Chapter 23 Problems

23.1 The square surface shown in Fig 23-26 measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude E = 1800 N/C. The field lines make an angle of 35 degrees with a normal to the surface as shown. Take the normal to be directed "outward" as though the surface were one face of a box. Calculate the electric flux through the surface.

The flux through this surface is

$$\varphi = E\,A\cos\theta$$
$$\theta = 180° - 35°$$
$$\varphi = (1800 N/C)\cdot(.0032 m)^2 \cdot \cos(180° - 35°)$$
$$= -1.51\times10^{-2}\,N\,m^2/C$$

Note that the angle is 180-35. This makes the flux negative--which means the flow is into the box. A net flow into a closed surface is taken to be negative.

23.2. The cube in Fig 23-27 has edge length of 1.4 m and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the field (in N/C) is given by (a) $6.00\hat{i}$, (b) $-2.00\hat{j}$ and (c) $-3.00\hat{i}+4.00\hat{k}$.

The area vector for the right face is

$$\vec{A} = (1.4m)^2\,\hat{j}$$

We can now compute flux.

(a) $\vec{E}\cdot\vec{A} = 6.00\hat{i}\cdot(1.4m)^2\hat{j} = 0$

(b) $\vec{E}\cdot\vec{A} = -2.00\hat{j}\cdot(1.4m)^2\hat{j} = -2.00\cdot(1.4m)^2 = -3.92 Nm^2/C$

(c) $\vec{E}\cdot\vec{A} = (-3.00\hat{i}+4.00\hat{k})\cdot(1.4m)^2\hat{j} = 0$

(d) The total flux through the cube is zero. A uniform field is present--every field line that enters onside of the cube leaves the other.

23.3 The cube in Fig 24-26 has edge length of 1.4 m and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the field (in N/C) is given by (a) $6.00\hat{i}$, (b) $-2.00\hat{j}$ and (c) $-3.00\hat{i}+4.00\hat{k}$.

The area vector for the right face is

$$\vec{A} = (1.4m)^2 \hat{j}$$

We can now compute flux.

(a) $\vec{E} \cdot \vec{A} = 6.00\hat{i} \cdot (1.4m)^2 \hat{j} = 0$

(b) $\vec{E} \cdot \vec{A} = -2.00\hat{j} \cdot (1.4m)^2 \hat{j} = -2.00 \cdot (1.4m)^2 = -3.92 Nm^2 / C$

(c) $\vec{E} \cdot \vec{A} = (-3.00\hat{i} + 4.00\hat{k}) \cdot (1.4m)^2 \hat{j} = 0$

(d) The total flux through the cube is zero. A uniform field is present--every field line that enters onside of the cube leaves the other.

23.5 A point charge of $1.8\mu C$ is at the center of a cubical Gaussian surface 55cm on edge. What is the net electric flux through the surface?

The net electric flux through a closed surface (like the cube that we have here) is given by

$$\varphi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{enc}}{\varepsilon_0} = \frac{1.8 \times 10^{-6} C}{\varepsilon_0} = 2.03 \times 10^5 \; Nm^2/C$$

23.7 In Fig. 23-29, a proton is a distance d/2 directly above the center of a square of side d. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge d).

If we think of the charge as enclosed by a cube with the charge at the center, we can use Gauss' Law to find the flux through the cube

$$\varphi_{cube} = \frac{q_{enc}}{\varepsilon_0} = 1.81 \times 10^{-8} Cm^2$$
$$\varphi_{side} = \frac{1}{6}\varphi_{cube} = 3.01 \times 10^{-9} \frac{N}{C} m^2$$

23.17 Space vehicles traveling through Earth's radiation belt can intercept a significant number of electrons. The resulting charge buildup can damage electronic components and disrupt operations. Suppose a spherical metallic satellite 1.3 m in diameters accumulates $2.4\,\mu C$ in one orbital revolution. (a) Find the resulting surface charge density. (b) Calculate the magnitude of the electric field just outside the surface of the satellite due to the surface charge.

$$\sigma = \frac{2.4\times10^{-6}C}{4\pi\cdot(1.3/2\ m)^2} = 4.52\times10^{-7}C/m^2$$
$$E = \frac{2.4\times10^{-6}C}{4\pi\varepsilon_0(1.3m/2)^2} = 5.1\times10^4 N/C$$

23.19 A uniformly charged conducting sphere of 1.2m diameter has a surface charge density of $8.1\times10^{-6}C/m^2$ (a) Find the net charge on the sphere. What is the total electric flux leaving the surface of the sphere.

(a) $q = \sigma\cdot 4\pi r^2 = 3.66\times10^{-5}C$

(b) $\varphi = \frac{q_{enc}}{\varepsilon_0} = \frac{3.66\times10^{-5}}{8.85\times10^{-12}}\frac{Nm^2}{C} = 4.14\times10^6\frac{Nm^2}{C}$

23.22 Figure 23-36 shows a section of a long, thin-walled metal tube of radius $R = 3.00cm$ with a charge per unit length $\lambda = 2.00\times10^{-8}C/m$. What is the magnitude of the electric field at radial distance (a) $r = R/2.00$ and (b) $r = 2.00R$. (c) Graph E versus r for the range $r = 0$ to $r = 2.00R$.

We can solve for the field using Gauss's law. We enclose with a cylindrical Gaussian surface.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$
$$\int_{ends}\vec{E}\cdot d\vec{A} + \int_{curve}\vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$
$$0 + \int_{curve}\vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$
$$\int_{curve}E\,dA = \frac{q_{enc}}{\varepsilon_0}$$
$$E\int_{curve}dA = \frac{q_{enc}}{\varepsilon_0}$$
$$E\cdot 2\pi r L = \frac{\lambda_{net}L}{\varepsilon_0}$$
$$E = \frac{\lambda_{net}}{2\pi\varepsilon_0 r}$$

Now we need to carefully compute the net charge/length enclosed. For $r = R/2.00$, no charge is enclosed, so $\lambda_{net} = 0$ and $E = 0$.

For $r = 2.00R$, $\lambda_{net} = \lambda$ and $E = \frac{\lambda}{2\pi\varepsilon_0 r}$.

The field is zero from 0 to R and then falls as 1/r from R to 2R.

23.23 An infinite line of charge produces a field of magnitude of $4.5 \times 10^4 N/C$ at a distance of 2.0m. Calculate the linear charge density.

We can solve for the field using Gauss's law. We enclose with a cylindrical Gaussian surface.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{ends} \vec{E} \cdot d\vec{A} + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curve} E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \int_{curve} dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r L = \frac{\lambda_{net} L}{\varepsilon_0}$$

$$E = \frac{\lambda_{net}}{2\pi \varepsilon_0 r}$$

$$\lambda_{net} = 2\pi \varepsilon_0 r E = 5.0 \times 10^{-6} N/C$$

23.27 Figure 23-37 is a section of a conduction rod of radius $R_1 = 1.30mm$ and length $L = 11.0m$ inside a thin-walled coaxial conducting cylindrical shell of radius $R_2 = 10.0R_1$ and the (same) length L. The net charge on the rod is $Q_1 = +3.40 \times 10^{-12} C$; that on the shell is $Q_2 = -2Q_1$ What are (a) the magnitude E and (b) direction (radially inward or outward) of electric field at a radial distance $r = 2R_2$? What are (c) E and (d) the direction at $r = 5R_1$ What is the charge on the (e)interior and (f) exterior surface of the shell.

For parts (a) and (b), we consider the field outside the outer shell. If we write out Gauss' law for a surface that is outside both the rod and shell

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{ends} \vec{E} \cdot d\vec{A} + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curve} E \, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \int_{curve} dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r L = \frac{\lambda_{net} L}{\varepsilon_0}$$

$$E = \frac{\lambda_{net}}{2\pi \varepsilon_0 r}$$

The net charge per unit length is the net charge divided by the length. The net charge inside the enclosing Gaussian surface is the total charge on the rod and the shell.

$$q_{net} = Q_1 + Q_2 = -3.40 \times 10^{-12} C$$

$$\lambda_{net} = \frac{q_{net}}{L} = \frac{-3.40 \times 10^{-12} C}{11.0m} = -3.09 \times 10^{-13} C / m$$

$$r = 2.00 R_2 = 0.026m$$

$$E = \frac{\lambda_{net}}{2\pi \varepsilon_0 r} = \frac{-3.09 \times 10^{-13} C / m}{2\pi \varepsilon_0 \cdot 0.026m} = -0.214 N / C$$

The minus sign indicates that the field points inward.

For parts c and d, we choose a Gaussian surface in the gap since $r = 5R_1$ is in the gap. The Gauss' Law calculation is exactly the same, except the charge per unit length enclosed will be different since we only enclose the rod.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{ends} \vec{E} \cdot d\vec{A} + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int_{curve} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curve} E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \int_{curve} dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi\, r\, L = \frac{\lambda_{net}\, L}{\varepsilon_0}$$

$$E = \frac{\lambda_{net}}{2\pi\, \varepsilon_0 r}$$

Now we use the charge on the rod only.

$$q_{net} = Q_1 = 3.40 \times 10^{-12} C$$

$$\lambda_{net} = \frac{q_{net}}{L} = \frac{3.40 \times 10^{-12} C}{11.0 m} = 3.09 \times 10^{-13} C / m$$

$$r = 5.00 R_1 = 0.0065 m$$

$$E = \frac{\lambda_{net}}{2\pi\, \varepsilon_0 r} = \frac{3.09 \times 10^{-13} C / m}{2\pi\, \varepsilon_0 \cdot 0.0065 m} = 0.855 N / C$$

The field points radially outward.

To find the charge on the inner surface of the shell, we choose a Gaussian Surface inside the shell. (in the conductor itself). If we choose a Gaussian Surface inside the conducting shell, we know that the E field and flux must be zero. If we solve for the charge on the inner surface of the shell, we find that it must be equal and opposite to the charge on the rod, since the charge enclosed must be zero.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 = \frac{q_{enc}}{\varepsilon_0}$$

$$0 = \frac{q_{inner} + q_{rod}}{\varepsilon_0}$$

$$q_{inner} = -q_{rod} = -Q_1$$
$$= -3.40 \times 10^{-12} C$$

If $-Q_1$ is on the inner surface of the shell and $Q_2 = -2Q_1$ is on the total shell, then $-Q_1 = -3.4 \times 10^{-12} C$ must be on the outer surface of the shell.

23.36 A large, flat, nonconducting surface has a uniform charge density σ. A small circular hole of radius R has been cut in the middle of the surface as shown in Fig. 23-42. Ignore fringing of the field lines around all edges and calculate the electric field at a point P a distance z from the center of the hole along its axis. (Hint: See Eq. 22-26 and use superposition)

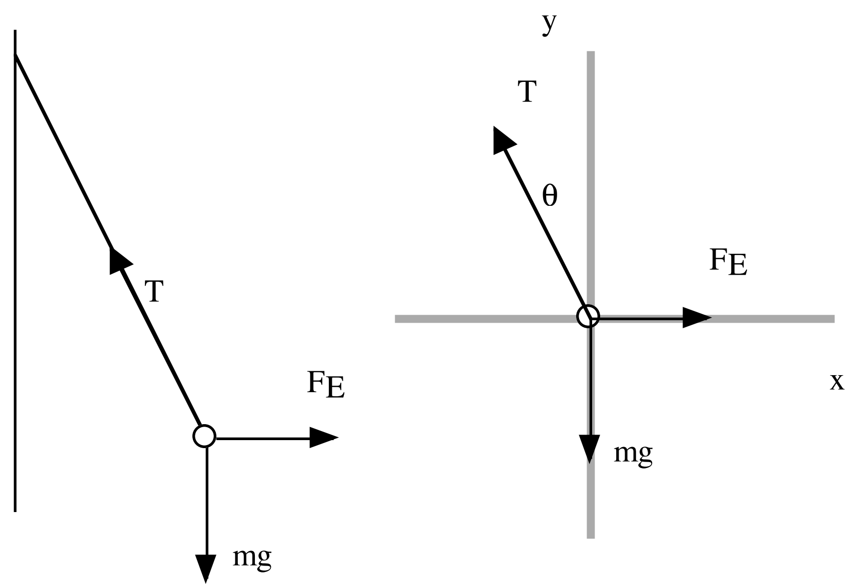
Superposition is the key to this problem Superposition is the principle that allows you to construct the field due to a number of charge distributions by adding the fields from each distribution together as vectors.

In this problem, the field at the point p will be the field due to an infinite plane - the field due to a disk. We think of this distribution as a plain and then we subtract the hole.

$$E = E_{plane} - E_{disk}$$
$$= \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$
$$= \frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{z^2 + R^2}}\right)$$

23.41 In Fig. 23-45, a small nonconducting ball of mass $m = 1mg$ and charge $q = 2 \times 10^{-8} C$ (distributed uniformly through its volume hangs from an insulating thread that makes an angle $\theta = 30°$ with a vertical uniformly charged nonconducting sheet (shown in cross section). Considering the gravitational force on the ball and assuming that the sheet extends far vertically and into and out of the page, calculate the surface charge density σ of the sheet.

We begin by drawing a free body diagram. We know that the forces must add to zero.



We solve for and eliminate the tension between the equations to find the electric force. We can then use the expression for the electric field for an infinite plane to solve for the charge density.

$$x - direction \qquad 0 = F_E - T\sin\theta$$

$$y - direction \qquad 0 = T\cos\theta - mg$$

$$T = \frac{mg}{\cos\theta}$$
$$F_E = T\sin\theta$$
$$= \frac{mg}{\cos\theta}\cdot\sin\theta$$
$$= mg\tan\theta$$

$$F_E = mg\tan\theta$$
$$F_E = qE = q\frac{\sigma}{2\varepsilon_0}$$
$$q\frac{\sigma}{2\varepsilon_0} = mg\tan\theta$$
$$\sigma = \frac{2\varepsilon_0 mg}{q}\tan\theta$$
$$= 5.0\times10^{-9} C/m^2$$

23.50 Figure 23-51 shows a spherical shell with uniform volume charge density $\rho = 1.84\times10^{-9} C/m^3$, inner radius $a = 10.0cm$ and outer radius $b = 2.00a$. What is the magnitude of the electric field at radial distances (a) $r = 0$, (b) $r = a/2.0$, (c) $r = a$, (d) $r = 1.5a$,
(e) $r = b$, (f) $r = 3.00b$

We begin by considering a spherical Gaussian Surface. For this spherical geometry, the electric field is parallel to the dA vector everywhere on the surface, so the dot product becomes a simple multiplication because the angle between the E vector and the dA vector is zero everywhere. The magnitude of E is the same everywhere by symmetry, so we can pull E out of the integration. We are left with integrating dA over our spherical Gaussian surface.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint E\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

For (a) $r = 0$, (b) $r = a/2.0$, and (c) $r = a$, the charge enclosed is zero so $E = 0$. For d) $r = 1.5a$, and (e) $r = b$, we need to calculate the charge enclosed.

$$q_{enc} = \rho \cdot V_{enc} = \rho \cdot \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3 \right)$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\rho \cdot \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3 \right)}{\varepsilon_0}$$

$$E = \frac{\rho (r^3 - a^3)}{3\varepsilon_0 r^2}$$

$$= \frac{1.84 \times 10^{-9} C/m^3 (r^3 - a^3)}{3\varepsilon_0 r^2}$$

$$r = 1.5a = 0.15m$$

$$E = 7.32 N/C$$

$$r = b = 0.20m$$

$$E = 12.13 N/C$$

For the last section (f), we enclose the entire shell so we use the entire charge on the shell.

$$q_{enc} = \rho \cdot V_{enc} = \rho \cdot \left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 \right)$$

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\rho \cdot \left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 \right)}{\varepsilon_0}$$

$$E = \frac{\rho (b^3 - a^3)}{3\varepsilon_0 r^2}$$

$$= \frac{1.84 \times 10^{-9} C / m^3 (b^3 - a^3)}{3\varepsilon_0 r^2}$$

$$r = 3b = 0.60m$$

$$E = 1.35 N / C$$