Chapter 25

25.2 The capacitor in Fig. 25-26 has a capacitance of $25\mu F$ and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it.

This question is really asking what the charge on the C is....

$$q = CV = 25\mu F \cdot 120V = 3000\mu C$$

25.4 You have two flat metal plates, each of area 1.00m2 with which to construct a a parallel plate capacitor. (a) If the capacitance of the device is to be 1.00 F, what must be the separation between the plates. (b) Could this capacitor actually be constructed....

$$C = \frac{\varepsilon_0 A}{d}$$
$$d = \frac{\varepsilon_0 A}{C} = \frac{\varepsilon_0 \cdot 1m^2}{1.00F} = 8.85 \times 10^{-12} m$$

...no way you could do this

25.7 What is the capacitance of a drop that results when two mercury spheres, each of radius R=2.00mm, merge?

$$V_{new} = 2 \cdot \frac{4}{3}\pi r^3 = 6.7 \times 10^{-8} m^3$$
$$r_{new} = 2.52 \times 10^{-2} m$$
$$C = 4\pi\varepsilon_0 r = 2.8 \times 10^{-13} F$$

We first calculate the volume of the resulting drop. This allows us to find the radius of the resulting drop

25.8 In Fig. 25-27 find the equivalent capacitance of the combination. Assume that $C_1 = 10\mu F$, $C_2 = 5\mu F$ and $C_3 = 4\mu F$.

Capacitors 1 and 2 are in *series*. We calculate the equivalent capacitance for this combination and then note that it is in *parallel* with 3.

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10\mu F} + \frac{1}{5\mu F} = \frac{3}{10\mu F}$$

$$C_{series} = \frac{10}{3}\mu F$$

$$C_{parallel} = \frac{10}{3}\mu F + 4\mu F$$

$$= \frac{22}{3}\mu F$$

25.9 In Fig. 25-29 find the equivalent capacitance of the combination. Assume that $C_1 = 10\mu F$, $C_2 = 5\mu F$ and $C_3 = 4\mu F$.

Capacitors 1 and 2 are in *parallel*. We calculate the equivalent capacitance for this combination and then note that it is in *series* with 3.

$$C_{parallel} = C_1 + C_2 = 15\mu F$$

$$\frac{1}{C_{series}} = \frac{1}{15\mu F} + \frac{1}{4\mu F} = \frac{4}{60\mu F} + \frac{15}{60\mu F} = \frac{19}{60\mu F}$$

$$C_{series} = \frac{60}{29}\mu F$$

25.11 Each of the uncharged capacitors in Figure 25-30 has a capacitance of $25\mu F$. A potential difference of V = 4200 V is established when the switch is closed. How many coulombs of charge pass through meter A.

The charge that passes through the meter is the charge that would end up on the equivalent capacitor. We first find the equivalent capacitance...

$$C_{eq} = 25\mu F + 25\mu F + 25\mu F = 75\mu F$$

We can now compute the charge on the equivalent capacitor

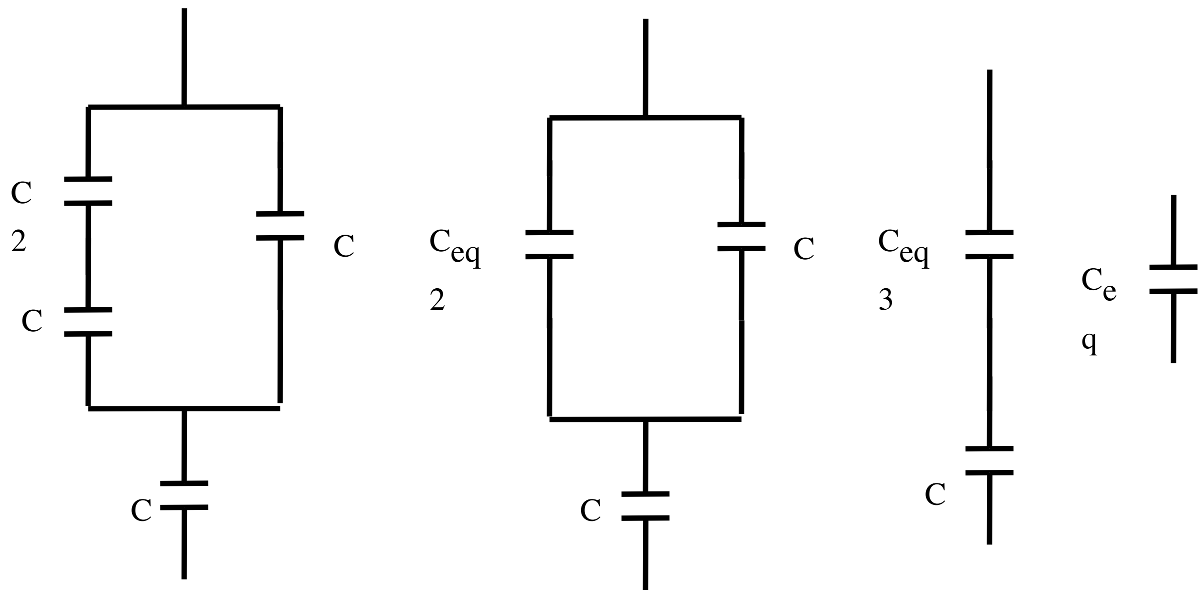
$$q_{eq} = C_{eq} V = 75 \times 10^{-6} F \cdot 4200V = 0.315\, C$$

25.12 In Fig. 25-30, the battery has a potential difference of V=10 V and the five capacitors each have a capacitance of $10\mu F$. What is the charge on (a) capacitor 1 and (b) capacitor 2.

We can calculate the charge on capacitor one very easily, since we know the voltage across it.

$$q = CV = 10.0\mu F \cdot 10.0V = 100\mu C$$

To tackle the charge on C_2, we need to do some substitutions. The picture below shows the successive substitutions.



$$\frac{1}{C_{eq2}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow C_{eq2} = \frac{C}{2} = 5\mu F$$
$$C_{eq3} = C_{eq2} + C = 5\mu F + 10\mu F = 15\mu F$$
$$\frac{1}{C_{eq}} = \frac{1}{C_{eq2}} + \frac{1}{C} = \frac{1}{15\mu F} + \frac{1}{10\mu F} \Rightarrow C_{eq} = 6\mu F$$

Now that we know the progression of substitutions, we can start with the final equivalent capacitor and work backward to find the charge on C_2. We know that the voltage across the equivalent capacitor is 10V.

$$q_{eq} = C_{eq} V = 6\mu F \cdot 10V = 60\mu C$$

The charge on C_{eq3} is the same as the charge on C_{eq} since C_{eq} arose from a series replacement. The charge on series capacitors and their replacement is always the same. This allows us to find the voltage across C_{eq3}.

$$V_{eq3} = \frac{q_{eq3}}{C_{eq3}} = \frac{60\mu C}{15\mu F} = 4V$$

This is the voltage across C_{eq2} since parallel capacitors and their equivalent replacement always have the same voltage. This allows us to compute the charge on C_{eq2}.

$$q_{eq2} = C_{eq2} \cdot V_{eq2} = 5\mu F \cdot 4V = 20\mu C$$

This is the charge on C_2. We know this because the charge on series capacitors and their replacement is the same.

$$q_2 = 20\mu C$$

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25.17 Did in problem session (I think).

25.23 In Fig. 25-39, the battery has potential difference V=9.0, $C_2 = 3.0\mu F$, $C_4 = 4.0\mu F$, and all the capacitors are initially uncharged. When switch S is closed, a total charge of $12\mu C$ passes through point a and a total charge $8\mu C$ passes through point b.

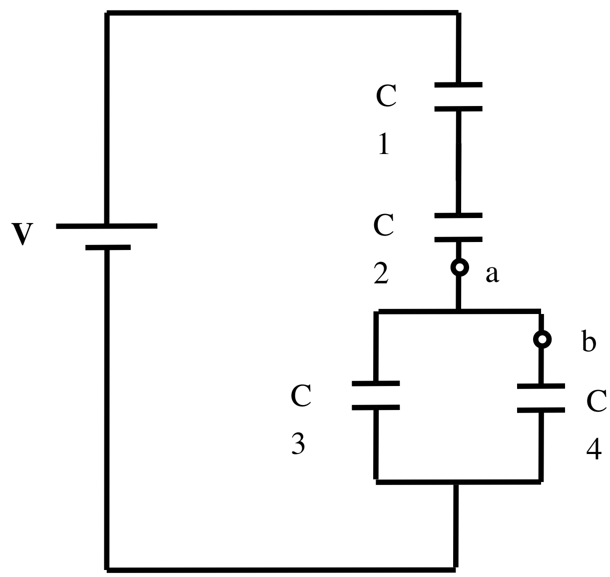
What are (a) C_1 and (b) C_3?

We know that the charge on C_4 must be $8\mu C$ since all of that charged passed through point b. We can calculate the voltage on C_4

$$V_4 = \frac{q_4}{C_4} = \frac{8\mu C}{4\mu F} = 2V$$

If $12\mu C$ passed through a and $8\mu C$ passed through (b), we know that $4\mu C$ must be on C_3. Using this charge and the voltage we computed, we can find C_3.

$$C_3 = \frac{q_3}{V_3} = \frac{4\mu C}{2V} = 2\mu F$$



Since $12\mu C$ passed through a , we know that the charge on both C_1 and C_2 must be $12\mu C$. We can compute the voltage across C_2.

$$V_2 = \frac{q_2}{C_2} = \frac{12\mu C}{3\mu F} = 4V$$

We know that the voltage across all of the capacitors is 9V, and the voltage across C_2 and C_4 are 4V and 2V respectively. This means that the remaining 3V drop occurs across C_1. Using the charge and voltage, we can now compute C_1.

$$C_1 = \frac{q_1}{V_1} = \frac{12\mu C}{3V} = 4\mu F$$

25.26 Figure 25-42 displays a 12.0 V battery and 3 uncharged capacitors of capacitances $C_1 = 4.00\mu F$, $C_2 = 6.00\mu F$, and $C_3 = 3.00\mu F$. The switch is thrown to the left side until capacitor 1 is fully charged. Then the switch is thrown to the right. What is the final charge on (a) capacitor 1, (b) capacitor 2, and (c) capacitor 3?

Initially we charge capacitor 1 by itself. It is straightforward how to compute its charge.

$$q_{1\,init} = C_1 V = 4.00\mu F \cdot 12V = 48\ \mu C$$

Now that we know the charge on capacitor 1, we can flip the switch. When we do this, charge will flow from 1 to 2 and 3. At the end, the voltage across capacitor 1 will equal the sum of the voltages across capacitors 2 and 3

$$V_{1final} = V_{2final} + V_{3final}$$

$$\frac{q_{1final}}{C_1} = \frac{q_{2final}}{C_2} + \frac{q_{3final}}{C_3}$$

$$Series\ Capacitors\!: \ q_{2final} = q_{3final} = q$$

$$\frac{q_{1final}}{C_1} = \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{q_{1final}}{C_1} = q\left(\frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$q_{1final} = qC_1\left(\frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$Charge\ Conservation\!: \ q_{1\,init} = q_{1final} + q$$

$$q_{1\,init} = q_{1final} + q = qC_1\left(\frac{1}{C_2} + \frac{1}{C_3}\right) + q$$

$$q_{1\,init} = q\left[C_1\left(\frac{1}{C_2} + \frac{1}{C_3}\right) + 1\right] = q\left[\left(\frac{C_1C_3}{C_2C_3} + \frac{C_1C_2}{C_2C_3}\right) + \frac{C_2C_3}{C_2C_3}\right]$$

$$q_{1\,init} = q\left[\frac{C_1C_2 + C_2C_3 + C_1C_3}{C_2C_3}\right]$$

$$q = \left[\frac{C_2C_3}{C_1C_2 + C_2C_3 + C_1C_3}\right]q_{1\,init}$$

$$q = \left[\frac{6\ \mu F \cdot 3\ \mu F}{4\ \mu F \cdot 6\ \mu F + 6\ \mu F \cdot 3\ \mu F + 4\ \mu F \cdot 3\ \mu F}\right]48\ \mu C$$

$$q = 16\ \mu C$$

$$q_{1final} = q_{1\,init} - q = 32 \mu C$$

25.29 A $2.0\mu F$ and a $4.0\mu F$ are connected in parallel across a 300V potential difference. Calculate the total energy stored in the capacitors.

$$U = \frac{1}{2}C_1V_1^{\,2} + \frac{1}{2}C_2V_2^{\,2} = \frac{1}{2}\cdot 2\times10^{-6}\cdot(300V)^2 + \frac{1}{2}\cdot 4\times10^{-6}\cdot(300V)^2 = 0.27J$$

25.31 What capacitance is required to store an energy for 10 kWh at a potential difference of 1000 V.

We convert to Joules first, and then find C.

$$U = 10\ kWh = 10{,}000W\cdot 3600s = 3.6\times10^7 J$$

$$U = \frac{1}{2}CV^2$$

$$C = \frac{2U}{V^2} = \frac{2\cdot 3.6\times10^7 J}{(1000V)^2} = 72\ F$$

25.39 A charged isolated metal sphere of diameter 10 cm has a potential of 8000V relative to V=0 at infinity. Calculate the energy density in the electric field near the surface of the sphere.

To compute the energy density in the field, we need to find the field. Since this charge is on a sphere, we can compute the field easily from the potential, assuming a point charge.

$$V = \frac{q}{4\pi\varepsilon_0 R}$$
$$q = 4\pi\varepsilon_0 R\,V$$
$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{4\pi\varepsilon_0 R\,V}{4\pi\varepsilon_0 R^2} = \frac{V}{R}$$
$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{R}\right)^2 = 0.113\,J/m^3$$

(Note: The radius is 0.05m)

25.41 Given a 7.4pF air-filled capacitor, you are asked to convert it to a capacitor that can store up to 7.4 μJ with a maximum potential difference of 652 V. Which dielectric in Table 25-1 should you use to fill the gap in the capacitor if you do not allow for a margin of error?

We need to find the required capacitance and then find the dielectric constant that we need to construct that capacitor from the air-filled capacitor that we have. We should then be able to identify the material.

$$U = \frac{1}{2}CV^2$$
$$C = \frac{2U}{V^2} = 3.4815\times 10^{-11}F$$
$$C = \kappa C_0$$
$$\kappa = \frac{C}{C_0} = \frac{34.815pF}{7.4pF} = 4.705$$

We need to you Pyrex

25.48 Figure 25-48 shows a parallel-plate capacitor with a plate area $A = 5.56\,cm^2$ and place separation $d = 5.56\,mm$. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 7.00$; the right half is filled with material of dielectric constant $\kappa_2 = 12.00$ What is the capacitance?

You can think of this as two capacitors in parallel--each with half the area and its own dielectric constant.

$$C_1 = \frac{\kappa_1 \varepsilon_0 A_1}{d} = \frac{\kappa_1 \varepsilon_0 (A/2)}{d}$$

$$C_2 = \frac{\kappa_2 \varepsilon_0 A_2}{d} = \frac{\kappa_2 \varepsilon_0 (A/2)}{d}$$

$$\begin{aligned} C_{parallel} &= C_1 + C_2 = \frac{\kappa_1 \varepsilon_0 (A/2)}{d} + \frac{\kappa_2 \varepsilon_0 (A/2)}{d} \\ &= (\kappa_1 + \kappa_2)\frac{\varepsilon_0 A}{2d} \\ &= (7.00 + 12.00)\frac{8.85\times 10^{-12} \cdot 5.56\times 10^{-4} m^2}{2 \cdot 5.56 \times 10^{-3} m} \\ &= 8.4074 \times 10^{-12} F \end{aligned}$$

25.49 Figure 25-49 shows a parallel plate capacitor with a plate area $A = 7.89\,cm^2$ and plate separation $d = 4.62\,mm$. The top half of the gap is filled with material of dielectric constant $\kappa_1 = 11.00$; the bottom half is filled with material of dielectric constant $\kappa_2 = 12.00$ What is the capacitance?

You can think of this as two capacitors in parallel--each with half the area and its own dielectric constant.

$$C_1 = \frac{\kappa_1 \varepsilon_0 A_1}{d_1} = \frac{\kappa_1 \varepsilon_0 A}{d/2}$$

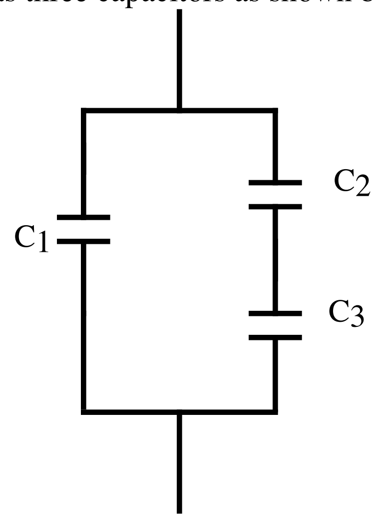
$$C_2 = \frac{\kappa_2 \varepsilon_0 A_2}{d_2} = \frac{\kappa_2 \varepsilon_0 A}{d/2}$$

$$\begin{aligned} \frac{1}{C_{series}} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/2}{\kappa_1 \varepsilon_0 A} + \frac{d/2}{\kappa_2 \varepsilon_0 A} \\ &= \frac{d/2}{\varepsilon_0 A}(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}) = \frac{d/2}{\varepsilon_0 A}(\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2}) \end{aligned}$$

$$\begin{aligned} C_{series} &= \frac{\varepsilon_0 A}{d/2} \cdot (\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}) \\ &= \frac{8.85 \times 10^{-12} \cdot 7.89 \times 10^{-4} m^2}{4.62 \times 10^{-3} m/2} \cdot (\frac{11 \cdot 12}{11 + 12}) \\ &= 1.734 \times 10^{-11} F \end{aligned}$$

25.50 Figure 25-50 shows a parallel-plate capacitor of plate area $A = 10.5cm^2$ and plate separation $2d = 7.12mm$. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 21.0$; the top of the right half is filled with material of dielectric constant $\kappa_2 = 42.0$; the bottom of the right half is filled with material of dielectric constant $\kappa_3 = 58.0$. What is the capacitance.

This capacitor can be thought of as three capacitors as shown below



$$C_1 = \frac{\kappa_1\, \varepsilon_0\, A/2}{2d} = 1.37\times 10^{-11} F$$

$$C_2 = \frac{\kappa_2\, \varepsilon_0\, A/2}{d} = 5.48\times 10^{-11} F$$

$$C_3 = \frac{\kappa_3\, \varepsilon_0\, A/2}{d} = 7.57\times 10^{-11} F$$

C_2 and C_3 in series

$$\frac{1}{C} = \frac{1}{5.48\times 10^{-11} F} + \frac{1}{7.57\times 10^{-11} F}$$

$$C = 3.18\times 10^{-11} F$$

C and C_1 in parallel

$$C_{eq} = 1.37\times 10^{-11} F + 3.18\times 10^{-11} F$$

$$= 4.59\times 10^{-11} F$$