Chapter 22

22.3 What is the magnitude of a point charge whose electric field 50 cm away has the magnitude of 2.00N/C.

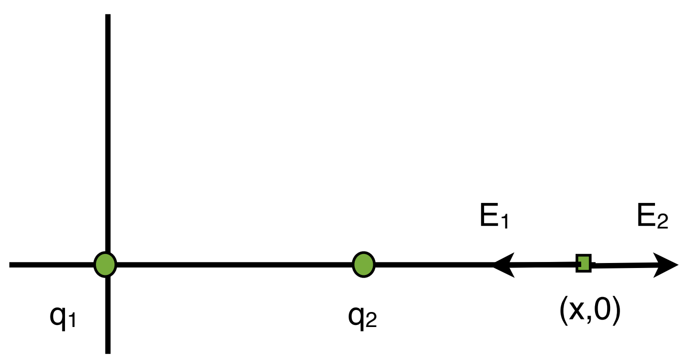
$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$
$$q = 4\pi\varepsilon_0 r^2 E$$
$$= 5.56\times10^{-11}C$$

22.5 An atom of plutonium-239 has a nuclear radius of 6.64 fm and atomic number Z=94. Assuming that the positive charge is distributed uniformly within the nucleus, what are the magnitude and direction of the electric field at the surface of the nucleus due to the positive charge.

Outside a uniformly charged sphere, the field looks like that of a point charge at the center of the sphere.

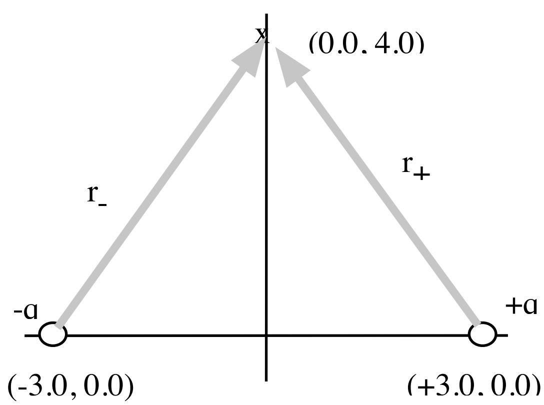
$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$
$$= \frac{94\cdot1.6\times10^{-19}C}{4\pi\varepsilon_0(6.64\times10^{-15}m)^2}$$
$$= 3.07\times10^{21}N/C!$$

22.8 In Fig. 22-31, particle 1 of charge $q_1 = -5q$ and particle 2 of charge $q_2 = +2q$ are fixed to an x-axis (a) As a multiple of distance L, at what coordinate on the axis is the net electric field of the particles zero? (a) Sketch the net electric field lines.



$$E_1 = \frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{5q}{4\pi\varepsilon_0 x^2}$$
$$E_2 = \frac{2q}{4\pi\varepsilon_0 (x-L)^2}$$
$$E_2 = E_1$$
$$\frac{5q}{4\pi\varepsilon_0 x^2} = \frac{2q}{4\pi\varepsilon_0 (x-L)^2}$$
$$\frac{5}{x^2} = \frac{2}{(x-L)^2}$$
$$x = \frac{1}{3}(5+\sqrt{10})L$$

22.11 Figure 22-34 shows two charged particles on an x axis: $-q = -3.2\times10^{-19}C$ at $x = -3.0m$ and $q = +3.2\times10^{-19}C$ at $x = +3.0m$. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the net electric field produced at point P at $y = +4.0m$

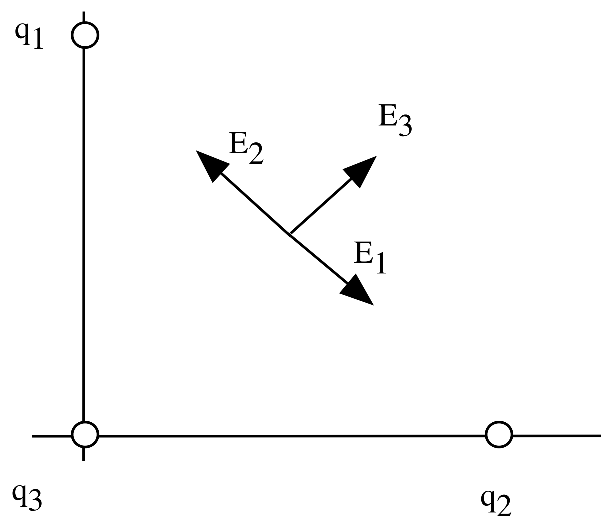


We can do this problem most easily by using the full vector form for the electric field due to a point charge.

$$\vec{r}_+ = (0-3.0)\hat{i} + (4.0-0)\hat{j} \qquad \vec{r}_- = (0-(-3))\hat{i} + (4-0)\hat{j}$$
$$r_+ = \sqrt{3^2+4^2} = 5m \qquad r_- = \sqrt{(-3)^2+4^2} = 5m$$
$$\hat{r}_+ = \frac{\vec{r}_+}{r_+} = \frac{-3\hat{i}+4\hat{j}}{5} \qquad \hat{r}_- = \frac{\vec{r}_-}{r_-} = \frac{3\hat{i}+4\hat{j}}{5}$$

$$\begin{aligned}\vec{E} &= \vec{E}_+ + \vec{E}_- \\ &= \frac{3.2\times10^{-19}C}{4\pi\varepsilon_0(5m)^2}\cdot\frac{-3\hat{i}+4\hat{j}}{5} - \frac{3.2\times10^{-19}C}{4\pi\varepsilon_0(5m)^2}\cdot\frac{3\hat{i}+4\hat{j}}{5} \\ &= \frac{3.2\times10^{-19}C}{4\pi\varepsilon_0(5m)^2}\cdot\frac{-6\hat{i}+0\hat{j}}{5} \\ &= -1.38\times10^{-10}N/C\hat{i}\end{aligned}$$

22.13 In Fig. 22-36, three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance $a = 6.00\mu m$. What are the (a)magnitude and (b) direction of the net electric field at point P due to the particles.

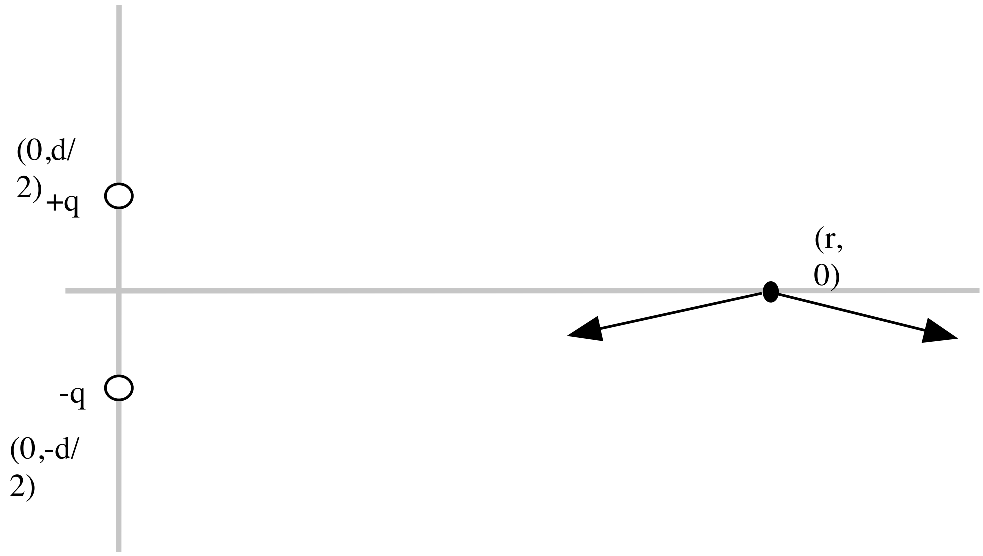


The fields due to charges 1 and 2 cancel exactly. This leaves us only to calculate the field due to 3.

$$r = \frac{1}{2}\sqrt{a^2+a^2} = \frac{\sqrt{2}}{2}a$$

$$E_3 = \frac{q_3}{4\pi\varepsilon_0\cdot a^2/2} = 159.8N/C\ at\ 45°$$

22.19. Find the magnitude and direction of the electric field at point P due to the electric dipole. You may assume that r>>d



We begin by writing the exact field.

$$\vec{r}_+ = (r-0)\hat{i} + (0-\frac{d}{2})\hat{j} \qquad \vec{r}_- = (r-0)\hat{i} + (0-(-\frac{d}{2}))\hat{j}$$

$$r_+ = \sqrt{r^2 + (\frac{d}{2})^2} \qquad r_- = \sqrt{r^2 + (\frac{d}{2})^2}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$= \frac{q}{4\pi\varepsilon_0 r_+^{\ 2}}\hat{r}_+ - \frac{q}{4\pi\varepsilon_0 r_-^{\ 2}}\hat{r}_-$$

$$\hat{r}_+ = \frac{\vec{r}_+}{r_+} = \frac{r\hat{i} - \frac{d}{2}\hat{j}}{\sqrt{r^2 + (\frac{d}{2})^2}} \qquad \hat{r}_- = \frac{\vec{r}_-}{r_-} = \frac{r\hat{i} + \frac{d}{2}\hat{j}}{\sqrt{r^2 + (\frac{d}{2})^2}}$$

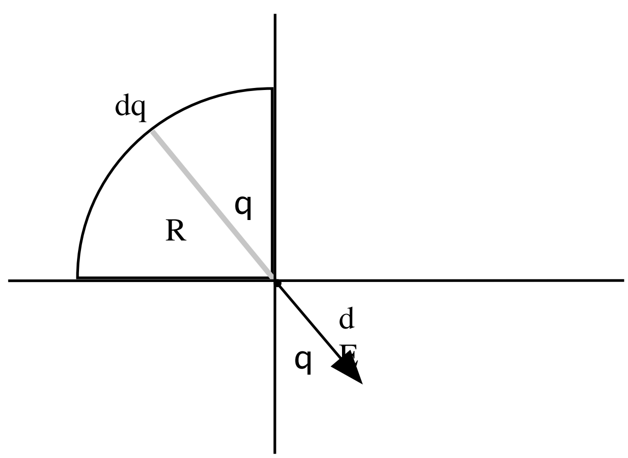
We can substitute in. In the last step, since d/2r is very small we ignore this term. Remember that the direction of p is in the +j direction in this problem (from - to + charge).

$$
\begin{aligned}
\vec{E} &= \vec{E}_+ + \vec{E}_- \\
&= \frac{q}{4\pi\varepsilon_0 r_+^2}\hat{r}_+ - \frac{q}{4\pi\varepsilon_0 r_-^2}\hat{r}_- \\
&= \frac{q}{4\pi\varepsilon_0 (r^2 + (\frac{d}{2})^2)} \cdot \frac{r\hat{i} - \frac{d}{2}\hat{j}}{\sqrt{r^2 + (\frac{d}{2})^2}} - \frac{q}{4\pi\varepsilon_0 (r^2 + (\frac{d}{2})^2)} \cdot \frac{r\hat{i} + \frac{d}{2}\hat{j}}{\sqrt{r^2 + (\frac{d}{2})^2}} \\
&= \frac{-qd}{4\pi\varepsilon_0 (r^2 + (\frac{d}{2})^2)^{3/2}}\hat{j} \\
&= \frac{-p}{4\pi\varepsilon_0 r^3 (1 + (\frac{d}{2r})^2)^{3/2}}\hat{j} \\
&\approx \frac{-\vec{p}}{4\pi\varepsilon_0 r^3}
\end{aligned}
$$

(- indicates downward)

22.24 In Fig 22-44, a thin glass rod forms a semicircle of radius $r = 5.00cm$. Charge is uniformly distributed along the rod, with $+q = 4.50\,pC$ in the upper half and $-q = -4.50\,pC$ in the l lower half. What are the (a) magnitude and (b) direction relative to the positive direction of the x axis of the electric field E a P, the center of the circle.

Because of the symmetry in the problem, we can see that the net field will point downward. We also can see that the contribution from the bottom quarter circle is equal to the contribution from the top quarter circle. Because of this, we only need to compute the downward component due to the top quarter circle and multiply by 2.



We begin by defining a charge per unit length

$$\lambda = \frac{q}{\pi R / 2}$$

We now find the component of interest and integrate...

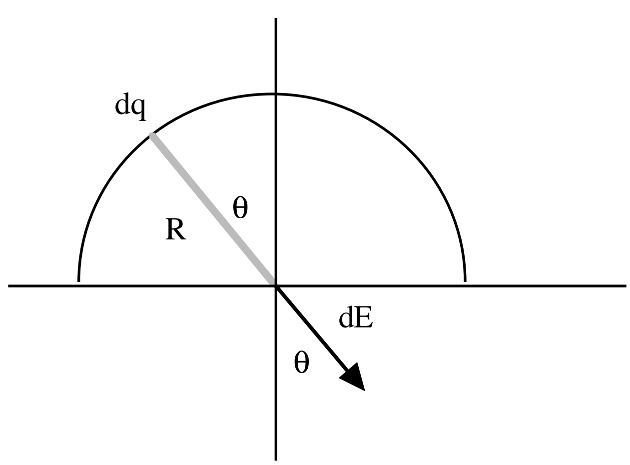
$$dE_y = -dE\cos\theta$$
$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$
$$r = R$$
$$dq = \lambda R\,d\theta$$

$$E_y = \int_0^{\pi/2} dE_y$$
$$E_y = \int_0^{\pi/2} \frac{\lambda R\,d\theta}{4\pi\varepsilon_0 R^2}\cdot\cos\theta$$
$$= \frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta\,d\theta$$
$$= \frac{\lambda}{4\pi\varepsilon_0 R}\cdot\frac{\pi}{2}$$
$$= \frac{\lambda}{8\varepsilon_0 R}$$
$$E_{y\,tot} = 2E_y = \frac{\lambda}{4\varepsilon_0 R}$$

The last factor of 2 is because there are two quarter rings.

22.25 In Fig 22-45, two curved plastic rods, one of charge +q and one of charge -q, for a circle of radius R in the XY plane.. Te x axis passes through their connection points and the charge is distributed uniformly on both rods. What are the magnitude and direction of the electric field E produces at P, the center of the circle.

Because of the symmetry in the problem, we can see that the net field will point downward. We also can see that the contribution from the bottom half circle is equal to the contribution from the top half circle. Because of this, we only need to compute the downward component due to the top half circle and multiply by 2.



We begin by defining a charge per unit length

$$\lambda = \frac{q}{\pi R}$$

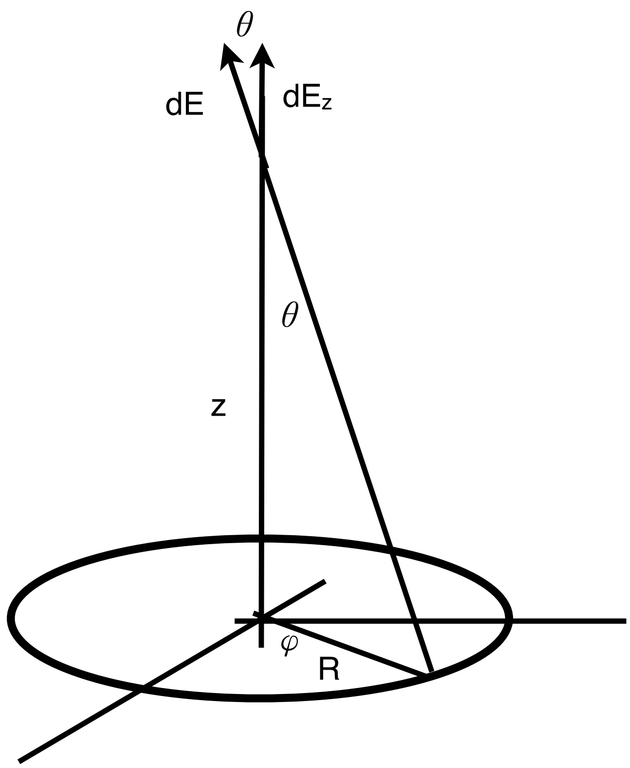
We now find the component of interest and integrate...

The last factor of 2 is because there are two half rings.

$$\begin{aligned} dE_y &= -dE\cos\theta \\ dE &= \frac{dq}{4\pi\varepsilon_0 r^2} \\ r &= R \\ dq &= \lambda R\,d\theta \end{aligned}$$

$$\begin{aligned} E_y &= \int_{-\pi/2}^{\pi/2} dE_y \\ E_y &= \int_{-\pi/2}^{\pi/2} -\frac{\lambda R\,d\theta}{4\pi\varepsilon_0 R^2}\cdot\cos\theta \\ &= -\frac{\lambda}{4\pi\varepsilon_0 R}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta \\ &= \frac{\lambda}{4\pi\varepsilon_0 R}\cdot 2 \\ &= \frac{\lambda}{2\pi\varepsilon_0 R} \\ E_{y\,tot} &= 2E_y = \frac{\lambda}{\pi\varepsilon_0 R} \end{aligned}$$

22.26



Because of the symmetry, we only need to compute the field along the axis (called z).

$$dE_z = dE\cos\theta$$

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$

$$r = \sqrt{R^2 + z^2}$$

$$dq = \lambda_1 R\, d\varphi$$

$$\cos\theta = \frac{z}{\sqrt{R^2 + z^2}}$$

$$E_z = \int_0^{2\pi} dE_z$$

$$E_z = \int_0^{2\pi} \frac{\lambda_1 R\, d\varphi}{4\pi\varepsilon_0 (R^2 + z^2)} \cdot \frac{z}{\sqrt{R^2 + z^2}}$$

$$= \int_0^{2\pi} \frac{\lambda_1 R z\, d\varphi}{4\pi\varepsilon_0 (R^2 + z^2)^{3/2}}$$

$$= \frac{\lambda_1 R z}{2\varepsilon_0 (R^2 + z^2)^{3/2}}$$

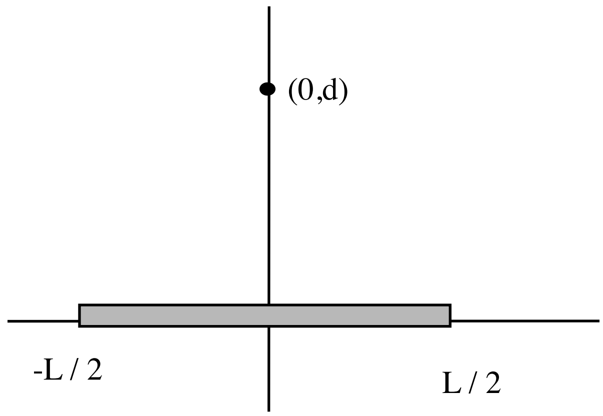
Now that we know the field, we take the derivative with respect to z and set it equal to zero to find the location of the maximum field.

$$E_z = \frac{\lambda_1 R z}{2\varepsilon_0 (R^2 + z^2)^{3/2}}$$

$$\frac{dE_z}{dz} = 0 = \frac{\lambda_1 R (R^2 - 2z^2)}{2\varepsilon_0 (R^2 + z^2)^{5/2}}$$

$$z = \frac{1}{\sqrt{2}} R$$

22-32. In Fig. 22-51, positive charge $q = 7.81pC$ is spread uniformly along a thin non-conducting rod of length $L = 14.5cm$. What are the (a) magnitude and (b) direction (relative to the x axis of the electric field produced at a distance $R = 6.00cm$ from the rod along its perpendicular bisector.



First we write dq for a little length of charge

$$dq = \lambda dx$$

We then write the $r, \vec{r}, \hat{r}$ for the charge dq.

$$r = \sqrt{(0-x)^2 + (d-0)^2}$$
$$\vec{r} = (0-x)\hat{i} + (d-0)\hat{j}$$
$$\hat{r} = \frac{\vec{r}}{r} = \frac{-x\hat{i} + d\hat{j}}{\sqrt{x^2 + d^2}}$$

We can then compute the x and y components of the Electric Field. You may need

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}}$$

$$\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r} = \int \frac{\lambda\, dx}{4\pi\varepsilon_0 (x^2+d^2)} \cdot \frac{-x\hat{i} + d\hat{j}}{\sqrt{x^2+d^2}}$$

$$E_x = 0$$

$$E_y = \int \frac{\lambda d\, dx}{4\pi\varepsilon_0 (x^2+d^2)^{3/2}} = \frac{\lambda d}{4\pi\varepsilon_0} \int \frac{dx}{(x^2+d^2)^{3/2}} = \frac{\lambda d}{4\pi\varepsilon_0} \cdot \left. \frac{x}{d^2\sqrt{d^2+x^2}} \right]_{-L/2}^{L/2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 d} \cdot \left[\frac{L/2}{\sqrt{d^2+L^2/4}} - \frac{-L/2}{\sqrt{d^2+L^2/4}} \right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0 d} \cdot \frac{L}{\sqrt{d^2+L^2/4}}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 d} \cdot \frac{L}{\sqrt{4d^2+L^2}}$$

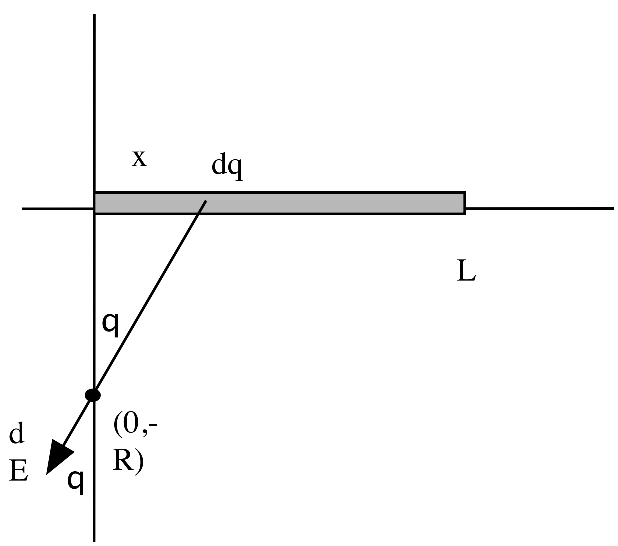
Now we can compute with numbers

$$\lambda = \frac{7.81 \times 10^{-12} C}{0.145 m} = 5.386 \times 10^{-11} C/m$$

$$d = 0.06 m$$

$$E_y == \frac{5.386 \times 10^{-11} C/m}{2\pi\varepsilon_0 \cdot 0.06 m} \cdot \frac{0.145 m}{\sqrt{4(0.06m)^2 + (0.145m)^2}} = 12.43 N/C$$

22.33 In Fig 22-52, a “semi infinite” non conducting rod (that is, infinite in one direction only), has uniform linear charge density λ. Show that the electric field $\vec{E}_p$ at point P makes an angle of 45 degrees with the rod and this result is independent of the the distance R.



$$r = \sqrt{(0-x)^2 + (-R-0)^2}$$
$$\vec{r} = (0-x)\hat{i} + (-R-0)\hat{j}$$
$$\hat{r} = \frac{\vec{r}}{r} = \frac{-x\hat{i} - R\hat{j}}{\sqrt{x^2+R^2}}$$

We can then compute the x and y components of the Electric Field. You may need

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

We can compute each integral and then take the limit as L goes to infinity. We find that in this limit, the components are identical for all R. This means that the ratio of these components is 1 and the angle is 45 degrees for all R.

$$\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r} = \int \frac{\lambda\, dx}{4\pi\varepsilon_0(x^2+R^2)} \cdot \frac{-x\hat{i} - R\hat{j}}{\sqrt{x^2+R^2}}$$

$$E_x = \int \frac{\lambda\, dx}{4\pi\varepsilon_0(x^2+R^2)} \cdot \frac{-x}{\sqrt{x^2+R^2}} = \int \frac{-x\lambda\, dx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}} = \frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{R^2+x^2}}\Bigg]_0^L$$
$$= \frac{\lambda}{4\pi\varepsilon_0} \cdot \left(\frac{1}{\sqrt{R^2+L^2}} - \frac{1}{R}\right)$$
$$\lim_{:L\to\infty} E_x = -\frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{1}{R}$$

$$E_y = \int \frac{-\lambda R\, dx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}} = -\frac{\lambda R}{4\pi\varepsilon_0}\int \frac{dx}{(x^2+R^2)^{3/2}} = -\frac{\lambda R}{4\pi\varepsilon_0} \cdot \frac{x}{R^2\sqrt{R^2+x^2}}\Bigg]_0^L$$
$$= -\frac{\lambda}{4\pi\varepsilon_0 R} \cdot \left[\frac{L}{\sqrt{R^2+L^2}} - 0\right]$$
$$= -\frac{\lambda}{4\pi\varepsilon_0 R} \cdot \frac{L}{\sqrt{R^2+L^2}}$$
$$\lim_{L\to\infty} E_y = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

22.34 This problem is worked out in detail in section 23-7 of the book. We also worked this problem in detail in class. Please review the derivation there.

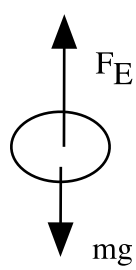
$$
\begin{aligned}
\sigma &= 5.3 \times 10^{-6} C/m^2 \\
R &= 2.5 \times 10^{-2} m \\
z &= 12 \times 10^{-2} m \\
E_z &= \frac{\sigma}{2\varepsilon_0} \cdot \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \\
&= 6.29 \times 10^3 N/C
\end{aligned}
$$

22.39 An electron is released from rest in a uniform electric field of magnitude $2.00 \times 10^4 N/C$. Calculate the acceleration of the electron (ignore gravitation).

$$F = ma = qE$$

$$a = \frac{q}{m}E = \frac{1.6 \times 10^{-19} C}{9.1 \times 10^{-31} kg} \cdot 2.00 \times 10^4 N/C = 3.5 \times 10^{15} m/s^2$$

22.47 In Millikan's experiment, an oil drop of radius $r = 1.64\,\mu m$ and density $\rho = 0.851\,g/cm^3$ is suspended in chamber C (Fig. 22-14) when a downward electric field $E = 1.92 \times 10^5 N/C$ is applied. Find the charge on the drop in terms of e



If the droplet is suspended, we know that the net force is zero. This means that the weight is balanced by an upward electric force. We write this balance

$$
\begin{aligned}
F_E &= mg \\
qE &= mg \\
q &= \frac{mg}{E}
\end{aligned}
$$

We can write the mass in terms of the volume of the drop.

$$m = \rho \cdot \frac{4}{3}\pi r^3 = 0.851\frac{g}{cm^3} \cdot \frac{4}{3}\pi (1.64 \times 10^{-4}\, cm)^3$$
$$= 1.57 \times 10^{-11}\, g$$
$$= 1.57 \times 10^{-14}\, kg$$
$$q = \frac{mg}{E} = \frac{1.57 \times 10^{-14}\, kg \cdot 9.8 m/s^2}{1.92 \times 10^5\, N/C} = 8.03 \times 10^{-19}\, C$$
$$= 5e$$