Chapter 25

25.8 In Fig. 25-27 find the equivalent capacitance of the combination. Assume that $C_1 = 10\mu F$, $C_2 = 5\mu F$ and $C_3 = 4\mu F$.

Capacitors 1 and 2 are in *series*. We calculate the equivalent capacitance for this combination and then note that it is in *parallel* with 3.

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10\mu F} + \frac{1}{5\mu F} = \frac{3}{10\mu F}$$

$$C_{series} = \frac{10}{3}\mu F$$

$$C_{parallel} = \frac{10}{3}\mu F + 4\mu F$$

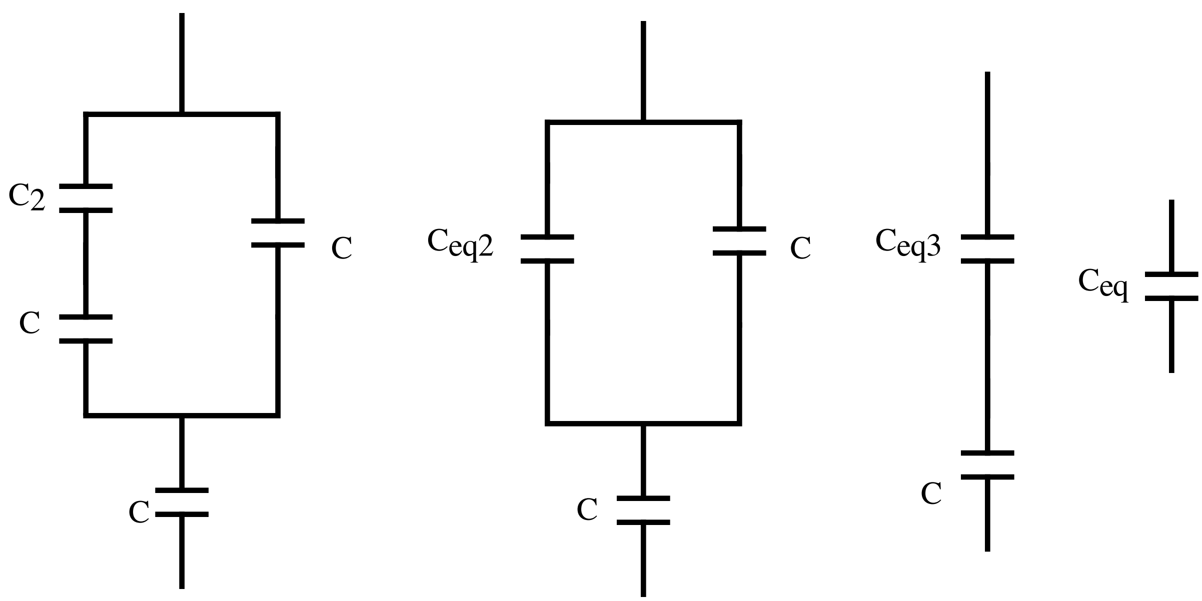
$$= \frac{22}{3}\mu F$$

25.12 In Fig. 25-30, the battery has a potential difference of V=10 V and the five capacitors each have a capacitance of $10\mu F$. What is the charge on (a) capacitor 1 and (b) capacitor 2.

We can calculate the charge on capacitor one very easily, since we know the voltage across it.

$$q = CV = 10.0\mu F \cdot 10.0V = 100\mu C$$

To tackle the charge on C_2, we need to do some substitutions. The picture below shows the successive substitutions.



$$\frac{1}{C_{eq2}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow C_{eq2} = \frac{C}{2} = 5\mu F$$

$$C_{eq3} = C_{eq2} + C = 5\mu F + 10\mu F = 15\mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq2}} + \frac{1}{C} = \frac{1}{15\mu F} + \frac{1}{10\mu F} \Rightarrow C_{eq} = 6\mu F$$

Now that we know the progression of substitutions, we can start with the final equivalent capacitor and work backward to find the charge on C_2. We know that the voltage across the equivalent capacitor is 10V.

$$q_{eq} = C_{eq}V = 6\mu F \cdot 10V = 60\mu C$$

The charge on C_{eq3} is the same as the charge on C_{eq} since C_{eq} arose from a series replacement. The charge on series capacitors and their replacement is always the same. This allows us to find the voltage across C_{eq3}.

$$V_{eq3} = \frac{q_{eq3}}{C_{eq3}} = \frac{60\mu C}{15\mu F} = 4V$$

This is the voltage across C_{eq2} since parallel capacitors and their equivalent replacement always have the same voltage. This allows us to compute the charge on C_{eq2}.

$$q_{eq2} = C_{eq2} \cdot V_{eq2} = 5\mu F \cdot 4V = 20\mu C$$

This is the charge on C_2. We know this because the charge on series capacitors and their replacement is the same.

$$q_2 = 20\mu C$$

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25.23 In Fig. 25-39, the battery has potential difference V=9.0, $C_2 = 3.0\mu F$, $C_4 = 4.0\mu F$, and all the capacitors are initially uncharged. When switch S is closed, a total charge of $12\mu C$ passes through point a and a total charge $8\mu C$ passes through point b.

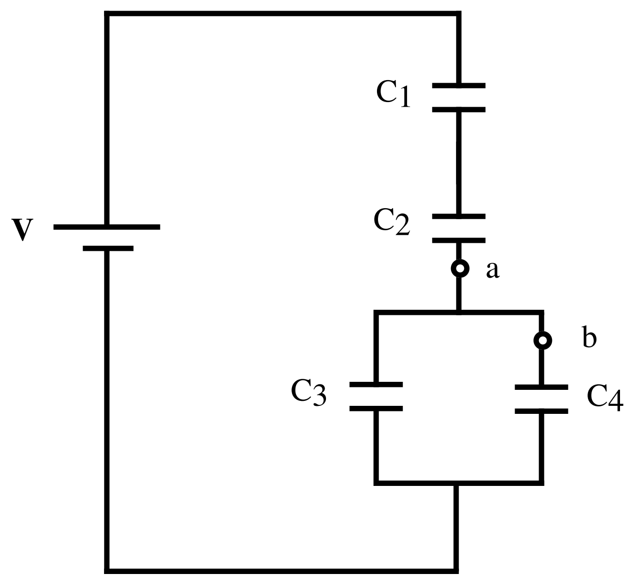
What are (a) C_1 and (b) C_3?

We know that the charge on C_4 must be $8\mu C$ since all of that charged passed through point b. We can calculate the voltage on C_4

$$V_4 = \frac{q_4}{C_4} = \frac{8\mu C}{4\mu F} = 2V$$

If $12\mu C$ passed through a and $8\mu C$ passed through (b), we know that $4\mu C$ must be on C_3. Using this charge and the voltage we computed, we can find C_3.

$$C_3 = \frac{q_3}{V_3} = \frac{4\mu C}{2V} = 2\mu F$$



Since $12\mu C$ passed through a , we know that the charge on both C_1 and C_2 must be $12\mu C$. We can compute the voltage across C_2.

$$V_2 = \frac{q_2}{C_2} = \frac{12\mu C}{3\mu F} = 4V$$

We know that the voltage across all of the capacitors is 9V, and the voltage across C_2 and C_4 are 4V and 2V respectively. This means that the remaining 3V drop occurs across C_1. Using the charge and voltage, we can now compute C_1.

$$C_1 = \frac{q_1}{V_1} = \frac{12\mu C}{3V} = 4\mu F$$

25.29 A $2.0\mu F$ and a $4.0\mu F$ are connected in parallel across a 300V potential difference. Calculate the total energy stored in the capacitors.

$$U = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = \frac{1}{2}\cdot 2\times10^{-6}\cdot(300V)^2 + \frac{1}{2}\cdot 4\times10^{-6}\cdot(300V)^2 = 0.27J$$

25.32 How much energy is stored in 1.00 m^3 of air due to the "fair weather" electric field of 150 V/m.

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = 9.957\times10^{-8}\, J/m^3$$
$$U = u_E \cdot V = 9.957\times10^{-8}\, J/m^3 \cdot 1\,m^3 = 9.957\times10^{-8}\, J$$

25.34 An air-filled parallel plate capacitor has a capacitance of 50pF. (a) If each of its plates has an area of $0.35m^2$, what is the separation? (b) If the region between the place is now filled with a dielectric with $\kappa = 5.6$ what is the capacitance?

$$C_i = \frac{\varepsilon_0 A}{d}$$
$$d = \frac{\varepsilon_0 A}{C_i} = 0.06195m^2$$
$$C_f = \frac{\kappa\varepsilon_0 A}{d} = 280pF$$

25.48 Figure 25-48 shows a parallel-plate capacitor with a plate area $A = 5.56cm^2$ and place separation $d = 5.56mm$. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 7.00$; the right half is filled with material of dielectric constant $\kappa_2 = 12.00$ What is the capacitance?

You can think of this as two capacitors in parallel--each with half the area and its own dielectric constant.

$$C_1 = \frac{\kappa_1\varepsilon_0 A_1}{d} = \frac{\kappa_1\varepsilon_0 (A/2)}{d}$$
$$C_2 = \frac{\kappa_2\varepsilon_0 A_2}{d} = \frac{\kappa_2\varepsilon_0 (A/2)}{d}$$
$$C_{parallel} = C_1 + C_2 = \frac{\kappa_1\varepsilon_0 (A/2)}{d} + \frac{\kappa_2\varepsilon_0 (A/2)}{d}$$
$$= (\kappa_1 + \kappa_2)\frac{\varepsilon_0 A}{2d}$$
$$= (7.00 + 12.00)\frac{8.85\times10^{-12}\cdot 5.56\times10^{-4}\, m^2}{2\cdot 5.56\times10^{-3}\, m}$$
$$= 8.4074\times10^{-12} F$$

25.49 Figure 25-49 shows a parallel plate capacitor with a plate area $A = 7.89cm^2$ and plate separation $d = 4.62\,mm$. The top half of the gap is filled with material of dielectric constant $\kappa_1 = 11.00$; the bottom half is filled with material of dielectric constant $\kappa_2 = 12.00$ What is the capacitance?

You can think of this as two capacitors in parallel--each with half the area and its own dielectric constant.

$$C_1 = \frac{\kappa_1 \varepsilon_0 A_1}{d_1} = \frac{\kappa_1 \varepsilon_0 A}{d/2}$$

$$C_2 = \frac{\kappa_2 \varepsilon_0 A_2}{d_2} = \frac{\kappa_2 \varepsilon_0 A}{d/2}$$

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/2}{\kappa_1 \varepsilon_0 A} + \frac{d/2}{\kappa_2 \varepsilon_0 A}$$

$$= \frac{d/2}{\varepsilon_0 A}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right) = \frac{d/2}{\varepsilon_0 A}\left(\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2}\right)$$

$$C_{series} = \frac{\varepsilon_0 A}{d/2} \cdot \left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)$$

$$= \frac{8.85 \times 10^{-12} \cdot 7.89 \times 10^{-4}\, m^2}{4.62 \times 10^{-3}\, m/2} \cdot \left(\frac{11 \cdot 12}{11 + 12}\right)$$

$$= 1.734 \times 10^{-11} F$$