Chapter 24

24.2 The electric potential difference between the ground and a cloud n a particular thunderstorm is $1.2\times10^{9}V$. In the unit electron volts, what is the magnitude of the change in the electric potential energy of an electron that moves between the ground and the cloud.

$$\begin{aligned} U &= q\Delta V \\ &= (1e)\cdot(1.2\times10^{9}V) \\ &= 1.2\times10^{9}eV \end{aligned}$$

24.3 A particular 12V car battery can send a total charge of 84 Ah through a circuit from one terminal to the other. (a) How many coulombs of charge does this represent? (b) If this entire charge undergoes a potential difference of 12V, how much energy is involved.

$$q = 84A\cdot h\cdot\frac{3600s}{1h} = 3.024\times10^{5}C$$

$$\Delta U = q\Delta V = 3.024\times10^{5}C\cdot12V = 3.63\times10^{6}J$$

24.5 An infinite nonconducting sheet has a surface charge density $\sigma = 0.10\mu C/m^{2}$ on one side. How far apart are equipotential surfaces whose potentials differ by 50V

$$\sigma = 0.10\mu C/m^{2}$$

$$V(x) - V_{0} = -\int_{0}^{x} E_{x}dx$$

$$E = \frac{\sigma}{2\varepsilon_{0}}$$

$$50V = -\int_{0}^{x} E_{x}dx = \frac{\sigma}{2\varepsilon_{0}}x$$

$$x = \frac{50V\cdot2\varepsilon_{0}}{\sigma} = 8.85mm$$

24.8 A graph of the x component of the electric field as a function of x in a region of space is shown in Fig. 24-30. The scale of the vertical axis is set by $E_{xs} = 20N/C$. The y and z components of the electric field are zero in this region. If the electric potential at the origin is 10V, (a) what is the electric potential at $x = 2.0m$ (b) What is the greatest positive value of the electric potential for points on the x axis for which $0.0 \le x \le 6.0$ and (c) for what value of x is the electric potential zero.

The change in the potential is given by.

$$V(x)-V_0 = -\int_0^x E_x\,dx$$

This is just minus the area under the curve. We can compute the area from the picture. (a) At $x = 3.0m$, Here $V_0 = 10V$ is the potential at the origin,

$$V(3.0m)-V_0 = -(\frac{1}{2}\cdot 3m \cdot -20N/C)$$
$$V(3.0m) = V_0 - (\frac{1}{2}\cdot 3m \cdot -20N/C) = 10V + 30V = 40V$$

The maximum positive potential occurs where the maximum negative area occurs. This would be at $x = 3.0m$ for this graph.

$$V(6.0m)-V_0 = -(\frac{1}{2}\cdot 3m \cdot -20N/C)$$
$$\begin{aligned} V(6.0m) &= V_0 - (\frac{1}{2}\cdot 3m \cdot -20N/C + \frac{1}{2}\cdot 1m \cdot +20N/C + 2\cdot +20N/C) \\ &= 10V - (-30V + 10V + 40V) \\ &= 0V \end{aligned}$$

The zero in the potential occurs when the net area is +10

$$V(6.0m)-V_0 = -(\frac{1}{2}\cdot 3m \cdot -20N/C)$$
$$\begin{aligned} V(6.0m) &= V_0 - (\frac{1}{2}\cdot 3m \cdot -20N/C + \frac{1}{2}\cdot 1m \cdot +20N/C + 2\cdot +20N/C) \\ &= 10V - (-30V + 10V + 40V) \\ &= 0V \end{aligned}$$

24.12 Consider a point charge $q = 1.0\mu C$, point A at a distance $d_1 = 2.0m$ from q and point B at a distance $d_2 = 1.0m$. (a) If these points are diametrically opposite each other as in 24-31a, what is the electric potential difference $V_A - V_B$? (b) What is the electric potential difference if points A and B are located as in 25-31b.

The potential at points A and B only depend on how far those points are from q, not the direction.

$$
\begin{aligned}
V_A - V_B &= \frac{q}{4\pi\varepsilon_0 r_A} - \frac{q}{4\pi\varepsilon_0 r_B} \\
&= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \\
&= \frac{1\times10^{-6}C}{4\pi\varepsilon_0}\left(\frac{1}{2m} - \frac{1}{1m}\right) \\
&-4.5\times10^{3}V
\end{aligned}
$$

The result is the same for both (a) and (b).

24.15 In Fig 24-33, what is the net potential at point P due to the four point charges if V=0 at infinity.

$$
\begin{aligned}
V &= \frac{+q}{4\pi\varepsilon_0 d} + \frac{+q}{4\pi\varepsilon_0 d} + \frac{-q}{4\pi\varepsilon_0 d} + \frac{-q}{4\pi\varepsilon_0 2d} \\
&= \frac{+q}{4\pi\varepsilon_0 d} + \frac{-q}{4\pi\varepsilon_0 2d} = \frac{+q}{4\pi\varepsilon_0 \cdot 2d} \\
&= 5.62\times10^{-4}V
\end{aligned}
$$

24.17 In Fig 24-33, particles of the charges $q_1 = 5e$ and $q_2 = -15e$ are fixed in place with a separation of $d = 24.0\ cm$. With V=0 at infinity, what are the finite (a) positive and (b) negative values of x at which the net electric potential on the x axis is zero.

To do this problem, we actually work it twice. First we assume that the zero is to the right of zero, and then to the left of zero.

$$q_1 = 5e$$
$$q_2 = -15e$$
$$d = 24.0\ cm$$
$$0 = \frac{q_1}{4\pi\varepsilon_0 x} + \frac{q_2}{4\pi\varepsilon_0 (d-x)}$$
$$0 = \frac{q_1}{x} + \frac{q_2}{d-x}$$
$$0 = \frac{q_1(d-x)}{x(d-x)} + \frac{q_2 x}{x(d-x)}$$
$$0 = q_1(d-x) + q_2 x = q_1 d + (q_2 - q_1)x$$
$$x = \frac{q_1 d}{(q_1 - q_2)} = \frac{5e \cdot d}{(5e - (-15e))} = \frac{d}{4} = 6cm\ (right\ of\ 0)$$
$$0 = \frac{q_1}{4\pi\varepsilon_0 x} + \frac{q_2}{4\pi\varepsilon_0 (d+x)}$$
$$0 = \frac{q_1}{x} + \frac{q_2}{d+x}$$
$$0 = \frac{q_1(d+x)}{x(d+x)} + \frac{q_2 x}{x(d+x)}$$
$$0 = q_1(d+x) + q_2 x = q_1 d + (q_2 + q_1)x$$
$$x = -\frac{q_1 d}{(q_1 + q_2)} = -\frac{5e \cdot d}{(5e + (-15e))} = \frac{d}{2} = 12cm\ (left\ of\ 0)$$

24.19 A spherical drop of water carrying a charge of 30 pC has a potential of 500V at its surface (with V=0 at infinity) (a) What is the radius of the drop? If two such drops of the same charge and radius combined to form a single drop, what is the potential at the surface of the new drop.

(a) The drop is a spherical conductor. From the outside, it looks like a point charge, so the potential at the surface is the potential at 1 radius from a point charge.

$$V = \frac{q}{4\pi\varepsilon_0 R}$$
$$R = \frac{q}{4\pi\varepsilon_0 V} = \frac{30\times 10^{-12}}{4\pi\varepsilon_0 \cdot 500V} = 5.4\times 10^{-4}\,m$$

(b) When we combine two drops, their volumes combine. We can use this to find the radius of the new drop. Their charges also combine.

$$\frac{4}{3}\pi R'^3 = 2\cdot\frac{4}{3}\pi R'^3$$
$$R'^3 = 2\cdot R^3$$
$$R' = 2^{1/3}\cdot R$$
$$V = \frac{q'}{4\pi\varepsilon_0 R'} = \frac{2q}{4\pi\varepsilon_0\cdot 2^{1/3}\cdot R} = \frac{2}{2^{1/3}}\frac{q}{4\pi\varepsilon_0 R}$$
$$= \frac{2}{2^{1/3}}\frac{q}{4\pi\varepsilon_0 R} = \frac{2}{2^{1/3}}\cdot 500V$$
$$= 793.7V$$

24.21 The ammonia molecule NH3 has a permanent electric dipole moment equal to 1.47D Calculate the electric potential due to an ammonia molecule at a point 52nm away along the axis of the dipole

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{1.47\times 3.34\times 10^{-30}\cos 0°}{4\pi\varepsilon_0 (52\times 10^{-9})^2} = 1.63\times 10^{-5}V$$

24.23 A plastic rod has been formed into a circle of radius R. It has a positive charge +Q uniformly distributed along one quarter of its circumference and a negative charge of -6Q uniformly distributed along the rest of the circumference. With V=0 at infinity, what is the electric potential at the center of the circle and (b) at point P, which is on the central axis of the circle at a distance z form the center?

We will begin with (b) since (a) is a special case of b. We proceed in the usual way by defining r and dq for bits of charge on the rod. We then integrate and simplify.

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{dq}{4\pi\varepsilon_0\sqrt{R^2+z^2}}$$
$$dq = \lambda dl = \lambda R d\theta$$
$$\lambda_+ = \frac{+Q}{2\pi R/4}$$
$$\lambda_- = \frac{-6Q}{6\pi R/4}$$
$$V = \int_0^{\pi/2} \frac{\lambda_+ R d\theta}{4\pi\varepsilon_0\sqrt{R^2+z^2}} + \int_{\pi/2}^{2\pi} \frac{\lambda_- R d\theta}{4\pi\varepsilon_0\sqrt{R^2+z^2}}$$
$$= \frac{\lambda_+ R}{4\pi\varepsilon_0\sqrt{R^2+z^2}}\left[\frac{\pi}{2}-0\right] + \frac{\lambda_- R}{4\pi\varepsilon_0\sqrt{R^2+z^2}}\left[2\pi-\frac{\pi}{2}\right]$$
$$= \frac{\lambda_+ R}{4\pi\varepsilon_0\sqrt{R^2+z^2}}\left[\frac{\pi}{2}\right] + \frac{\lambda_- R}{4\pi\varepsilon_0\sqrt{R^2+z^2}}\left[\frac{3\pi}{2}\right]$$
$$= \frac{\frac{+Q}{2\pi R/4}\cdot R}{4\pi\varepsilon_0\sqrt{R^2+z^2}}\left[\frac{\pi}{2}\right] + \frac{\frac{-6Q}{6\pi R/4}\cdot R}{4\pi\varepsilon_0\sqrt{R^2+z^2}}\left[\frac{3\pi}{2}\right]$$
$$= \frac{+Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}} + \frac{-6Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}$$
$$= \frac{-5Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}$$

We set z=0 to get the solution to part (a).

$$V = \frac{-5Q}{4\pi\varepsilon_0 R}$$

24.25 (a) Figure 24-39a shows a nonconducting rod of length $L = 6.00cm$ and uniform charge linear charge density $\lambda = +3.68 pC/m$. Take $V = 0$ at infinity. What is V at point P at distance $d = 8.00cm$ along the rod's perpendicular bisector. (b) Figure 24-39b shows an identical rod except that one half is now negatively charged. What is V at P?

This problem is the problem worked out in the book with different limits... We can work it again.

Write an expression for the electric potential due to a small charge dq at the point p?

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{dq}{4\pi\varepsilon_0\sqrt{x^2+d^2}}$$

Write an expression for the dq and the r in terms of x and the distance y.

$$dq = \lambda dx$$

The potential at the point indicated?

$$V = \int_{-L/2}^{L/2} \frac{\lambda dx}{4\pi\varepsilon_0 \sqrt{x^2 + d^2}} = \frac{\lambda}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{dx}{\sqrt{x^2 + d^2}}$$
$$= \frac{\lambda}{4\pi\varepsilon_0} \cdot \ln\left[\frac{L/2 + \sqrt{(L/2)^2 + d^2}}{-L/2 + \sqrt{(L/2)^2 + d^2}}\right]$$

The potential for the second drawing is zero, since the equal and opposite charge contributions cancel exactly.

24.27 In Fig 24-40, what is the net electric potential at the origin due to the circular arc of charge $Q_1 = 7.21pC$ and the two particles of charges $Q_2 = 4.00Q_1$ and $Q_3 = -2.00Q_1$. The Arc's center of curvature is at the origin and its radius is $R = 2.00m$; the angle indicated is $\theta = 20°$.

I think that this picture is mislabeled. You do not need to know the angle shown.

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{dq}{4\pi\varepsilon_0 R}$$
$$\lambda = \frac{Q_1}{R\theta}$$
$$dq = \lambda dl = \lambda R d\theta$$
$$V_{arc} = \int_0^\theta \frac{\lambda R d\theta}{4\pi\varepsilon_0 R} = \frac{\lambda R\theta}{4\pi\varepsilon_0 R} = \frac{\left(\frac{Q_1}{R\theta}\right) R\theta}{4\pi\varepsilon_0 R} = \frac{Q_1}{4\pi\varepsilon_0 R}$$

$$V_{total} = V_{arc} + V_2 + V_3 = \frac{Q_1}{4\pi\varepsilon_0 R} + \frac{Q_2}{4\pi\varepsilon_0 2R} + \frac{Q_3}{4\pi\varepsilon_0 R}$$
$$= \frac{Q_1}{4\pi\varepsilon_0 R} + \frac{4Q_1}{4\pi\varepsilon_0 2R} + \frac{-2Q_1}{4\pi\varepsilon_0 R}$$
$$= \frac{Q_1}{4\pi\varepsilon_0 R} = 0.0324V$$

24.30 Find the potential at the point indicated. This is just problem 25 with different limits.

Write an expression for the electric potential due to a small charge dq at the point p?

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{dq}{4\pi\varepsilon_0 (x+d)}$$

Write an expression for the dq and the r in terms of x and the distance y.

$$dq = \lambda dx$$

The potential at the point indicated?

$$V = \int_0^L \frac{\lambda\, dx}{4\pi\varepsilon_0 (x+d)} = \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(x+d)}$$
$$= \frac{\lambda}{4\pi\varepsilon_0} \cdot \ln\left[\frac{(L+d)}{d}\right]$$

24.34 The electric potential V in the sace between two flat parallel plates 1 and 2 is given (in volts) by $V = 1500\, x^2$ where x in meters is the perpendicular distance from plate 1. At $x = 1.3\, cm$ (a) what is the magnitude of the electric field directed toward or away from plate 1.

$$E_x = -\frac{\partial V}{\partial x} = -3000x$$
$$at\ x = 1.3 \times 10^{-2} m$$
$$E_x = -39\ V/m$$

Field points toward plate 1.

24.35 The electric potential at points in an xy plane is given by

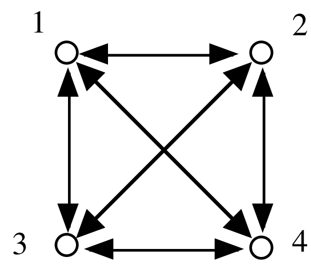
$$V = (2.0V\ /m^2)x^2 - (3.0\,V\,/\,m^2)\,y^2$$

In unit-vector notation, what is the electric field at the point (3.0 m, 2.0m).

$$E_x = -\frac{\partial V}{\partial x} = -(4.0\,V\,/\,m^2)\,x$$
$$E_y = -\frac{\partial V}{\partial y} = (6.0\,V\,/\,m^2)\,y$$
$$\vec{E} = -(4.0\,V\,/\,m^2)\,x\,\hat{i} + (6.0V\ /m^2)\,y\,\hat{j}$$
$$= -(12V\ /m)\,\hat{i} + (12.0\,V\,/\,m)\ \hat{j}$$

24.41 How much work is required to set up the arrangement of Fig-24.46 if

$q = 2.3pC, a = 64.0\ cm$



$$U = \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_1q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_1q_4}{4\pi\varepsilon_0 r_{14}} + \frac{q_2q_3}{4\pi\varepsilon_0 r_{23}} + \frac{q_2q_4}{4\pi\varepsilon_0 r_{24}} + \frac{q_3q_4}{4\pi\varepsilon_0 r_{34}}$$

$$= \frac{-q^2}{4\pi\varepsilon_0 a} + \frac{-q^2}{4\pi\varepsilon_0 a} + \frac{q^2}{4\pi\varepsilon_0\sqrt{2}\,a} + \frac{q^2}{4\pi\varepsilon_0\sqrt{2}\,a} + \frac{-q^2}{4\pi\varepsilon_0 a} + \frac{-q^2}{4\pi\varepsilon_0 a}$$

$$= \frac{-4q^2}{4\pi\varepsilon_0 a} + \frac{2q^2}{4\pi\varepsilon_0\sqrt{2}\,a}$$