^2 R_2 = (-0.5A)^2 \cdot 8\Omega = 2W$$

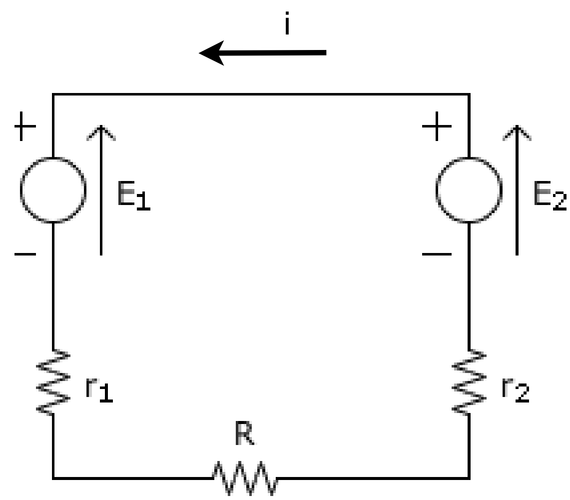
The energy transfer rate in each battery is

$$P_1 = i\,\varepsilon_1 = (0.5A)\cdot 12V = 6W$$
$$P_2 = i\,\varepsilon_2 = (0.5A)\cdot 6V = 3W$$

Energy is supplied by ε_1 and absorbed by ε_2. We know this because the current is flowing in the "correct" direction through ε_1 and "backward" through ε_2.

27.12 (a) In Fig. 27-29, what value much R have if the current in the circuit is to be 1mA. Take $\varepsilon_1 = 2.0\,V$, $\varepsilon_2 = 3.0\,V$ and $r_1 = r_2 = 3\,\Omega$. (b) What is the rate at which thermal energy appears in R

We can replace the resistors with a single resistor that is the series replacement of the internal resistors and the load resistor

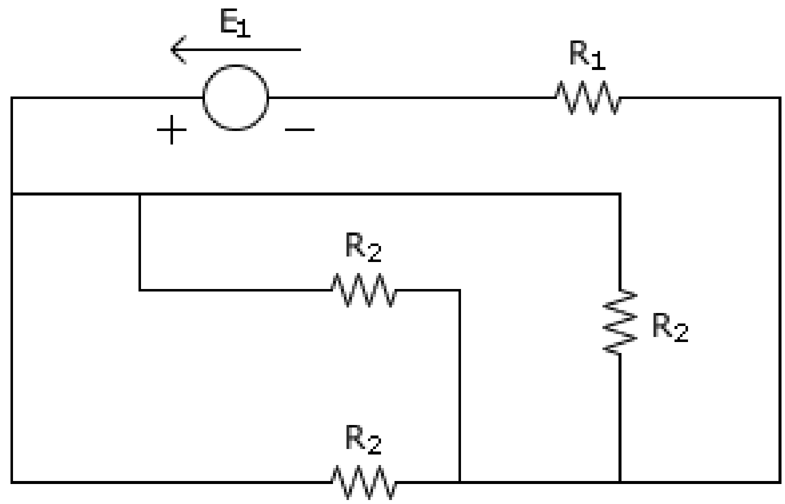


$$
\begin{aligned}
0 &= \varepsilon_1 - \varepsilon_2 + i(r_1 + r_2 + R) \\
r_1 + r_2 + R &= \frac{\varepsilon_2 - \varepsilon_1}{i} \\
R &= \frac{\varepsilon_2 - \varepsilon_1}{i} - (r_1 + r_2) \\
&= \frac{3V - 2V}{1\times 10^{-3}A} - (6\Omega) \\
&= 994\Omega
\end{aligned}
$$

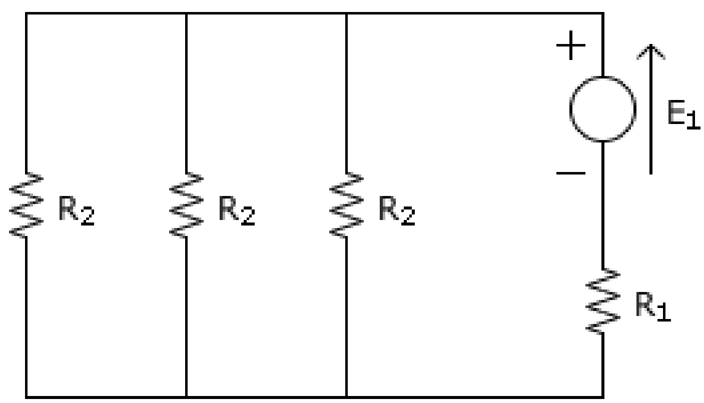
The power is given by

$$P = i^2 R = (1\times 10^{-3}A)^2 \cdot (994\Omega) = 9.94\times 10^{-4}\,W$$

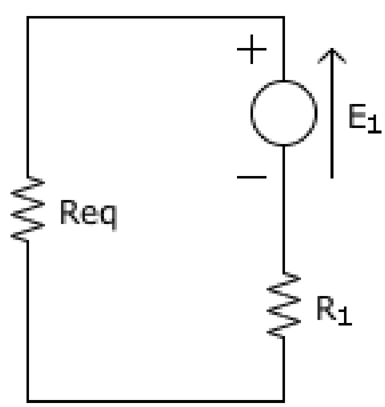
27.17 In Fig. 27-34, $R_1 = 6.00\Omega$, $R_2 = 18.00\Omega$, and the ideal battery has an emf $\varepsilon = 12.0V$. What are the (a) size and (b) direction of the current shown. (How much energy os dissipated by all four resistors in 1 min?



We can redraw this circuit



And find an equivalent resistance for the parallel R's.



$$\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_2} + \frac{1}{R_2} \Longrightarrow R_{eq} = \frac{R_2}{3}$$

We can now find the current in the circuit. We can compute the voltage drop across the equivalent resistor now that we know V.

$$i_{eq} = \frac{\varepsilon}{R_{eq} + R_1} = \frac{12.0V}{12\Omega} = 1A$$
$$V_{eq} = \varepsilon - i_{eq} R_1 = 6V$$

Now we can find the current through each parallel resistor.

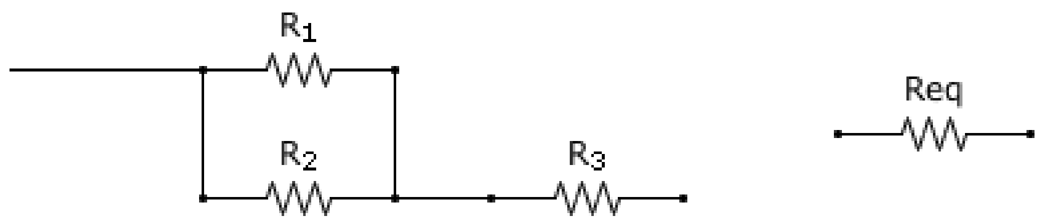
$$i_1 = \frac{6V}{18\Omega} = \frac{1}{3}A$$

Current is to the right.

The energy dissipated can be computed from the equivalent circuit

$$P = i_{eq}^2 R_1 + i_{eq}^2 R_{eq} = 12W$$
$$U = P \cdot t = 12W \cdot 60s = 720J$$

27.23 In Fig. 27-38, $R_1 = R_2 = 4\Omega$, and $R_3 = 2.5\Omega$. Find the equivalent resistance of this circuit.

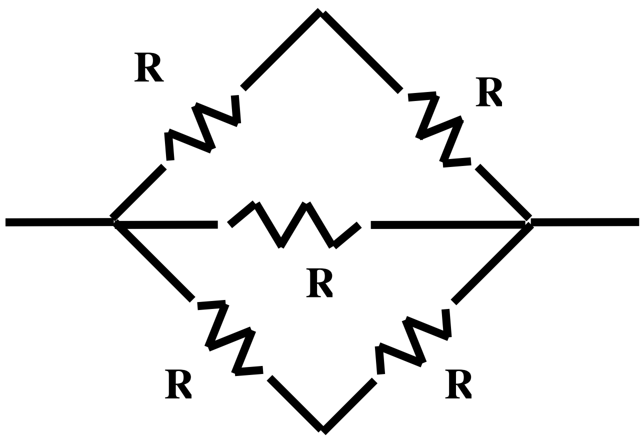


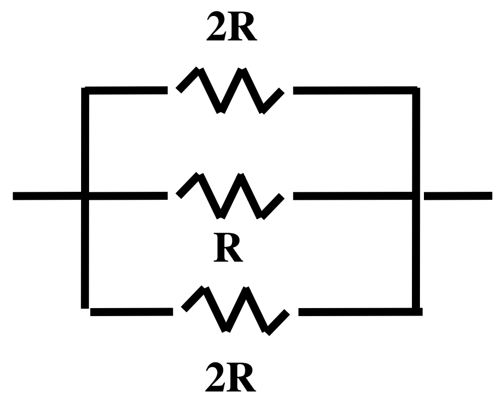
Here we combine R_1 and R_2 in parallel (to yield $R = 2\ \Omega$). This equivalent resistance is in series with R_3 and the final equivalent resistance is $R_{eq} = 4.5\Omega$

26.26 Figure 27-33 shows five 5Ω resistors. Find the equivalent resistance between points (a) F and H and (b) F and G. (Hint: For each pair of points, imagine that a battery is connected across the pair.)

The sequence of replacement and redrawing for F and H is shown below. In the last step, the three resistors in parallel are combined via

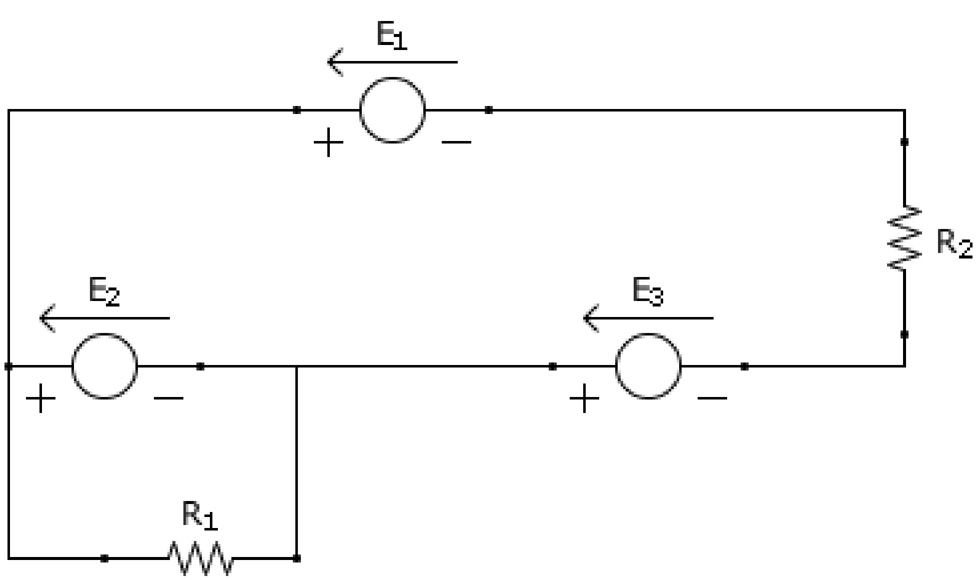
$$\begin{aligned}\frac{1}{R_{eq}} &= \frac{1}{2R} + \frac{1}{R} + \frac{1}{2R} \\ &= \frac{1}{2R} + \frac{2}{2R} + \frac{1}{2R} \\ &= \frac{4}{2R} \\ R_{eq} &= \frac{R}{2}\end{aligned}$$





Now we consider the resistance between F and G. There are a number of parallel and series replacements. You should check to see that I have done them correctly!

27.27 In Fig. 27-40, $R_1 = 100\Omega$, $R_2 = 50\Omega$ and the ideal batteries have emfs $\varepsilon_1 = 6.0\,V$, $\varepsilon_2 = 5.0\,V$, and $\varepsilon_3 = 4.0\,V$. Find (a) the current in resistor 1, (b) The current in resistor 2, and (c) the potential difference between points a and b.



a) The current in resistor 1 is entirely determined by the potential difference across it--given by emf 2

$$i = \frac{\varepsilon_2}{R_1} = \frac{5V}{100\Omega} = 0.05A$$

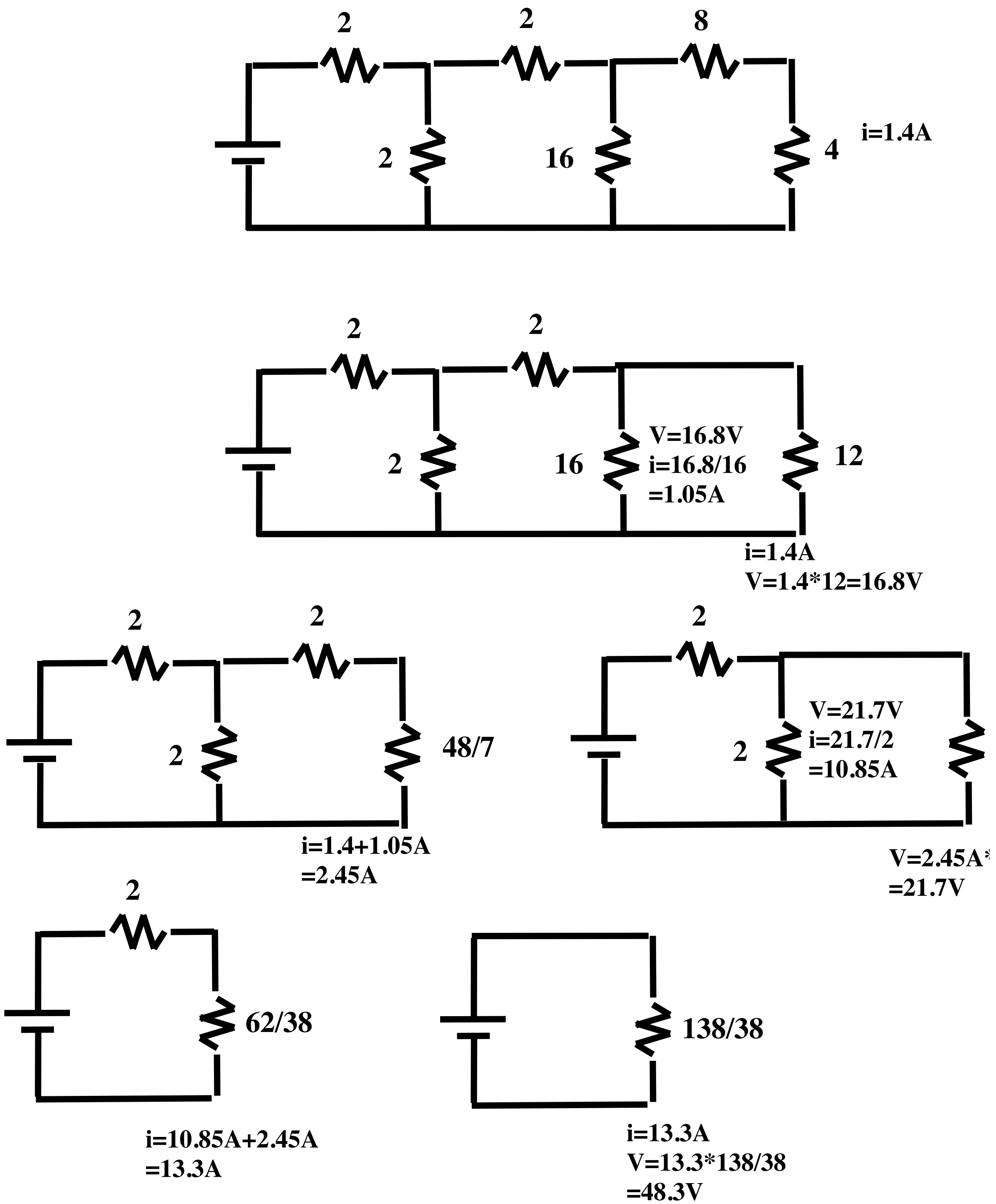
b) The current in resistor is given by

$$i = \frac{-\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{R_{21}} = \frac{3V}{50\Omega} = 0.06A$$

c)   The potential difference is just given by the sum of emfs 2 and 3.

$$V = \varepsilon_2 + \varepsilon_3 = 9V$$

27.31 In Fig. 27-42, the current in resistance 6 is $i_6 = 1.4A$ and the resistances are $R_1 = R_2 = R_3 = 2.0\Omega$, $R_4 = 16.0\Omega$, $R_5 = 8.0\Omega$ and $R_6 = 4.0\Omega$. What is the emf of the ideal battery.

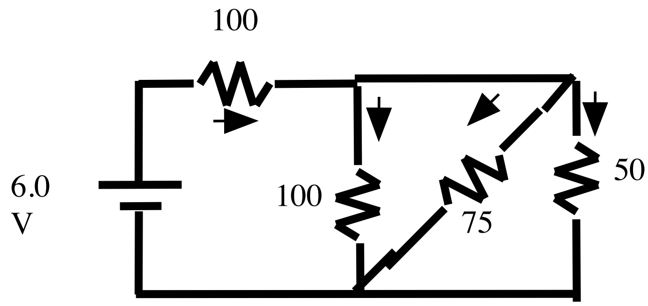


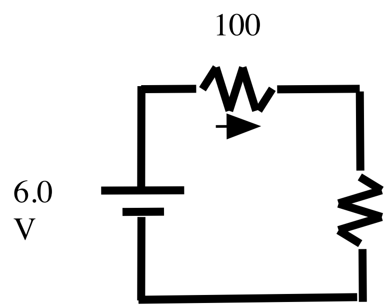
The emf voltage is 48.3V!

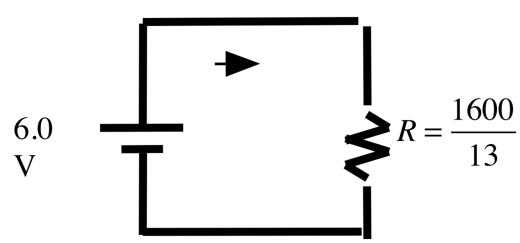
27.32. This circuit is the same as the one that we built in lab, with different numerical values for the resistors and emfs. See your notes from lab.

27.37 Same as 27.32, except for the additional emf.

27.40 In Fig 27-51, $R_1 = 100\Omega$, $R_2 = R_3 = 50\Omega, R_4 = 75\Omega$ and the ideal battery has emf $\varepsilon = 6.0V$
(a) What is the equivalent resistance? What is i in (b) resistor 1, (c) resistor 2, (d), resistor 3, and (e) resistor 4?







Now that we know the equivalent resistance, we can find the current that flows through the equivalent resistance. This is also the current through resistor 1.

$$i = \frac{6V}{1600/13\Omega} = 0.04875A$$

Using this current, we can compute the voltage that occurs across the three resistors in parallel. We do this by using the equivalent resistance for the parallel combination as shown in the middle drawing.

$$V = i\,R = 0.04875A \cdot \frac{300}{13}\Omega = 1.125V$$

Now that we know this voltage, we can find the current through each of the resistors

$$i_2 = \frac{1.125V}{100} = 0.01125A$$
$$i_3 = \frac{1.125V}{50} = 0.0225A$$
$$i_4 = \frac{1.125V}{100} = 0.015A$$

27.59 A 15 $k\Omega$ resistor and a capacitor are connected in series and then a 12.0 V potential difference is suddenly applied across hem. The potential difference across the capacitor rises to 5.0 V in $1.3\mu s$ (a) Calculate the time constant of the circuit. (b) Find the capacitance of the capacitor.

$$V = V_0(1 - e^{-t/\tau})$$
$$\frac{V}{V_0} = (1 - e^{-t/\tau})$$
$$\frac{5}{12} = (1 - e^{-t/\tau})$$
$$e^{-t/\tau} = 1 - \frac{5}{12} = \frac{7}{12}$$
$$-\frac{t}{\tau} = \ln(\frac{7}{12})$$
$$\tau = -\frac{t}{\ln(\frac{7}{12})} = 2.41 \times 10^{-6} s$$
$$RC = \tau$$
$$C = \frac{\tau}{R} = 1.61 \times 10^{-10} F$$

27.61 Switch S in Fig. 27-52 is closed at time t=0, to begin charging an initially uncharged capacitor of capacitance $C = 15\mu F$ through a resistor of resistance $R = 20\Omega$. At what time is the potential across the capacitor equal to that across the resistor.

$$
\begin{aligned}
q &= q_0(1-e^{-t/RC}) \\
i &= \frac{dq}{dt} = \frac{q_0}{RC}e^{-t/RC} \\
V_R &= V_C \\
i\,R &= \frac{1}{C}q \\
\frac{q_0}{RC}e^{-t/RC}R &= \frac{1}{C}q_0(1-e^{-t/RC}) \\
e^{-t/RC} &= (1-e^{-t/RC}) \\
e^{-t/RC} &= \frac{1}{2} \\
t &= -\ln(\frac{1}{2})RC \\
&= -\ln(\frac{1}{2})\cdot 20\cdot 15\times 10^{-6} \\
&= 2.08\times 10^{-4}\,s
\end{aligned}
$$

27.62